

MATHEMATICAL MODEL OF HYDROGEN-POWERED BICYCLE ELECTRIFICATION

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Abstract

Mathematical modeling plays an essential role in investigating the behavior, design, and analysis of power systems. This article proposes a time-invariant mathematical model of the power system of a hydrogen-powered electric bicycle. The proposed mathematical model is derived using the generalized state-space averaging method. The proposed model is validated through computer-based simulation using MATLAB/SIMULINK. The system responses obtained using the proposed model agree well with the simulation results under both transient and steady-state conditions. Furthermore, the proposed mathematical model significantly reduces the computation time compared with direct simulation. Thus, the proposed model is both accurate and efficient. The proposed averaging model is also suitable for artificial intelligence applications requiring iterative searching due to its fast computational time.

Keywords: Bidirectional DC/DC Converter; Cascade PI Controller; Constant Power Load; Electric Bike; Generalized State-Space Averaging Method; Hydrogen Fuel cell; Mathematical Model

Introduction

The use of fossil fuels has led to numerous environmental issues, particularly global warming, which has become a critical concern worldwide. Consequently, both current and future trends indicate an increasing demand for clean energy. Hydrogen-based electricity is a promising clean energy source because hydrogen provides a higher energy density than other energy sources. In addition, the electricity generation process using hydrogen fuel cells is an electrochemical reaction that delivers high efficiency, produces only water as a byproduct, and emits no carbon dioxide

(Bai *et al.*, 2022; Sharma and Garg, 2023; Hamedani *et al.*, 2024; Putala and Ferencey, 2025). The application of hydrogen energy in vehicle propulsion systems presents a potential solution for reducing dependency on fossil fuels in the transportation sector. Hydrogen-powered electric bicycles represent an early stage in the development of hydrogen-powered electric vehicles. The primary components responsible for power generation, distribution, and control in such systems are the hydrogen fuel cell and the power conversion circuit. The hydrogen fuel cell generates electrical

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energy from hydrogen, and the power converter ensures that the power supplied to the system load is appropriate (Touahria *et al.*, 2024; Mtumka and Krishnamurthy, 2025). Therefore, the design, behavioral study, and analysis of this type of power system are essential aspects in advancing hydrogen-powered electric vehicles. However, such studies rely on accurate mathematical models of the system. In practical applications, the power system exhibits time-varying behavior because of the switching operations of the bidirectional DC/DC converter, which introduces considerable complexity to the analysis (Emadi, 2004; Azer and Emadi, 2020; Chaicharoenadomrung *et al.*, 2022). To address this issue, this article proposes a time-invariant mathematical model for the power system of a hydrogen-powered electric bicycle. The proposed model can subsequently be used for system analysis and design purposes. The derivation of the mathematical model employs the generalized state-space averaging (GSSA) method to eliminate the switching effects of the bidirectional DC/DC converter with cascaded proportional-integral (PI) control used in the powertrain system. The hydrogen fuel cell is modeled using a polynomial equation derived by curve fitting, based on manufacturer-provided characteristic data. The drivetrain, which consists of a brushless DC motor and its associated controller, is assumed to behave as an ideal constant power load (CPL) (Emadi *et al.*, 2006; Cespedes *et al.*, 2011; Hossain *et al.*, 2018). The proposed mathematical model is validated by computer-based simulations using the SimPowerSystem® toolbox in MATLAB/SIMULINK. The system responses obtained from the proposed time-invariant model agree well with the simulation results under both transient and steady-state conditions. In addition, the proposed model significantly reduces computation time by up to 99.98% compared with the direct simulation of the MATLAB/SIMULINK exact topology model. In conclusion, the proposed

mathematical model provides high accuracy and computational efficiency.

This paper is organized as follows: Section II describes the configuration of the hydrogen-powered electric bicycle under consideration. Section III presents the derivation of the mathematical model. Section IV discusses the model validation through computer simulations.

Finally, Section V concludes the study and suggests directions for future research.

Considered Hydrogen-Powered Bicycle Electrification

The power system of the hydrogen-powered electric bicycle considered in this study is illustrated in Figure 1. The system comprises three primary components. The first component is the hydrogen fuel cell, which is used to generate electrical energy from hydrogen.

In this study, a commercially available 500 W hydrogen fuel cell (model H-500, Horizon Fuel Cell Technologies) is selected as the power source. The second component comprises a bidirectional half-bridge DC/DC converter, which is regulated by a cascaded PI controller. The converter regulates power delivery and maintains a constant output voltage of 48 V, which is essential for the brushless DC motor used in the propulsion system. The drivetrain, which consists of a brushless DC motor, is the third component. In this study, the motor with its speed control is assumed to behave as an ideal CPL (Emadi *et al.*, 2006; Cespedes *et al.*, 2011; Hossain *et al.*, 2018). Typically, an electric bicycle operates in the driving mode. Therefore, the operation of the DC/DC converter is analyzed under this condition. The detailed derivation of the proposed mathematical model for the power system is presented in the following section.

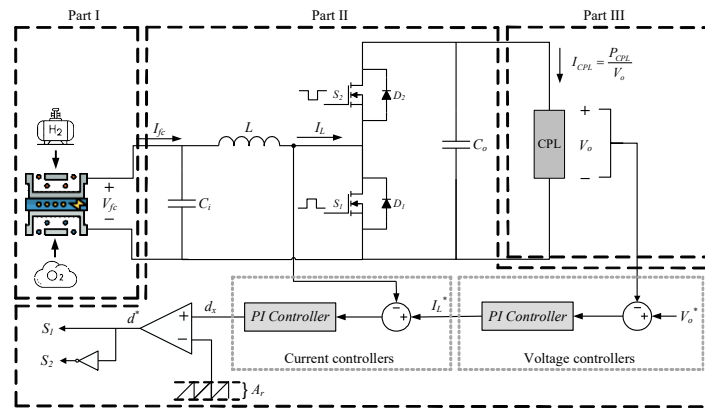


Figure 1. The considered hydrogen-powered bicycle electrification

Mathematical Model Derivation

The derivation of the proposed mathematical model of the power system under consideration is divided into two parts. The first part focuses on the modeling of the hydrogen fuel cell. The output voltage of the fuel cell is not constant and varies depending on the load current; thus, this study uses an appropriate simplified model. The fuel cell behavior is represented by a polynomial equation obtained by curve fitting, which is based on the characteristic data provided by the manufacturer. The second part involves the derivation of the mathematical model for the bidirectional DC/DC converter. The GSSA method eliminates the switching behavior and derives a time-invariant representation of the converter. The detailed modeling procedure for both components is presented in following section.

A. Simplified Hydrogen Fuel Cell Model

Based on the characteristic data of current and voltage for the Horizon Fuel Cell Technologies H-500 model (rated at 500 W) (Horizon Educational, 2024), the following key parameters are obtained:

- Voltage at 0 A and 1 A $[E_{oc}, V_I] = [23.5, 21]$
- Nominal operating point $[I_{nom} \text{ (A)}, V_{nom} \text{ (V)}] = [20, 16.2]$
- Maximum operating point $[I_{end} \text{ (A)}, V_{end} \text{ (V)}] = [40, 13.4]$

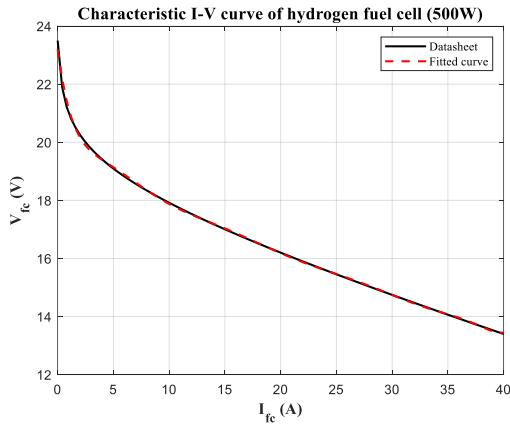


Figure 2. I-V characteristic of the considered hydrogen fuel cell

These data are applied to the Fuel Cell Stack block in SimPowerSystem®, a toolbox of MATLAB/SIMULINK, to construct the current-voltage (I-V) characteristic curve, which is represented by the black line in Figure 2. Subsequently, a polynomial curve fitting method is used to derive an appropriate mathematical representation of the I-V relationship. Consequently, a 10th-order polynomial equation (1) is obtained, where I_{fc} denotes the current of the fuel cell stack. The corresponding fitted curve (the red line in Figure 2) shows strong agreement with the manufacturer-provided characteristic curve. Therefore, (1) is considered the simplified mathematical model of the hydrogen fuel cell used in this study. In addition, the 10th-order polynomial representing the fuel cell stack model is the minimum order required to achieve satisfactory accuracy. This order ensures accurate dynamic responses compared with the exact topological model.

B. Half-Bridge Bidirectional DC/DC Converter Model

The mathematical model of the bidirectional half-bridge DC/DC converter (part II of Figure 1) begins with the analysis of the circuit without the cascaded PI controller. In this modeling process, two state variables are defined: the inductor current (I_L) and the output capacitor voltage (V_o). The converter operates in boost mode because the operation of the converter is limited to the driving mode, as discussed in Section II. Under this condition, the two power switches S_1 and S_2 alternate between conducting (ON) and non-conducting (OFF) states, which can be categorized into two operating modes:

Mode 1: Switch S_1 is conducting (ON), whereas switch S_2 is non-conducting (OFF), as shown in Figure 3.

Mode 2: Switch S_1 is non-conducting (OFF), whereas switch S_2 is conducting (ON), as shown in Figure 4.

The mathematical model for each mode is detailed in the following section. As shown in Figure 3, when switch S_1 is conducting (ON) and switch S_2 is non-conducting (OFF), the hydrogen

$$V_{fc} = 3.10351 \cdot 10^{-12} I_{fc}^{10} - 6.5893 \cdot 10^{-10} I_{fc}^9 + 6.0226 \cdot 10^{-8} I_{fc}^8 - 3.09986 \cdot 10^{-6} I_{fc}^7 + 9.86889 \cdot 10^{-5} I_{fc}^6 - 20.10212 \cdot 10^{-4} I_{fc}^5 + 26.20722 \cdot 10^{-3} I_{fc}^4 - 21.30091 \cdot 10^{-2} I_{fc}^3 + 10.25093 \cdot 10^{-1} I_{fc}^2 - 28.95844 \cdot 10^{-1} I_{fc} + 23.19806 \quad (1)$$

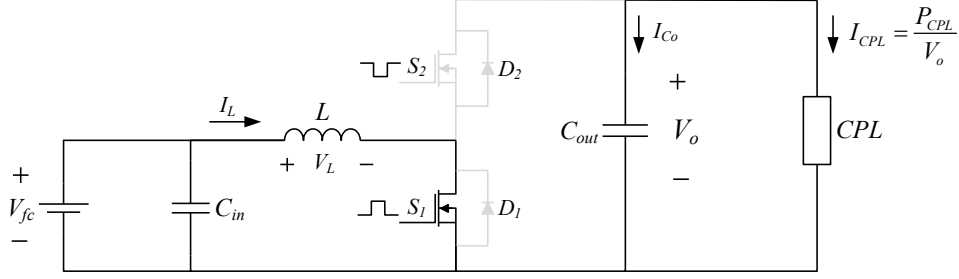


Figure 3. Bidirectional half-bridge DC/DC converter operates in mode 1

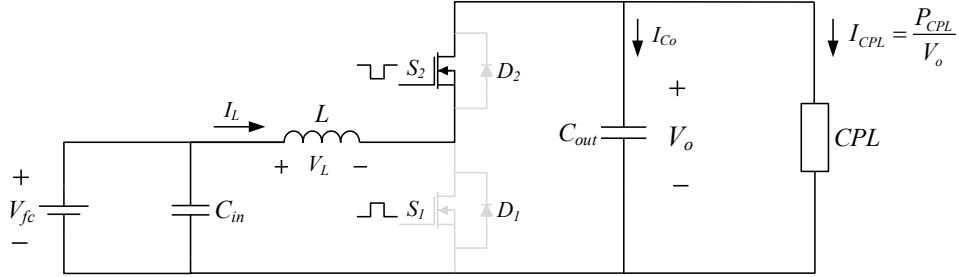


Figure 4. Bidirectional half-bridge DC/DC converter operates in mode 2

fuel cell supplies electrical energy to the inductor L , which stores the energy in the form of a magnetic field. The charge stored in the capacitor C_{out} is discharged to supply energy to the load, which is considered an ideal CPL . Accordingly, the circuit analysis of the configuration shown in Figure 3 yields the following differential equations that describe the system behavior in Model 1, as expressed in (2) and (3).

$$\frac{di_L}{dt} = \frac{V_{fc}}{L} \quad (2)$$

$$\frac{dv_o}{dt} = \frac{-P_{CPL}}{V_o C_o} \quad (3)$$

As shown in Figure 4, when switch S_1 is non-conducting (OFF) and switch S_2 is conducting (ON), both the hydrogen fuel cell and the energy previously stored in the inductor L supply electrical energy to charge the capacitor C_{out} to deliver power to the CPL . Based on the circuit analysis of the configuration shown in Figure 4, the differential equations that describe the system behavior in Mode 2 can be derived and are expressed as (4) and (5).

$$\frac{di_L}{dt} = \frac{V_{fc} - V_o}{L} \quad (4)$$

$$\frac{dv_o}{dt} = \frac{I_L}{C_o} - \frac{P_{CPL}}{V_o C_o} \quad (5)$$

The switching function of the bidirectional DC/DC converter is then considered, which describes the relationship expressed in (6) as follows:

$$u(t) = \begin{cases} 1 & ; 0 < t < dT_s \\ 0 & ; dT_s < t < T_s \end{cases} \quad (6)$$

The switched model of the considered DC/DC converter can be obtained by combining the differential equations of both operating modes, utilizing a validation function (h_i) that depends on the operational mode of the circuit. In this case, two functions are involved: h_1 and h_2 . For Mode 1, when switch S_1 is conducting and switch S_2 is non-conducting, the values of h_1 and h_2 are 1 and 0, respectively. For Mode 2, when switch S_1 is non-conducting and switch S_2 is conducting, the values of h_1 and h_2 are 0 and 1, respectively. Based on these relationships, the system can be expressed in a pair of linear equations $h_1 = u(t)$ and $h_2 = 1 - u(t)$. Therefore, the switched model of the converter is described by (7).

$$\begin{cases} \dot{i}_L = \frac{1}{L} \cdot V_{fc} - \frac{1}{L} (1 - u(t)) \cdot v_o \\ \dot{v}_o = \frac{1}{C_o} (1 - u(t)) \cdot i_L - \frac{P_{CPL}}{v_o C_o} \end{cases} \quad (7)$$

Based on (7), the obtained switched model is time-dependent due to the behavior of the switching devices in the circuit, as indicated by the switching function $u(t)$. Therefore, this study employs the GSSA method to eliminate the effects of the switching action, resulting in a time-invariant model. The GSSA method uses the complex Fourier coefficients of the state variables of the circuit as the state variables for the model. The process is briefly described as follows:

In general, $f(t)$, any periodic signal with a period T can be expressed in the form of a complex Fourier series, as expressed in (8).

$$f(t) = \sum_{k=-\infty}^{\infty} \langle x \rangle_k(t) e^{jk\omega_s t} \quad (8)$$

where $\omega_s = \frac{2\pi}{T_s} = 2\pi f_s$ and $\langle x \rangle_k(t)$ is complex Fourier coefficients.

The GSSA method uses the Fourier series coefficients $\langle x \rangle_k(t)$ of the signal to represent the state variables of the system. The complex Fourier coefficients in (8) can be obtained using (9) as follows:

$$\langle x \rangle_k(t) = \frac{1}{T_s} \int_{t-T_s}^t f(t) e^{-jk\omega_s t} dt \quad (9)$$

The following properties of the complex Fourier coefficient $\langle x \rangle_k(t)$ are required for deriving the mathematical model:

Property 1: The property of the time rate of change can be expressed as shown in (10).

$$\frac{d}{dt} \langle x \rangle_k = \left\langle \frac{dx}{dt} \right\rangle_k - jk\omega_s \langle x \rangle_k \quad (10)$$

Property 2: The property of the multiplication relationship can be expressed as shown in (11).

$$\langle xy \rangle_k = \sum_i \langle x \rangle_{k-i} \langle y \rangle_i \quad (11)$$

Property 3: If $f(t)$ is the real value, the complex Fourier coefficient with a negative value of k can be expressed as shown in (12).

$$\langle x \rangle_{-k} = \overline{\langle x \rangle_k} = \langle x \rangle_k^* \quad (12)$$

Based on (8) and (9), the value of k depends on the accuracy of the model. In theory, if k approaches infinity, the model error will approach zero. However, assuming that the filter of the proposed system is well-designed and there are no signal ripples, k can be set to 0, a method known as zero-order approximation. If the signal oscillates with the fundamental frequency and ignores harmonics, k can be set to -1, 0, or 1, known as a first-order

approximation (Emadi, 2004; Ngamkong *et al.*, 2012; Azer and Emadi, 2020). In this study, the mathematical model is derived using a zero-order approximation, which is sufficient for system analysis and design. Therefore, based on (8), when considering the zero-order approximation, the state variables of the circuit can be expanded into the terms of the complex Fourier series, as shown in (13).

$$\begin{cases} I_L = \langle I_L \rangle_0 e^{j0\omega_s t} = \langle I_L \rangle_0 \\ V_o = \langle V_o \rangle_0 e^{j0\omega_s t} = \langle V_o \rangle_0 \end{cases} \quad (13)$$

Based on (7), by applying Property 1 of the complex Fourier coefficients and considering the zero-order approximation ($k = 0$), it can be expressed as shown in (14).

$$\begin{cases} \dot{\langle I_L \rangle}_0 = \frac{1}{L} \cdot V_{fc} - \frac{1}{L} \cdot \langle V_o \rangle_0 + \frac{1}{L} \cdot \langle V_o \rangle_0 \langle u(t) \rangle_0 \\ \dot{\langle V_o \rangle}_0 = \frac{1}{C_o} \cdot \langle I_L \rangle_0 - \frac{1}{C_o} \cdot \langle I_L \rangle_0 \langle u(t) \rangle_0 - \frac{P_{CPL}}{\langle V_o \rangle_0 C_o} \end{cases} \quad (14)$$

Based on (9), the complex Fourier coefficients can be obtained by considering the zero-order approximation ($k = 0$) of the switching function $u(t)$, as shown in (15).

$$\langle u(t) \rangle_0 = \frac{1}{T_s} \int_0^{dT_s} 1 e^0 dt = d \quad (15)$$

where d is the duty cycle of the bidirectional DC/DC power converter.

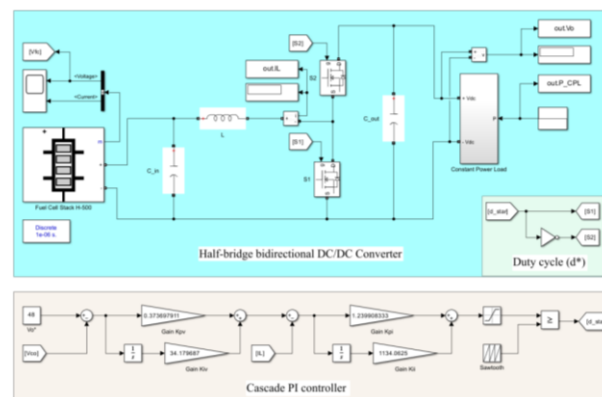
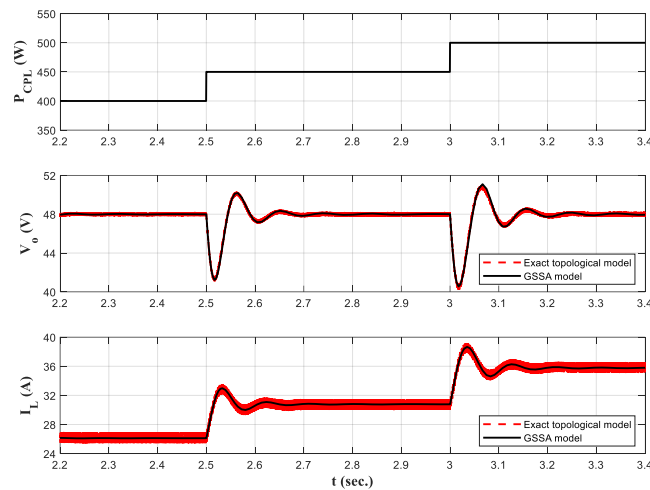
Therefore, based on (13) and (15), (14) can be rewritten in terms of the state variables of the model, as shown in (16). Equation (16) represents the time-invariant mathematical model of the bidirectional DC/DC power converter in the case without a cascade PI controller.

$$\begin{cases} \dot{I}_L = \frac{1}{L} \cdot V_{fc} - \frac{(1-d)}{L} \cdot V_o \\ \dot{V}_o = \frac{(1-d)}{C_o} \cdot I_L - \frac{P_{CPL}}{V_o C_o} \end{cases} \quad (16)$$

Consider the controller; the cascade PI controller for the system shown in Figure 5 is analyzed. The controller comprises two control loops. The first loop (the inner loop) regulates the current flowing through the inductor, and the second loop (the outer loop) controls the voltage across the output capacitor. This controller is responsible for managing the power supply and maintaining a stable output voltage of 48 V, which is appropriate for a brushless DC motor in the drive system, serving as the load of the electrical system under consideration. The details of the controller design can be found in (Surakan *et al.*, 2025).

Table 1. The parameters of the system

Parameter	Values	Descriptions
V_{fc}	14.4 V	Fuel cell stack voltage of the rated power 500 W
V_o	48 V	Output voltage
L ($\Delta I_L = 0.69$ A)	0.7258 mH	The inductance of the bidirectional half-bridge DC/DC converter
C_i ($\Delta V_i = 2.48$ V)	1.5 μ F	The input capacitance of the bidirectional half-bridge DC/DC converter
C_o ($\Delta V_o = 0.24$ V)	1519.097 μ F	The output capacitance of the bidirectional half-bridge DC/DC converter
A_r	10	Amplitude of the carrier signal (sawtooth) for PWM
K_{pv}	0.373697911	Proportional gain of the voltage loop
K_{pi}	1.239908333	Proportional gain of the current loop
K_{iv}	34.179687	Integral gain of the voltage loop
K_{ii}	1134.0625	Integral gain of the current loop

**Figure 6. The SimPowerSystem® block for the considered hydroge-powered electric bicycle****Figure 7. The responses of I_L and V_o when the P_{CPL} is changed**

Conclusions

This study presents the derivation of a time-invariant mathematical model of the power system of a hydrogen-powered electric bicycle. The model of the hydrogen fuel cell was derived by polynomial equation fitting using characteristic data provided by the manufacturer. The model of the bidirectional DC/DC converter circuit was derived using the

GSSA method. The results of the validation of the model, which involves comparing the responses calculated from the derived model with those obtained from a computer simulation using MATLAB/SIMULINK, demonstrate that the proposed model yields system responses consistent with the simulation results under both transient and steady-state conditions. In addition, the use of the mathematical model significantly reduces the

computation time compared to the simulation. Therefore, the derived model is accurate and suitable for use in related studies. In future studies, the proposed mathematical model will be used to analyze the stability and predict instability points of the system. This is important because it contributes to the research of modern electric vehicle systems, specifically in terms of stability, an area that has not been extensively studied or published.

Acknowledgement

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