

A NEW APPROACH FOR MULTIMODAL MEDICAL IMAGE FUSION USING PDE-BASED TECHNIQUE

Gargi Jeegar Trivedi* and Rajesh Sanghvi

Received: March 28, 2023; Revised: July 07, 2023; Accepted: July 10, 2023

Abstract

Accurate diagnosis of diseases is essential for effective medical treatment, and medical image fusion plays a crucial role in achieving this goal. This article proposes a novel technique for multimodal medical image fusion based on fourth order PDE-based techniques. The proposed technique uses inputted CT and brain MRI images, and applies a PDE-based technique for noise reduction and edge preservation. Subsequently, a computed fusion rule using principal component analysis is applied to generate a fused image of the brain that presents clear and essential components for medical diagnosis and analysis. The article provides a comparative study of the proposed technique with other existing techniques, demonstrating its optimal performance. The proposed technique can significantly enhance medical image diagnosis and analysis for accurate disease detection and treatment.

Keywords: Image fusion; partial differential equation; principal component analysis

Introduction

Cancer, which is becoming more prevalent in the aging population, has become a global health issue. Medical imaging (Trivedi and Sanghvi, 2022a). techniques such as computed tomography, positron emission tomography, and magnetic resonance Imaging are utilized for diagnosing brain cancer cells. Image fusion is a method that fuses multiple image of the same entity into one and provides detailed information about the entity being examined or observed from every feasible perspective and input source. This single fused image contains detailed data and information, making visual perception analysis straight forward (Xiao *et al.*, 2020). Medical professional requires

both spectral and spatial information in a single image for a research, monitoring, correct disease diagnosis, and therapy. More information from a fused image can be deduced if the noise in the image is optimally reduced. Second-order as well as fourth-order partial differential equations are a valuable tool for image noise reduction and scale-space analysis for image improvement. It has been shown that these methods may effectively balance edge detection and noise reduction implementation in image processing. PDE is an image processing tool that treats images as continuous objects (Ibrahim *et al.*, 2022; Trivedi *et al.*, 2023). Several fourth order PDE models have been proposed in the

Department of Mathematics, GH Patel College of Engineering & Technology, CVM University, Vallabh Vidhyanagar-388120, India, Tel.: +91-9426149522; E-mail: gargi1488@gmail.com; rajeshsanghvi@gcet.ac.in

* Corresponding author

DOI: <https://doi.org/10.55766/sujst-2023-04-e0843>

Suranaree J. Sci. Technol. 30(4):030132(1-7)

literature* for edge identification and picture denoising (Gonzalez *et al.*, 2010) in the expectation that these approaches would outperform their second-order predecessors.

Materials and Methods

Principal Component Analysis (PCA)

It is a mathematical technique used to reduce the dimensionality of a large dataset while retaining as much information as possible. It is a commonly used technique in data analysis and machine learning, particularly in image processing, signal processing, and feature extraction (Rajinikanth *et al.*, 2018; Trivedi and Sanghvi, 2023b).

PCA works by finding the principal components of the dataset, which are the directions that capture the largest amount of variance in the data. In other words, these components are the linear combinations of the original variables that explain the most significant amount of variability in the dataset. The first principal component is the direction of maximum variance, and each subsequent principal component is orthogonal to the previous components and has the next highest variance.

PCA is mathematically defined as a linear transformation that maps the data to a new coordinate system, where the axes are the principal components. Let X be an $n \times p$ matrix, where n is the number of observations and p is the number of variables. The goal of PCA is to find a linear transformation of X to a new set of variables Y , such that the covariance matrix of Y is diagonal. This means that the variables in Y are uncorrelated, and each variable captures the maximum amount of variance in the data.

The transformation from X to Y is achieved by computing the eigen vectors and eigenvalues of the covariance matrix of X . The eigen vectors are the principal components, and the corresponding eigenvalues represent the amount of variance explained by each component. The first principal component is the eigenvector with the highest eigenvalue, and each subsequent component is the eigen vector with the next highest eigenvalue.

PCA can be used for various purposes, including dimensionality reduction, feature extraction, and data compression. It is particularly useful for visualizing high-dimensional data in a lower-dimensional space, where it can be easily analyzed and interpreted. PCA is widely used in image processing, where it is used to extract features from images and reduce the dimensionality of the feature space.

In medical image fusion, PCA is often used to combine the information from different modalities, such as CT and MRI images. The computed fusion rule using principal component analysis helps to generate a fused image that captures the essential components of each modality and presents a clear and accurate representation of the underlying pathology.

In conclusion, PCA is a powerful statistical technique that is widely used in data analysis, machine learning, and image processing. It helps to reduce the dimensionality of a large dataset while retaining the essential information and is particularly useful for visualizing high-dimensional data in a lower-dimensional space. The computed fusion rule using principal component analysis can significantly enhance medical image diagnosis and analysis for accurate disease detection and treatment.

Partial Differential Equation

In Second-order partial differential equations the use of Gaussian filter in image analysis led to the initial introduction of the de-noising theory of partial differential Equations (5).

$$\begin{cases} \frac{\partial u}{\partial t} = \text{div} \left(c(x, y, t) \nabla u(x, y, t) \right) \\ \frac{1}{2\pi\sigma^2} e^{-\left(\frac{x^2+y^2}{2\sigma^2}\right)} \end{cases} \quad (1)$$

Where ∇ signifies gradient operation, and div signifies divergence operation, $c(x, y, t)$ presents diffusion conductance in the equation. To remove inconsistencies in filtering of local image features and noise by the Gauss filter (Perona and Malik, 1990; Getreuer, 2012) addressed this and presented a technique called non-linear anisotropic diffusion, here smoothing effect is calculated as under:

$$\begin{cases} \frac{\partial u}{\partial t} = \text{div}[c(|\nabla u|) \cdot \nabla u] \\ u(x, y, 0) = u_0(x, y) \\ u(x, y, t) = u_0(x, y) * G(x, y, t) \end{cases} \quad (2)$$

where they consider two values c the diffusion factor.

$$c(x, y, t) = e^{-\left(\frac{|\nabla u|}{k}\right)^2} \text{ or } c(x, y, t) = \frac{1}{1 + \left(\frac{|\nabla u|}{k}\right)^2} \quad (3)$$

The diffusion factor c in the following equations varies at different locations in the image. This component has a low value in locations where the image's gradient is large. As a result, the diffusion factor is minimal at the margins, and edges

are prevented from smoothing. Here k is used as conductance parameter that adjust diffusion factor in Equation (3).

To put it another way the gradient ∇u is substituted in Equation (2). After smoothing the picture at time t with a gaussian convolution r is represented as a normalization parameter and defines consistent smoothing method that determines the gradient of the picture taken at time interval t .

$$\begin{cases} \nabla u = \nabla(u(x, y, t) * G_{\sigma_r}) \\ u(t) = \text{div}(c|\nabla u * G_{\sigma_r}|)\nabla u \end{cases} \quad (4)$$

To address this shortcoming, (Rudin *et al.*, 1992) suggested procedure on the reduction of Total-Variation (TV) model to reduce noise. It provides straight forward results on efficient image de-blurring and de-noising.

$$\text{Min } E(u), E(u) = \iint_{\Omega} (|\Delta u| + \frac{\lambda}{2} \|u_0 - u\|^2) dx dy \quad (5)$$

From the preceding equation, image is restored which employed parabolic equation using time as evolution parameter also referred as gradient descent technique.

$$\frac{\partial u}{\partial t} - \Delta \left(\frac{\Delta u}{|\Delta u|} \right) = \lambda (u_0 - u) \quad (6)$$

Bavirisetti *et al.* (2019); Mikula (2002); Cui (2021); and Zhang (2022). However, in case of backward-diffusion (You and Kaveh, 2000) any step picture will be global minimum of the energy function, hence blocks are generated in the early states of dissemination.

Curvature gradient flow technique is used for these approaches, which are fourth order PDEs:

$$\frac{\partial u}{\partial t} = -\nabla^2 [c(\nabla^2 u) \nabla^2 u] \quad (7)$$

where is ∇^2 Laplacian operator. PDEs seek elimination of noise and maintain edges by approximation an observed image with a piece wise planar image because the Laplacian of image at given pixel is zero if the image is planar in its vicinity (Vanitha *et al.*, 2019). The following energy functional was related to Equation (7)

$$E(u) = \int_{\Omega} \sqrt{((\sqrt{u_x})^2 + (\sqrt{u_{xx}})^2)^2 + ((\sqrt{u_y})^2 + (\sqrt{u_{yy}})^2)^2} + \frac{\lambda}{2} (u - u_0)^2 dx dy \quad (8)$$

The numerical solution (Pop *et al.*, 2007) of image smoothing, and edge preservation can be obtained through an iterative strategy that can be

used to approximate the solution of a differential Equation (7). Given as

$$u_{i,j}^{n+1} = u_{i,j}^n - \Delta t \nabla^2 g_{i,j}^n(\cdot) \quad (9)$$

where Laplacian of $g(\cdot)$ can be computed by

$$\nabla^2 g_{i,j}^n = g_{i+1,j}^n + g_{i-1,j}^n + g_{i,j+1}^n + g_{i,j-1}^n - 4 g_{i,j}^n \quad (10)$$

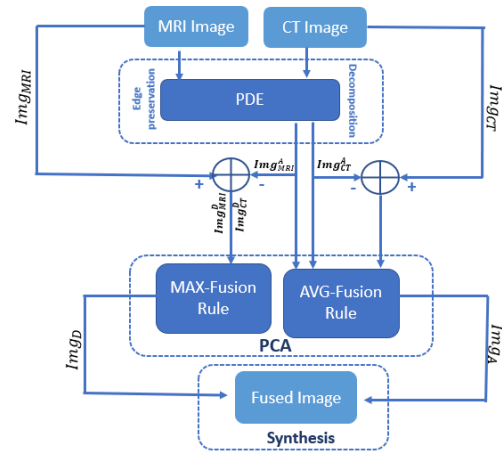


Figure 1. Diagram of proposed method

Proposed Methodology

In this section, we outline the novel Partial Differential Equation (PDE) and Principal component analysis (PCA) image fusion technique, which comprises several essential steps. The proposed methodology is visually represented in Figure 1, illustrating the key components of the process. The novel PDE image fusion technique is as follows:

Step1:

Edge-preserving Fourth PDE shown in Equations (7)-(10) are used to decompose each input image into Approximate Coefficient (A) and Detailed coefficients (D). Using this technique Input Images are divided into approximation images.

$$\begin{cases} \text{Img}_{CT}^A = \text{PDE}(\text{Img}_{CT}) \\ \text{Img}_{MRI}^A = \text{PDE}(\text{Img}_{MRI}) \end{cases}$$

and

Detail images generated from the average of Input Image and Approximate Image calculated from Fourth order PDE using the below formula:

$$\begin{bmatrix} \text{Img}_{CT}^D = \frac{\text{Img}_{CT}^A + \text{Img}_{CT}}{2} ; \\ \text{Img}_{MRI}^D = \frac{\text{Img}_{MRI}^A + \text{Img}_{MRI}}{2} \end{bmatrix}$$

Step 2:

Detailed images contain visually meaningful information like edges, lines, and contours, and Approximate Images have blurry information, they convey information less effectively than detailed ones. Hence an effective fusion-rule formulated through principal component analysis is applied on detail images and approximate images by MAX - AVG Rule. This fusion technique determines the ideal weights based on primary components, ensuring that the necessary data from detail pictures are integrated into the finished detail image. Since approximate pictures have blurry information, they convey information less effectively than detailed ones. Therefore, combining approximation pictures into a final approximation image just requires a straightforward average fusion method.

- Make two vectors $(\text{Img}_{MRI}^D, \text{Img}_{CT}^D)^T$ and $(\text{Img}_{MRI}^A, \text{Img}_{CT}^A)^T$ of Detailed images and Approximated Images to create the matrix B_1 and B_2 respectively.

$$B_1 = \begin{bmatrix} \text{Img}_{CT}^D \\ \text{Img}_{MRI}^D \end{bmatrix}, B_2 = \begin{bmatrix} \text{Img}_{CT}^A \\ \text{Img}_{MRI}^A \end{bmatrix}$$

- Build Covariance Matrix of $C(i)$ of B_1 and B_2 by taking rows as observation mean vectors and columns as a variable.
- Calculate the Eigenvalue corresponding to the Eigenvector of $C(i)$.
- Divide each element by the mean of the eigenvector that corresponds to a well-built eigenvalue. This results in the first main component P_1 . To obtain the second principal component P_2 , repeat the technique with an eigenvector corresponding to a lesser eigen value.

$$P_1 = \frac{C(i)}{\Sigma C}, P_2 = \frac{C(i)}{\Sigma C}, \text{ where } i = 1, 2$$

- High-frequency and Low-frequency coefficients are fused using MAX- AVG method

$$\text{Img}_D = \text{MAX} \{ \text{abs}(\text{Img}_{CT}^D), \text{abs}(\text{Img}_{MRI}^D) \}$$

$$\text{Img}_A = \text{AVG} \{ \text{abs}(\text{Img}_{CT}^A), \text{abs}(\text{Img}_{MRI}^A) \}$$

Step 3:

Fused image is reconstructed by taking a linear combination of final detailed and approximated images.

$$I_f = P_1 \text{Img}_D + P_2 \text{Img}_A$$

Results and Discussion

This Section is divided in to three different subsections

- Dataset used for the implementation.
- Image quantitative analysis by quality evaluation matrices.
- Image visual quality analysis.

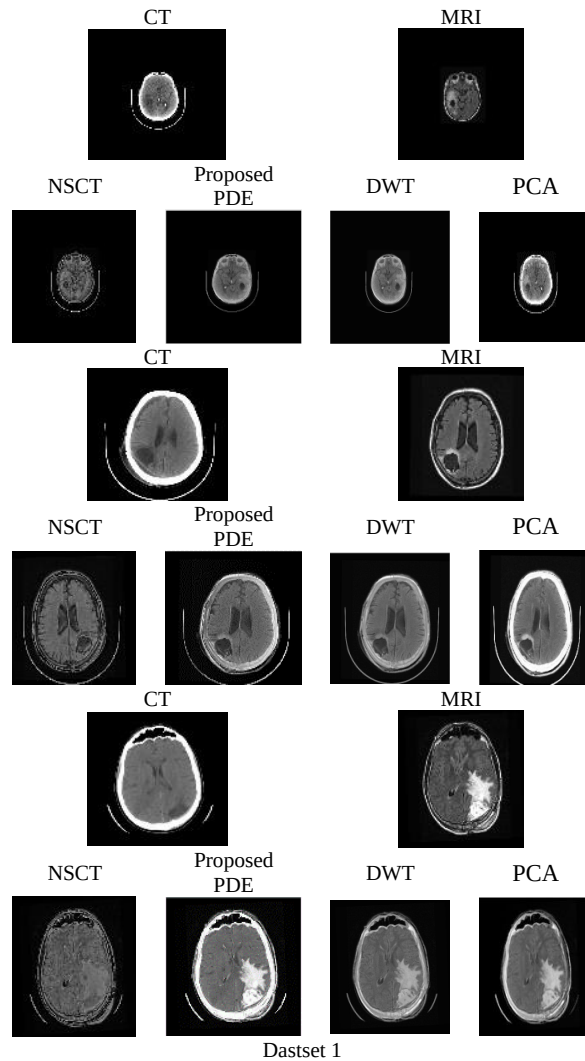


Figure 2. Visual quality comparison of fused images from different techniques

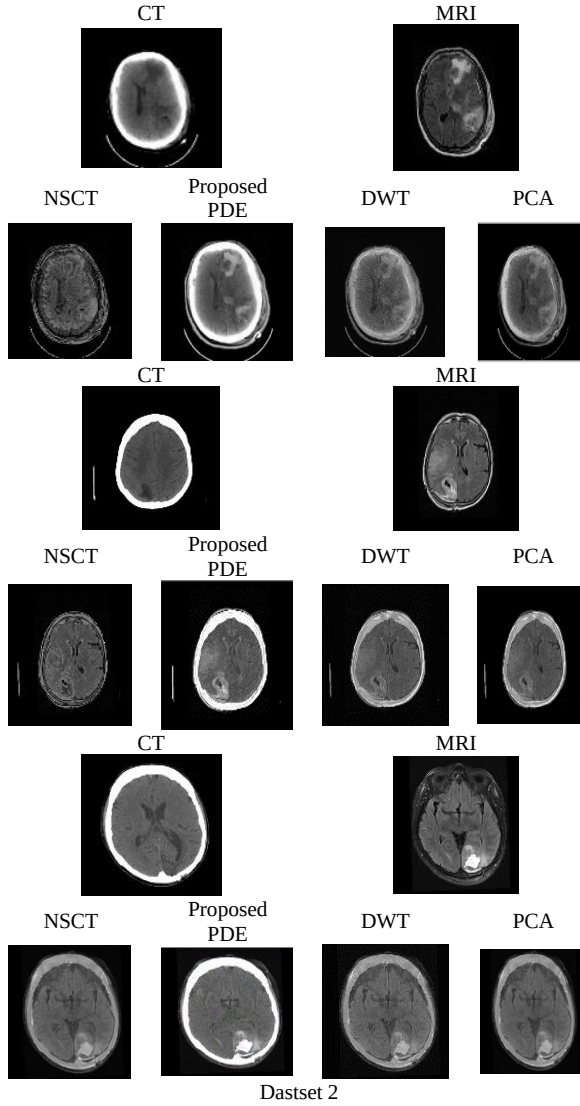


Figure 3. Visual quality comparison of fused images from different techniques

Datasets used for Images

Implementations of CT and MRI image datasets are depicted in Figures 2 and 3. These datasets were obtained through the websites <http://www.med.harvard.edu/AANLIB/home.html> and www.Kaggle.com

Different Quality Assessment Parameters for Measuring Fusion Performance

Evaluating fusion interpretation is an effortful task, various parameters have been proposed by researchers (Trivedi and Sanghvi, 2023c; 2023d) for evaluating performance of image fusion. Among them, some traditional fusion techniques have been used.

- a. **Entropy (EN)** measures information presented by fused image and is calculated as

$E = - \sum_{i=0}^{255} p_i \log_2(p_i)$, where p_i represents probability for intensity-value i in 8-bit image.

- b. **Mutual information (MI)** evaluates performance between for inputted images and the merged image and is calculated as

$$MI = \sum_k \sum_l p_{I_1, F}(k, l) \log_2 \left(\frac{p_{I_1, F}(k, l)}{p_{I_1(k)} p_F} \right) + \sum_k \sum_l p_{I_2, F}(k, l) \log_2 \left(\frac{p_{I_2, F}(k, l)}{p_{I_2(k)} p_F} \right).$$

where MI is the mutual information between a source image I_1, I_2 and fused-image F.

- c. **The correlation coefficient (CC)** measures the relationship between inputted images and fused images and

$$CC = \frac{\sum_i \sum_j (I_1(i, j) - \bar{I}_1) (I_2(i, j) - \bar{I}_2) + \sum_i \sum_j (I_2(i, j) - \bar{I}_2) (I_1(i, j) - \bar{I}_1)}{\sqrt{(\sum_i \sum_j (I_1(i, j) - \bar{I}_1)^2) (\sum_i \sum_j (I_2(i, j) - \bar{I}_2)^2)}}$$

- d. **Spatial frequency (SF)** computes the total information level in the regions of an image and is calculated as

$$SF = \sqrt{\left(\frac{\sum_{i=1}^m \sum_{j=1}^n (I_2(i, j) - I_1(i, j-1))^2}{m \cdot n} \right)^2 + \left(\frac{\sum_{i=1}^m \sum_{j=1}^n (I_2(i, j) - I_1(i-1, j))^2}{m \cdot n} \right)^2}$$

Table 1. Quality measurement of image fusion performance of data set 1

	EN	MI	CC	SF
Proposed Method	0.8524	0.9122	0.7840	12.9465
DWT	0.7645	0.8256	0.6514	13.5015
PCA	0.7845	0.8623	0.7312	11.3625
NSCT	0.8347	0.9024	0.8064	13.4305

Table 2. Quality measurement of image fusion performance of data set 2

	EN	MI	CC	SF
Proposed Method	1.9236	1.5863	0.7123	15.2385
DWT	1.4508	1.0492	0.6693	10.0410
PCA	1.1096	1.0687	0.6958	14.4437
NSCT	0.8489	1.1766	0.5994	17.1667

Table 1 presents the evaluation results for the MRI and CT dataset. The techniques considered in the study are Proposed PDE, DWT, PCA, and NSCT. The evaluation metrics used are Entropy (EN), Mutual Information (MI), Cross-Correlation (CC), and Spatial Frequency (SF). Looking at the table, it is observed that the Proposed PDE technique achieves the highest scores for Entropy (0.85), MI (0.91), and CC (0.78) compared to other techniques. This suggests that the PDE-based method preserves more information, exhibits stronger alignment with the original images, and captures higher pixel intensity similarities in the fused CT images. However, in terms of SF (12.94), the PDE technique has a lower score compared to DWT (13.50). This indicates that the DWT method may better preserve the fine details and sharpness in the fused CT images results for the MRI dataset.

In Table 2, the PDE-based technique continues to demonstrate promising results. It achieves the highest scores for Entropy (1.92), MI (1.58), and CC (0.71) among the evaluated techniques. This indicates that the PDE-based fusion method successfully preserves more information, aligns well with the original MRI and CT images, and exhibits higher pixel intensity similarities. However, for SF (15.23), the PDE-based technique has a lower score compared to (17.16), suggesting that the NSCT method may better capture the fine details and sharpness in the fused images.

Overall, both tables provide a comprehensive comparison of the evaluated techniques in terms of their performance on different evaluation metrics for CT and MRI image fusion. The results indicate that the PDE technique consistently shows promising performance across multiple metrics for both datasets, suggesting its effectiveness in preserving information, alignment, and pixel intensity similarities.

Different Fusion Techniques Were Applied for Comparisons

The suggested approach is contrasted with four other conventional fusion procedures. Discrete wavelet transformation, Principal component analysis, and non-subsampled contourlet transformation are among these techniques (NSCT).

The goal of every fusion procedure is to create a dependable and trustworthy image that transfers meaningful information along with the reduction of source information loss and fusion artifacts. Research work presents a novel image fusion strategy based on partial differential equations and principal component analysis; in comparison to other traditional methods, the following are the benefits of the suggested method:

- The suggested technique may transfer most of the necessary details from the inputted images to fused image.
- Less information is lost during the fusion process.
- Fusion artifacts are reduced.
- Implemented techniques calculation time is equitable.

When compared to DWT, PCA, and NSCT techniques our suggested technique can fuse most of the information from both input images, as demonstrated in Figures 2 and 3 for visual quality, our suggested technique performed well in terms of EN, MI, CC, and SF in quantitative parameters as shown in Tables 1 and 2.

Conclusions

Implemented Novel PDE Method has been proposed for the fusion of multimodal images. This paper highlights the most recent developments in PDE based - image fusion and suggests some avenues for further research in this area. The following points might be used to describe the main contributions of this study.

- Novel PDE based image fusion techniques can automatically extract the most useful characteristics from data. To avoid the complexity of human design.
- The suggested strategy, according to experimental findings, performs at the cutting edge in terms of both visual quality and quantitative assessment criteria.
- In conclusion, the current work on image fusion by PDE based medical image fusion demonstrates a bright direction in the field of picture fusion with plenty of scope for advancement in the future. Further similar studies will likely carry on in the upcoming years to support the growth of image fusion.

References

- Bavirisetti, D.P., Xiao, G., and Liu, G. (2019). Multi-sensor image fusion based on fourth order partial differential equations. In: 2019 10th International Conference on Computing, Communication and Networking Technologies (ICCCNT); Kanpur, India, July 6-8, 2019. <https://doi.org/10.1109/ICCCNT45670.2019>
- Cui, D. (2021). Image segmentation algorithm based on partial differential equation. *Journal of Intelligent and Fuzzy Systems*, 40(4):5945-5952. <https://doi.org/10.3233/JIFS-189434>
- Getreuer, P. (2012). Rudin-osher-fatemi total variation denoising using split bregman. *Image Processing on Line*, 2:74-95. <https://doi.org/10.5201/ipol.2012.g-tvd>

- Gonzalez, R.C., Woods, R.E., and Eddins, S.L. (2010). Digital Image Processing Using MATLAB. 620p.
- Ibrahim, R.W., Jalab, H.A., Karim, F.K., Alabdulkreem, E., and Ayub, M.N. (2022). A medical image enhancement based on generalized class of fractional partial differential equations. *Quantitative Imaging in Medicine and Surgery*, 12(1):172-183. <https://doi.org/10.21037/qims-21-15>
- Mikula, K. (2002). Image processing with partial differential equations. In: *Modern Methods in Scientific Computing and Applications*. Bourlioux, A., Gander, M.J., and Sabidussi, G. (eds). NATO Science Series, Springer, Dordrecht, 175:283-321. https://doi.org/10.1007/978-94-010-0510-4_8
- Perona, P. and Malik, J. (1990). Scale-space and edge detection using anisotropic diffusion. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 12(7):629-639. <https://doi.org/10.1109/34.56205>
- Pop, S., Lavialle, O., Terebes, R., Borda, M. (2007). A PDE-based approach for image fusion. In: *Advanced Concepts for Intelligent Vision Systems*. Blanc-Talon, J., Philips, W., Popescu, D., and Scheunders, P. (eds). ACIVS 2007. Lecture Notes in Computer Science, Springer, Berlin, Heidelberg, 4678:121-131. https://doi.org/10.1007/978-3-540-74607-2_11
- Rajinikanth, V., Satapathy, S.C., Dey, N., and Vijayarajan, R. (2018). DWT-PCA Image Fusion Technique to Improve Segmentation Accuracy in Brain Tumor Analysis. In: *Microelectronics, Electromagnetics and Telecommunications*. Anguera, J., Satapathy, S., Bhateja, V., Sunitha, K. (eds). Lecture Notes in Electrical Engineering, Springer, Singapore, 471:453-462. https://doi.org/10.1007/978-981-10-7329-8_46
- Rudin, L.I., Osher, S., and Fatemi, E. (1992). Nonlinear total variation based noise removal algorithms. *Physica D: Nonlinear Phenomena*, 60(1-4):259-268. [https://doi.org/10.1016/0167-2789\(92\)90242-F](https://doi.org/10.1016/0167-2789(92)90242-F)
- Trivedi, G.J. and Sanghvi, R. (2022a). Medical image fusion using cnn with automated pooling. *Indian Journal of Science and Technology*, 15(42):2267-2274. <https://doi.org/10.17485/IJST/v15i42.1812>
- Trivedi, G.J. and Sanghvi, R. (2023b). Optimizing Image Fusion Using Modified Principal Component Analysis Algorithm and Adaptive Weighting Scheme, *International Journal of Advanced Networking and Applications* 15(1) : 5769-5774. 10.35444/IJANA.2023.1510
- Trivedi, G.J. and Sanghvi, R. (2023c). Novel Approach to Multi-Modal Image Fusion using Modified Convolutional Layers. *Journal of Innovative Image Processing*, 5(3) : 229. <https://doi.org/10.36548/jiip.2023.3.002>
- Trivedi, G.J. and Sanghvi, R. (2023d). Fusessharp: A Multi-Image Focus Fusion Method Using Discrete Wavelet Transform And Unsharp Masking. *J. Appl. Math. & Informatics* Vol. 41(5) :115- 1128. <https://doi.org/10.14317/jami.2023.1115>
- Trivedi, G.J., Shah, V., Sharma, J., and Sanghvi, R. (2023a). On solution of non-instantaneous impulsive Hilfer fractional integro-differential evolution system. *Mathematica Applicanda*, 51(1):3-20. <https://doi.org/10.14708/ma.v51i1.7167>
- Vanitha, K., Satyanarayana, D., and Prasad, M.N.G. (2019). A new hybrid medical image fusion method based on fourth-order partial differential equations decomposition and DCT in SWT domain. In: *2019 10th International Conference on Computing, Communication and Networking Technologies (ICCCNT)*; Kanpur, India, 2019, p. 1-6. <https://doi.org/10.1109/ICCCNT45670.2019>
- Xiao, G., Bavirisetti, D.P., Liu, G., and Zhang, X. (2020). *Image Fusion*. 1st ed. Springer Singapore, 404p. <https://doi.org/10.1007/978-981-15-4867-3>
- You, Y.L. and Kaveh, M. (2000). Fourth-order partial differential equations for noise removal. *IEEE Transactions on Image Processing*, 9(10):1723-1730. <https://doi.org/10.1109/83.869184>
- Zhang, Q. (2022). Construction of robot computer image segmentation model based on partial differential equation. *Journal of Sensors*, 2022:Article ID 6216423. <https://doi.org/10.1155/2022/6216423>