

CROSS STYLE IMAGE SYNTHESIS THROUGH SEMI COUPLED DICTIONARY LEARNING AND GENERALIZED CROSS VALIDATION ALGORITHM

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Abstract

In various Computer vision applications for better visualization, interpretation and better recognition, we frequently want to change an image in one style into another one which is a large research area of compressive sensing. Algorithms on compressed sensing are in demand to create better reconstruction of the images while taking less computational time and requiring less storage capacity. In the present work attempts are put in the same direction and brought an algorithm which includes semi coupled dictionary learning (SCDL) model of compressed sensing with Singular Value Decomposition (SVD) based Generalized Cross Validation (GCV) algorithm. Using this algorithm one can find the different value of regularization parameters for different sizes of images and that helps to get better reconstruction of the images with ease while doing experiments on cross style image synthesis problems.

Keywords: Compressed sensing, Cross style image processing, Dictionary learning, Semi Coupled Dictionary Learning, Generalized Cross Validation

Introduction

In most of the applications of image processing we are receiving the images of the same scene from different origins and on that basis we require to change those images from one domain to another. For example in applications of the pattern recognition we want to recognize particular patterns of the images having same scene. To solve this purpose we need to construct a low resolution image (LR image) into a high resolution image (HR Image), by up-converting the LR image captured by one device into a HR image for better interpretation and visualization, which is known as process of

cross style image synthesis. In cross style image synthesis there are two main operations learning and reconstruction. Based on reconstruction operation many algorithms like Cubic- Splines interpolation (Hou and Andrews, 1978) Projections onto Convex sets (Stark and Oskoui, 1989), Iterative back projection (Irani and Peleg, 1991), Regularization algorithm (Peleg, 1919), etc. are studied. Not only these, as a part of cross-style image synthesis problem in applications of image processing and computer vision, methods are designed like image Super Resolution (Li and Orchard, 2001, Nhat *et al.*,

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2001, Zhang and Wu 2008), Artistic Rendering (Mallat and Yu 2010), (Hertzmann *et al.*, 2001, Efros and Freeman, 2011), Photo- Sketch Synthesis (Lin and Tang, 2005), and Multi Modal Biometrics (Wang and Tang, 2008, Lei and Li 2009). Many different algorithms are also examined to solve any given problems by using Patch-based Matching, Coupled Sunspace Learning, and Coupled Dictionary Learning (Sharma and Jacobs, 2011) techniques. But to find the complex transformation between the different styles and in the process of conversion to reconstruct the local structures related to different styles all these methods have limitations.

So to fulfill the need in the present work, a well-defined model with advanced algorithm are proposed which gives better calculations for the reconstruction of the images of cross style image synthesis problems properly by overcoming the earlier limitations.

Materials and Methods

In the proposed work to solve the application of compressed sensing (Yang *et al.*, 2010) like cross style image synthesis problems for one image, more specific model that is Semi Coupled Dictionary Learning (SCDL) model with Singular Value Decomposition (SVD) based Generalized Cross Validation (GCV) algorithm is used. By a SCDL model, a pair of dictionaries and a transformation (mapping function) will be learned at the same time of calculations. The pair of dictionaries designs to identify the two structural domains of the two different styles and at the same time, the transformation is to show the relationship between the two styles for synthesis. In SCDL, the two dictionaries will not be fully coupled, and therefore more flexibility is given to the transformation for an appropriate result (conversion). The Sparse Coding Super Resolution (ScSR) algorithm (Belgaonkar, 2022) is available to learn the coupled dictionaries of HR and LR patches. Then the patch of LR image over the LR dictionary is coded and one can reconstruct the patch of HR image with HR dictionary with the assumed sparse representation

coefficients. So by this strong assumption, the quality of reconstruction is affected and therefore, in the SCDL algorithm (Yang *et al.*, 2008, Wang *et al.*, 2012) a transformation between the sparse representation coefficients over the pair of dictionaries is taken.

Theory/Calculation

Once the dictionary pair and mapping are learned, SCDL based cross style image synthesis process can be applied, as given in Figure (1) below:

In general each of this stage is described step by step below:

Stage 1: Sparse Coding

Stage one is an Input stage, in which we have to input a LR image to get the HR image of it as reconstruction. In this stage the given input image is converted into a data form as coefficient matrix A_x with the help of dictionary D_x as shown in Figure 2, where the coefficient matrix A_x is in the form of sparse matrix.

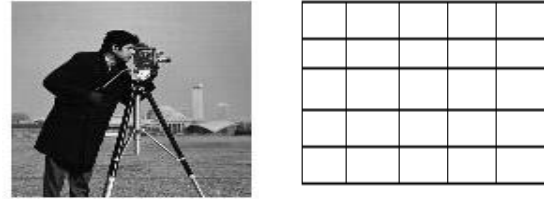


Figure 2. Input Image in style S_x $\rightarrow$ Sparse Coding $\rightarrow$ Coefficient Sparse Matrix A_x by D_x

Stage 2: Sparse Domain Transform

Stage two is as Mapping stage in which we have the converted sparse coefficient matrix A_x in one domain X and we have to convert this matrix into the another sparse coefficient matrix A_y in another domain Y with the help of pre learned mapping W as shown in Figure 3.

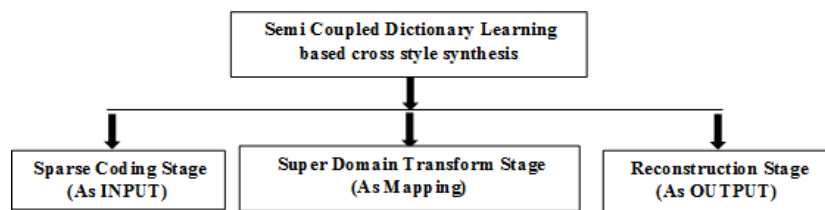


Figure 1. Flowchart of the image cross style synthesis based on SCDL

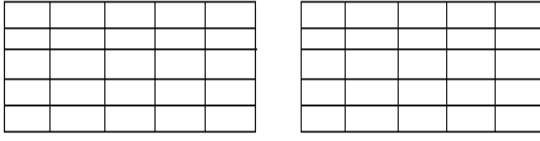


Figure 3. Coefficient Sparse Matrix $A_x \rightarrow$ Pre learned Mapping $W \rightarrow$ Coefficient Sparse Matrix A_y

Stage 3: Reconstruction

Stage three is as output stage, in which we have the converted sparse coefficient matrix A_y with the help of mapping W . Now we have to convert this A_y sparse matrix in to the image form of style S_y in another domain as our reconstruction image with the help of dictionary D_y as shown in Figure 4.

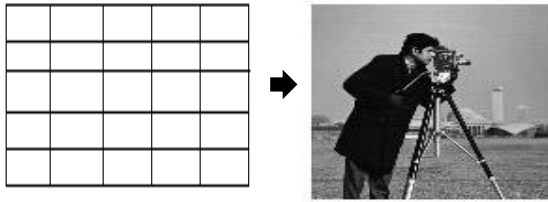


Figure 4. Coefficient Sparse Matrix A_y by $D_y \rightarrow$ Synthesis $\rightarrow$ Reconstructed Image in style S_y

Now let us be familiar with some particular terms which are going to use in the problem:

Given an image y_l in the style S_l ,

Recover the image y_h in the style S_h .

Space of all the images in style S_l is Y_l ,

Space of all images in style S_h is Y_h .

D_h and D_l are dictionaries for HR and LR image patches respectively.

A_h and A_l are sparse coefficients respectively.

$y_h = [y_{h1}, y_{h2}, \dots, y_{hn}]$ is patch matrix of HR images, $y_l = [y_{l1}, y_{l2}, \dots, y_{ln}]$ is patch matrix of LR images. $\{y_{hi}, y_{li}\}$ is the corresponding image patch pair.

There exists a transformation $f(\cdot)$ from Y_l to y_h : $y_h = f(y_l)$ and if the inverse transformation of $f(\cdot)$ is exist and known, then it's simple to transform between Y_l and Y_h . But mostly this type of mapping is invertible and not easy to learn directly (Sarvaiya, 2016).

Accordingly, the linear transformation in SCDL algorithm where the sparse coefficients are not equal can be presented as:

$$A_h = f(\cdot)A_l = WA_l \quad (1)$$

To find the required Semi Coupled Dictionaries and mappings, we design to minimize the energy function:

$$\min_{\{D_l, D_h, f(\cdot)\}} E_{data}(D_l, y_l) + E_{data}(D_h, y_h) + \gamma E_{map}(f(A_l), A_h) + \lambda E_{reg}(A_l, A_h, f(\cdot), D_l, D_h) \quad (2)$$

Where, $E_{data}(\cdot, \cdot)$ is the data accuracy term which shows the error in data, $E_{map}(\cdot, \cdot)$ is the mapping accuracy term which shows the mapping error between the sparse coding coefficients of two styles, E_{reg} is the regularization term to regularize the coding coefficients and transformation.

From the equation (1) we can say that the sparse coding coefficients of y_l and y_h over D_l and D_h will be related by a transformation $f(\cdot)$ so, D_l and D_h dictionaries and the $f(\cdot)$ transformation will be optimized jointly.

From the equations (1) and (2) the framework of SCDL can be converted into the dictionary learning and ridge regression problem as:

$$\min_{\{D_l, D_h, f(\cdot)\}} \|y_l - D_l A_l\|_F^2 + \|y_h - D_h A_h\|_F^2 + \gamma \|A_h - f(\cdot)A_l\|_F^2 + \lambda_l \|A_l\|_1 + \lambda_h \|A_h\|_1 + \lambda_{f(\cdot)} \|f(\cdot)\|_F^2$$

such that $\|d_{h,i}\|_{l_2} \leq 1, \|d_{l,i}\|_{l_2} \leq 1,$

Let, $f(\cdot) = W$ in the above equation we get,

$$\min_{\{D_l, D_h, W\}} \|y_l - D_l A_l\|_F^2 + \|y_h - D_h A_h\|_F^2 + \gamma \|A_h - WA_l\|_F^2 + \lambda_l \|A_l\|_1 + \lambda_h \|A_h\|_1 + \lambda_W \|W\|_F^2$$

such that

$$\|d_{h,i}\|_{l_2} \leq 1, \|d_{l,i}\|_{l_2} \leq 1 \quad (3)$$

To adjust the different terms in the objective function given in equation (3) some regularization parameters are included in it like $\gamma, \lambda_h, \lambda_l, \lambda_W$ and the atoms are $d_{h,i}, d_{l,i}$ of the dictionaries D_h, D_l respectively.

The objective function in equation (3) is not jointly convex to D_h, D_l, W . If other terms are fixed then it is convex with respect to each of them. To optimize the variables one by one alternatively, we can propose an iterative algorithm. The transformation W is defined as an identity matrix initially. And the coding coefficients A_l and A_h are supposed to be equal. Here we found that the SCDL model is performed using two main algorithms, one is LARS algorithm (Kowsalya, 2021) to solve l_1 norm optimization problem, and second one is

Metaphase learning (MFL) algorithm (Hastie *et al.*, 2004) to learn the dictionary. But for complex and large data set, this model has limitation to reproduce the images in inverse mapping and it spends more time and limits to the training accuracy.

Therefore, where the transformation between different styles are complex, nonlinear, and vary from the space, one need to find the relation between the styles. It is not enough to use a single mapping to solve this purpose. So to improve the stability and hardness (robustness) of SCDL, here for one image we are using a new model selection i.e., clustering method and we club it with SCDL model. This kind of model can separate the data into different clusters (bunch of similar things) and in this process the calculation can be learned in the each cluster one by one. Hence one can learn a stable linear transformation between the different two styles.

So, in comparison of previous SCDL model with new SCDL + clustering model approach, we find that new model is allowing the mapping errors between the coding coefficients and thus it relaxes the connection of dictionaries. In previous method, we do apply clustering in the signal domain, but as in we do clustering in the style specific sparse domains, it enhances the capability of the style conversion.

Here we first initialize dictionary pair D_h, D_l and mapping W . Initially mapping W can be as the identity matrix. To initialize D_h, D_l , we have many ways to adopt like random matrix, PCA basis (principal component analysis basis), DCT basis (Discrete cosine transform basis), etc. Using l_1 minimization the sparse coefficients A_l and A_h can then be obtained and by assuming that W is linear and bidirectional transform learning strategy [22], we can learn the transform from A_l to A_h and vice versa.

Now to face the minimization of energy function in equation (3), we separate the problem into three sub-problems as: (1) sparse coding for training data, (2) dictionary updating and (3) mapping updating.

- 1) First we initialize of mapping W and dictionary pair D_h, D_l , we can update the A_l and A_h sparse coding coefficients as,

$$\begin{aligned} & \min_{\{A_l\}} \|y_l - D_l A_l\|_F^2 + \gamma \|A_h - W A_l\|_F^2 + \lambda_l \|A_l\|_1 \\ & \& \min_{\{A_h\}} \|y_h - D_h A_h\|_F^2 + \gamma \|A_l - W A_h\|_F^2 + \lambda_h \|A_h\|_1 \end{aligned} \quad (4)$$

Equation (4) is a multi-task Lasso problem. To solve l_1 optimization here we used LARS for its stability and efficiency.

- 2) Fix A_l and A_h , dictionary pair D_h, D_l can be updated as,

$$\min_{\{D_l, D_h\}} \|y_l - D_l A_l\|_F^2 + \|y_h - D_h A_h\|_F^2$$

Such that

$$\forall i, \|d_{l,i}\|_{l_2} \leq 1, \|d_{h,i}\|_{l_2} \leq 1 \quad (5)$$

Equation (5) is constrained quadratic programming problem and to solve this we choose a one by one updated strategy.

- 3) Fix dictionary pair D_h, D_l and coding coefficients A_l and A_h , we can update the transformation W as,

$$\min_{\{W\}} \|A_h - W A_l\|_F^2 + \frac{\lambda_W}{\gamma} \|W\|_F^2 \quad (6)$$

Equation (6) is a ridge regression problem and solved analytically by,

$$W = A_h A_l^T \left(A_l A_l^T + \left(\frac{\lambda_W}{\gamma} \right) I \right)^{-1} \quad (7)$$

where, I is an identity matrix.

After completion of the calculations of the dictionaries D_l, D_h and the linear mapping W , for a change in given input image y_l in style s_l to the image y_h of style s_h we solve the equation,

$$\begin{aligned} & \min_{\{\alpha_{l,i}, \alpha_{h,i}\}} \|y_{l,i} - D_l \alpha_{l,i}\|_F^2 + \|y_{h,i} - D_h \alpha_{h,i}\|_F^2 + \\ & \gamma \|\alpha_{h,i} - W \alpha_{l,i}\|_F^2 + \lambda_l \|\alpha_{l,i}\|_1 + \lambda_h \|\alpha_{h,i}\|_1 \end{aligned} \quad (8)$$

where, y_{li} and y_{hi} are the patches of the y_l and y_h respectively. Equation (8) can be solved by alternately updating $\alpha_{l,i}$ and $\alpha_{h,i}$.

So each pair of y_h can be reproduced as,

$$y_{hi} = D_h \alpha_{h,i} \quad (9)$$

when all the patches are estimated one by one then the estimation of the required image y_h can be obtained.

In this process, an initial estimation of y_h is needed. But for different problem there are different ideas to initialize y_h , for example in image super resolution problem, y_h can be found initially by Bicubic interpolation, but the regularized parameters and the clusters are most affected values

for this SCDL model to give accurate reconstruction of super resolution image.

Approche to find values of Regularization Parameters

There is a crucial problem to select the regularization parameter λ from the data in method of regularization. In this work we have proposed Generalized Cross Validation (GCV) (Zhu and Xiezhang, 2019), as a technique to find good value of the Ridge Parameter λ . GCV was presented by Golub *et al.* (1979a) (Luk, 1986, Hemmerle, 1975), it is one of the most important technique used to determine the regularization parameter (Marquardt, 1970; Horel, 1976; Swindel, 1976; Golub, 1979b), for the inverse problems and regularization techniques.

For regression problem, GCV is also used in selection of subset and also applicable with singular value decomposition (SVD) methods. GCV can be used for the problems of having n-p data (where n is no. of data and p is no. omitted data) is small, means with very small data like n - p = k, we can find the value of ridge parameter.

Minimize the GCV function (Wahaba, 1976).

$$G(\lambda) = \frac{\|AX_\lambda - b\|_2^2}{\text{trace}(I_m - AB)^2}$$

$$\text{Where, } B = (A^T A + \lambda I)^{-1} A^T$$

It is easy to compute the trace term when the SVD is available.

SVD for inconsistent system:

Let $AX = b$ is an inconsistent system (that means system has no solution and infinite many solutions) at that time for matrix factorization we need SVD as below:

$$\text{We have, } AX = b \quad (10)$$

$$\begin{aligned} \Rightarrow USV^T X &= b \text{ (by SVD, } A = USV^T) \\ \Rightarrow SV^T X &= U^{-1}b \\ \Rightarrow SV^T X &= U^T b \end{aligned}$$

(U is orthonormal unitary matrix so, $U^{-1} = U^T$)

$$\begin{aligned} \Rightarrow X &= (V^T)^{-1}(S)^{-1}U^T b \\ \Rightarrow X &= V(S)^{-1}U^T b \end{aligned} \quad (11)$$

(V is also orthonormal unitary matrix so $V^{-1} = V^T$)

Any digital image is in the form of matrix and the entries are in the form of the pixel values. SVD is applied on the data form of those matrix, by deleting some of nonzero singular values and take in

consideration only the first k large singular values out of the r nonzero singular values. So, the image dimension can be reduced to get good outputs with less storage.

Discrete Picard condition

To get good regularized solutions necessary condition is the right-hand side of equation (11) is decay to zero faster than the generalized singular values, when the equation (11) is in the expression of generalized SVD. This condition is known as the discrete Picard condition.

Implementation of GCV with SCDL to find the value of regularization parameters is explained in below Table 1:

Table 1. Implementation of GCV and SCDL

GCV	SCDL
Problem as $b = AX$ inconsistent system, (from $AX = b$) where X is unknown vector to be found.	Problem as $A_h = A_i W$ inconsistent system, where W is unknown vector to be found.
By least square technique we minimize the sum of squared residuals, which is written as $\min \ AX - b\ _2^2$	By least square technique we minimize the sum of squared residuals, which is written as $\min \ A_i W - A_h\ _2^2$
To solve this inconsistent system, we use ridge regression, as now problem is, minimize $\ AX - b\ _2^2 + \lambda \ X\ _2^2$ here, $\lambda \ X\ _2^2$ is penalty term	To solve this inconsistent system, we use ridge regression, as now problem is, minimize $\ A_i W - A_h\ _2^2 + \lambda \ W\ _2^2$ here, $\lambda \ W\ _2^2$ is penalty term
Explicit solution of this problem is: $x(\lambda) = (A^T A + \lambda I)^{-1} A^T b$	Explicit solution of this problem is: $w(\lambda) = \left(A_i^T A_i + \left(\frac{\lambda_w}{\gamma} \right) I \right)^{-1} A_i^T A_h$
Noise model as $b = b^{exact} + e$ where e is error. Also $b^{exact} = AX^{exact}$	Noise model as $A_h = (A_h)^{exact} + e$ where e is error. Also $(A_h)^{exact} = A(w)^{exact}$
Minimize the GCV function $G(\lambda) = \frac{\ AX_\lambda - b\ _2^2}{\text{trace}(I_m - AB)^2}$ Where, $B = (A^T A + \lambda I)^{-1} A^T$	Minimize the GCV function $G(\lambda) = \frac{\ A_{i\lambda} w_\lambda - A_h\ _2^2}{\text{trace}(I_m - wB)^2}$ Where, $B = \left(A_i^T A_i + \left(\frac{\lambda_w}{\gamma} \right) I \right)^{-1} A_i^T$

Results

Algorithm of SCDL with SVD & GCV is applied on certain sample images of different sizes and results obtained are kept as Figure 5 to Figure 8, for image size 32, 64, 128 & 256 respectively. Reconstructed images by SCDL with regulation parameter values by GCV for butterfly image of size [260,260], cameraman image of size [256,256] and Parthenon Image of size [464, 295] are shown in Figure 9, Figure 10 and Figure 11 respectively.

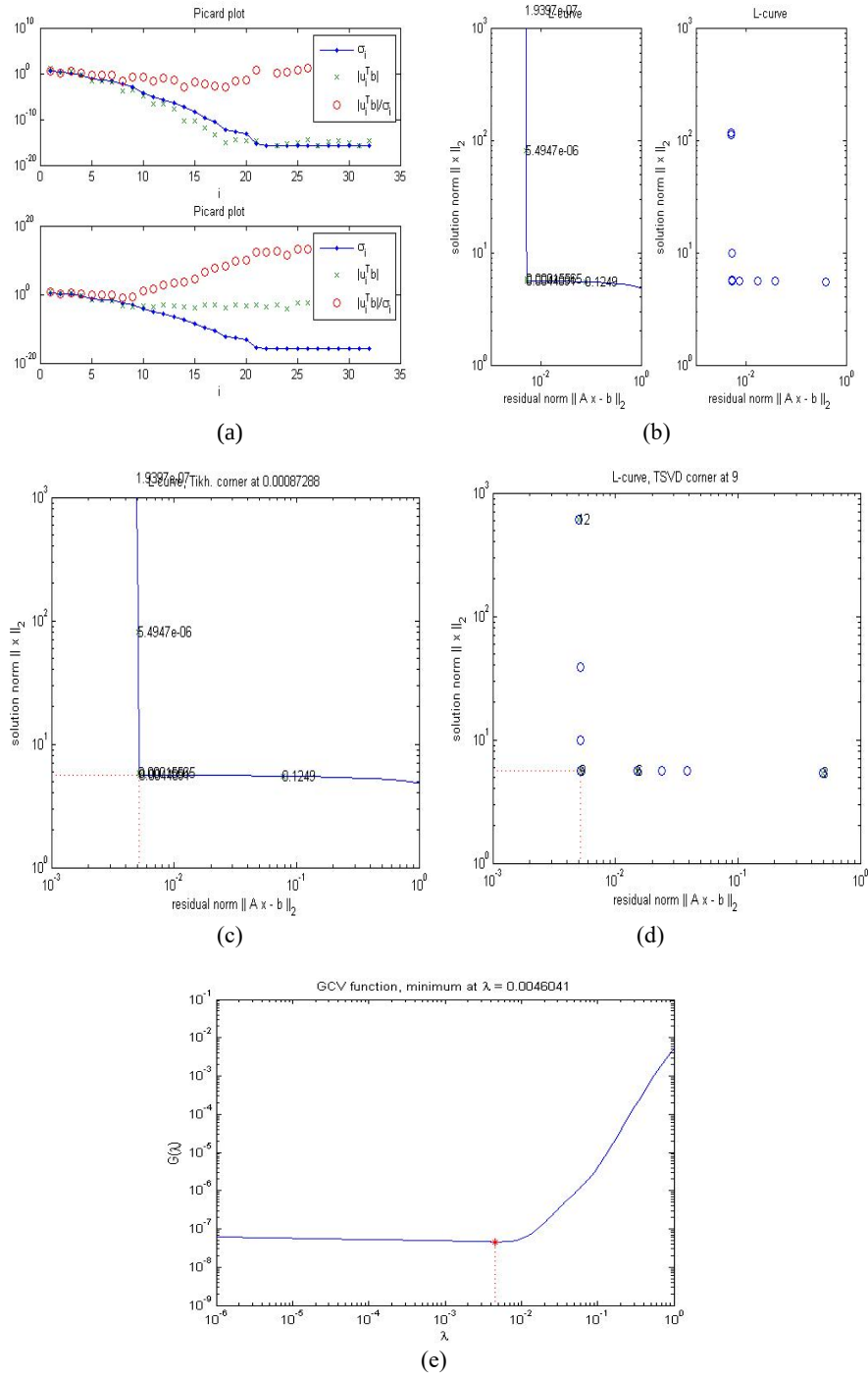


Figure 5. (a) Graph of Picard condition (b) Norm graph (c) Norm graph by Tikhonov corner (d) Norm graph by TSVD corner (e) Graph of minimum good regularized parameter lambda by GCV

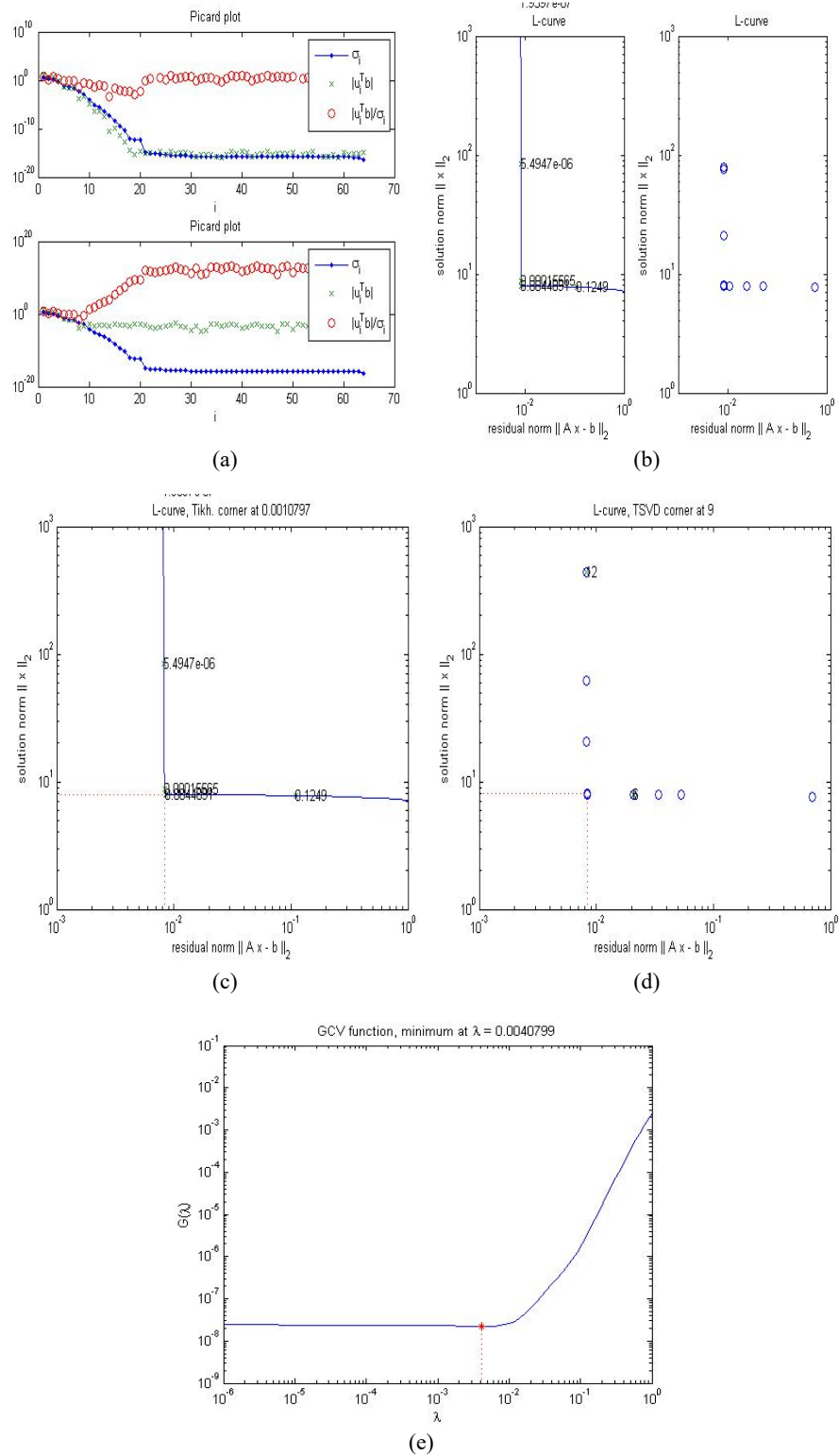


Figure 6. (a) Graph of Picard condition (b) Norm graph (c) Norm graph by Tikhonov corner (d) Norm graph by TSVD corner (e) Graph of minimum good regularized parameter lambda by GCV

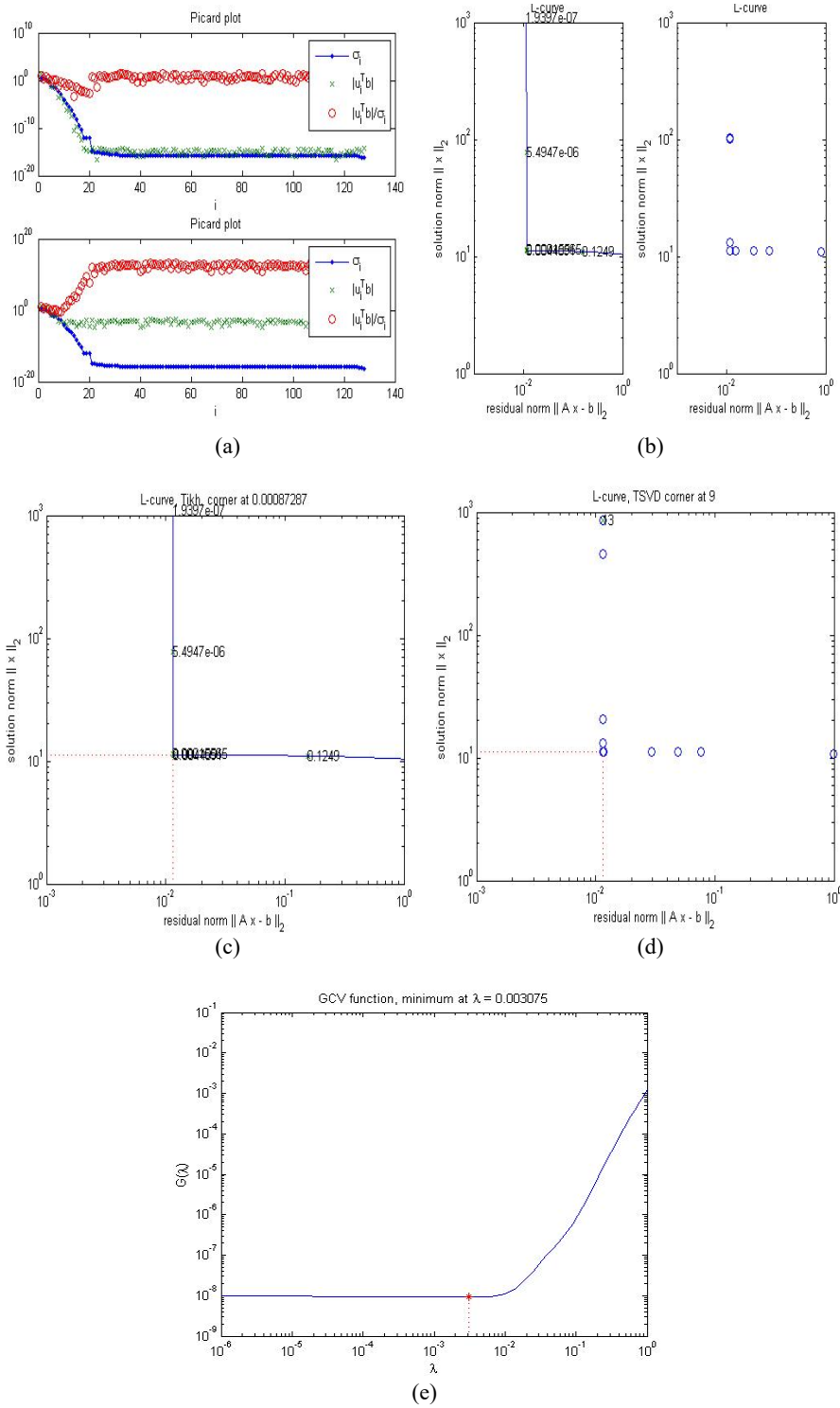


Figure 7. (a) Graph of Picard's condition (b) Norm graph (c) Norm graph by Tikhonov corner (d) Norm graph by TSVD corner (e) Graph of minimum good regularized parameter lambda by GCV

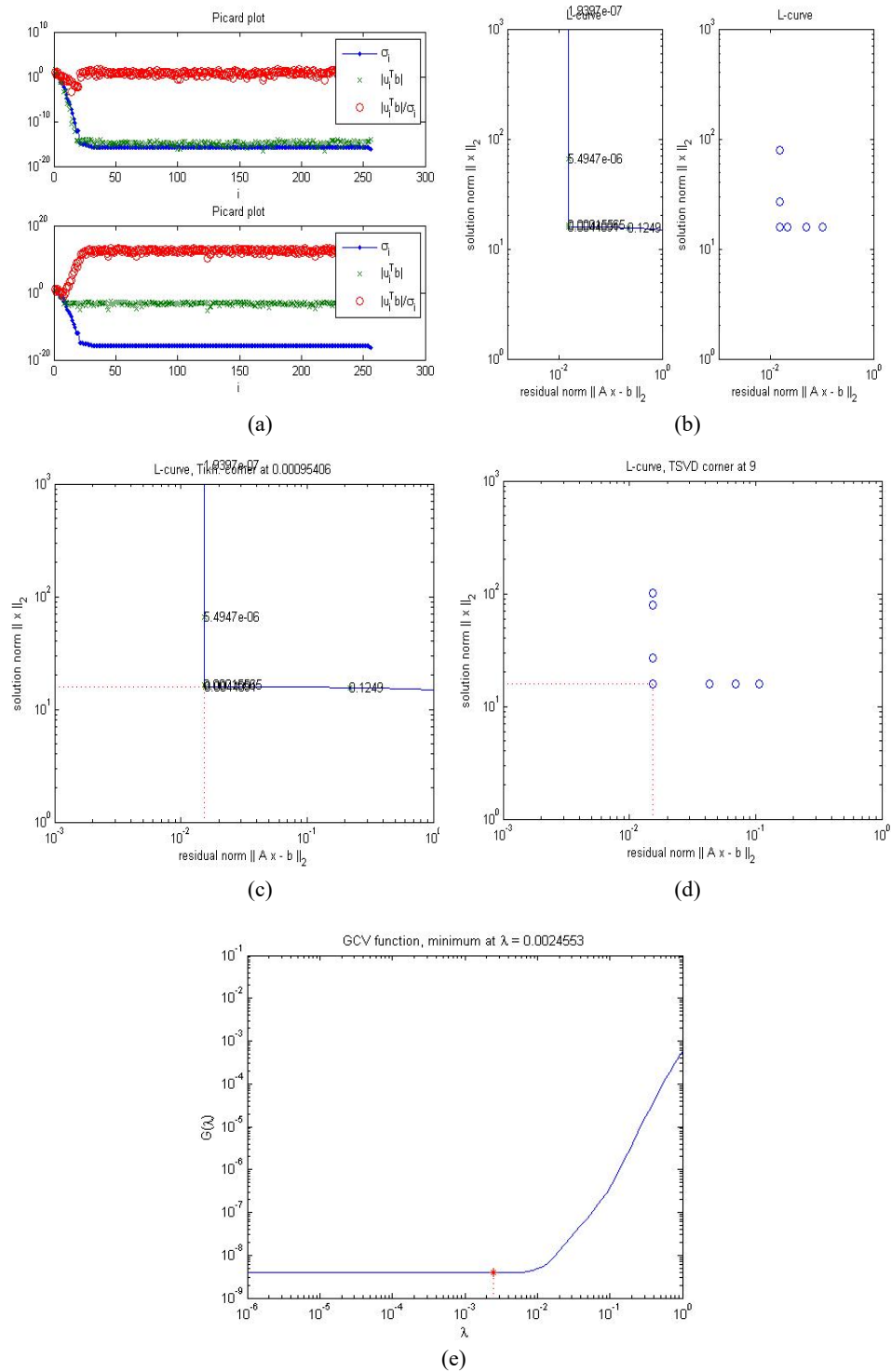


Figure 8. (a) Graph of Picard condition (b) Norm graph (c) Norm graph by Tikhonov corner (d) Norm graph by TSVD corner (e) Graph of minimum good regularized parameter lambda by GCV

Reconstructed images by SCDL with regulation parameter values by GCV:

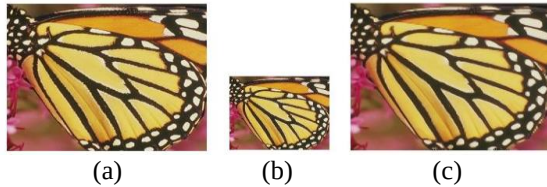


Figure 9. (a) HR image (b) LR Image (c) Reconstructed Image by SCDL with $\lambda=0.00242$ by GCV



Figure 10. (a) HR image (b) LR Image (c) Reconstructed Image by SCDL with $\lambda=0.0024553$ by GCV

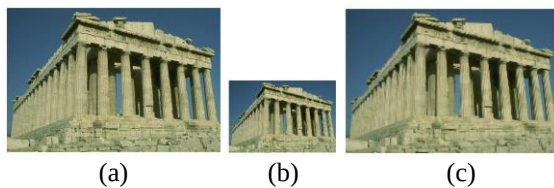


Figure 11. (a) HR image (b) LR Image (c) Reconstructed Image by SCDL with $\lambda=0.001161$ by GCV

Conclusions

In this work SVD based GCV algorithm is used to find the right value of regularization parameter for different images. Here it is found that for different size of images one can find the different value of regularization parameter using the proposed Mathematical algorithm and these regularization parameters values can be taken in SCDL for better reconstruction of the images. Here one can observe that for accurate reconstruction of super resolution images not only regularized parameter but clusters are also playing important role. During this work attempts are made to add clusters which improved reconstruction of the image but it is taking much time for the separate calculations of regularized parameter using GCV in such case. To overcome

this limitation in future GCV algorithm can be inbuilt in SCDL to produce results fast compare to existing ones.

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