

TEMPORAL NEUTROSOPHIC TOPOLOGY

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Abstract

This paper introduces temporal neutrosophic sets and discusses their basic properties. It presents the concept of temporal neutrosophic topology in Chang's sense and outlines its fundamental characteristics. The overall neutrosophic topology in Chang's sense is also defined. Finally, the study explores the relationship between this topology and both the temporal and overall neutrosophic topologies in Sostak's sense.

Keywords: Intuitionistic Fuzzy Set ($\mathcal{IFS}$); Neutrosophic set ($\mathcal{NS}$); Neutrosophic Topology; Temporal Neutrosophic Topology in Chang's sense ($\mathcal{CT-TNS}$); Temporal Neutrosophic Topology in Sostak's sense ($\mathcal{ST-TNS}$); Overall Neutrosophic Topological Spaces; Temporal Neutrosophic Interior; Temporal Neutrosophic Closure; Temporal Neutrosophic Continuous (TNCont)

Introduction

Zadeh L. A. described the fuzzy set ($\mathcal{FS}$) (Zadeh, 1965). In resolving environmental practical issues, numerous fuzzy set generalisations have been defined. The intuitionistic fuzzy set ($\mathcal{IFS}$) (Atanassov, 1983), which was presented by Atanassov k. T. in 1983, is one of these generalizations. Neutrosophic Sets ($\mathcal{NS}$ s), (Smarandache, 1999) presented by Florentin Smarandache, are an extension of $\mathcal{FS}$ s and $\mathcal{IFS}$ and it is strong tool to deal with the conflicting, ambiguous, and inadequate data that occurs in the actual world. The degree of indeterminacy was first presented by Smarandache as an independent component Zadeh L. A. described the fuzzy set ($\mathcal{FS}$) (Zadeh, 1965). In resolving environmental practical issues, numerous fuzzy set generalisations

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Topology is the mathematical study of the properties of spaces that are preserved under continuous deformations. Fuzzy topology extends the concepts of classical topology by incorporating

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the idea of fuzziness, where the boundaries of set are not sharply defined. The concept of fuzzy topology was first introduced by Chang, C. L. (Chang, 1968), who defined fuzzy open sets as fuzzy subsets satisfying axioms similar to those of classical open sets. In Chang's definition constant functions between fuzzy topological spaces are not necessarily continuous. To address this, Lowen (Lowen, 1976) introduced fuzzy topology to clarify the connection between fuzzy topological spaces and classical topological spaces.

Subsequently, A. Sostak (Sostak, 1985) introduced fuzzy topology using the notion of fuzzy neighborhood systems in L-topological spaces, offering a unified, general, and categorical framework for fuzzy topological spaces. Unlike Chang and Lowen, who focused on fuzzy open sets, Šostak's approach supported defining topology through neighborhoods just like in classical topology. There is also additional research on fuzzy topology, including work on fuzzy m -open sets (Wadei Faris and Alomeri, 2024) and totally continuous mappings, which has further developed the area.

Intuitionistic fuzzy topology was developed as a topological extension by Atanassov K. T (Atanassov, 1986). Following Atanassov's work, Samanta, Coker, Fatih Kutlu, and Tunay Bilgin, Feride Tugrul, Bilgin, Mondal, further developed and defined (Coker, 1997; Samanta *et al.*, 1997; Kutlu *et al.* 2015; Kutlu, 2019; Tugrul, 2021) Intuitionistic fuzzy topological spaces.

Neutrosophic topology introduced by Florentin Smarandache (Smarandache, 2005). Smarandache and other researchers (Salama & Alblowi, 2012; Salama *et al.*, 2014; Iswarya & Bageerathi, 2016; Saber *et al.*, 2020; Abdulhusein *et al.*, 2025) examined NSs and their associated topological structures, then presented the idea of a Neutrosophic Topological Space (NTS), drawing on the concepts of Chang, Lowen, and Šostak. There is also other research on neutrosophic sets and their extensions. Al-Omeri and collaborators have advanced neutrosophic theory through new fixed point results (Al-Omeri, 2023a), applications of neutrosophic graphs to earthquake response (Al-Omeri and Kaviyarasu, 2024) and service classification (Wadei *et al.*, 2024), and studies on neutrosophic g -closed sets and topological spaces (Al-Omeri, 2023b).

A temporal set models elements that change over time, capturing real-world dynamics. This helps in applications like weather, health, economics, and image-video processing. Using fuzzy and intuitionistic fuzzy approaches in temporal models handles uncertainty better, adds realism, and supports better decision-making.

Topology in temporal settings helps analyze how elements change over time with continuity and structure. It enables the study of evolving relationships and smooth transitions, making it essential for modeling dynamic, real-world systems. Atanassov defined the temporal intuitionistic fuzzy set ($\mathcal{TJFS}$) (Atanassov, 1990). According to his concept, an element's degree of membership or non-membership totally depends on its composition and the present time period. He also developed the concept of relations on temporal intuitionistic fuzzy sets (Atanassov, 2001). This extension has been recognized as one of the most significant developments in the field (Hassaballah & Ghareeb, 2017; Garg & Kumar, 2020; Kutlu, 2019, 2021; Tugrul, 2021). Furthermore, level operators on JFSs were described by Çuvalcıoğlu and Yılmaz in 2014 (Yılmaz & Çuvalcıoğlu, 2014), and fuzzy functions for JFSs were developed by Parvathi and Radhamani (Parvathi & Radhamani, 2019).

For temporal intuitionistic fuzzy sets, Kutlu, F., Atan, and Bilgin (Kutlu *et al.*, 2016) introduced similarity, entropy, inclusion, and distance measures and looked into some of these measures' characteristics. Additionally, such ideas were defined both temporally and generally, and the relationship between these definitions was studied. Kutlu and collaborators developed temporal intuitionistic fuzzy metric spaces (Kutlu *et al.*, 2015) and extended intuitionistic fuzzy topology to the temporal case in Šostak's sense.

The most fundamental ideas of Temporal Neutrosophic Sets ($\mathcal{TNS}$ s) are still undefined. In real-world problems involving a time component, there is a high likelihood of encountering indeterminate cases. To address this, there is a need for the development of Temporal Neutrosophic Sets. $\mathcal{TNS}$ is incorporates temporal factors into the calculation of the truth $\mathcal{T}_M$, indeterminacy $\mathcal{I}_M$, and falsity $\mathcal{F}_M$ membership degrees. It represents one of the most significant expansions of neutrosophic sets.

Incorporating temporal aspects into neutrosophic topology enables the modeling of uncertainty that changes over time. By assigning time-dependent grades to open sets, this approach offers a more realistic and flexible framework for analyzing dynamic systems in fields such as engineering, medicine, and economics.

In this work, $\mathcal{TNS}$ and its topology are defined, with descriptions of $\mathcal{TNS}$ s provided. $\mathcal{TNS}$ is introduced in Chang's, Lowen's, and Šostak's sense, along with its fundamental characteristics.

Preliminaries

Definition 2.1 (Smarandache, 2005)

Consider E be a set which is not empty (universal set). M is a $\mathcal{NS}$ on E is of the form:

$$M = \{\langle m, \mathcal{T}_M(m), \mathcal{I}_M(m), \mathcal{F}_M(m) \rangle | m \in E\},$$

where $\mathcal{T}_M(m), \mathcal{I}_M(m), \mathcal{F}_M(m) \in [0,1]$,
 $0 \leq \mathcal{T}_M(m) + \mathcal{I}_M(m) + \mathcal{F}_M(m) \leq 2$,

for all m in E . $\mathcal{T}_M(m)$ is membership, $\mathcal{I}_M(m)$ indeterminacy and $\mathcal{F}_M(m)$ non-membership functions where $\mathcal{T}_M(m)$ and $\mathcal{F}_M(m)$ are dependent components and $\mathcal{I}_M(m)$ is an independent component.

Definition 2.2 (Smarandache, 2005)

If $M = \{\langle p, \mathcal{T}_M(p), \mathcal{I}_M(p), \mathcal{F}_M(p) \rangle | p \in E\}$ and $N = \{\langle p, \mathcal{T}_N(p), \mathcal{I}_N(p), \mathcal{F}_N(p) \rangle | p \in E\}$ are any two $\mathcal{NS}$ s of E then

- $M \subseteq N$ iff $\mathcal{T}_M(p) \leq \mathcal{T}_N(p)$; $\mathcal{I}_M(p) \leq \mathcal{I}_N(p)$; and $\mathcal{F}_M(p) \geq \mathcal{F}_N(p)$, $\forall p \in E$
- $M = N$ iff $\mathcal{T}_M(p) = \mathcal{T}_N(p)$; $\mathcal{I}_M(p) = \mathcal{I}_N(p)$; and $\mathcal{F}_M(p) = \mathcal{F}_N(p)$, $\forall p \in E$
- $M \cap N = \left\{ \left\langle p, \mathcal{T}_{M \cap N}(p), \mathcal{I}_{M \cap N}(p), \mathcal{F}_{M \cap N}(p) \right\rangle \mid p \in E \right\}$

where,

- $\mathcal{T}_{M \cap N}(p) = \min\{\mathcal{T}_M(p), \mathcal{T}_N(p)\}$
- $\mathcal{I}_{M \cap N}(p) = \min\{\mathcal{I}_M(p), \mathcal{I}_N(p)\}$
- $\mathcal{F}_{M \cap N}(p) = \max\{\mathcal{F}_M(p), \mathcal{F}_N(p)\}$

$$d. M \cup N = \left\{ \left\langle p, \mathcal{T}_{M \cup N}(p), \mathcal{I}_{M \cup N}(p), \mathcal{F}_{M \cup N}(p) \right\rangle \mid p \in E \right\}$$

where,

- $\mathcal{T}_{M \cup N}(p) = \max\{\mathcal{T}_M(p), \mathcal{T}_N(p)\}$
- $\mathcal{I}_{M \cup N}(p) = \max\{\mathcal{I}_M(p), \mathcal{I}_N(p)\}$
- $\mathcal{F}_{M \cup N}(p) = \min\{\mathcal{F}_M(p), \mathcal{F}_N(p)\}$

Definition 2.3 (Smarandache, 2005)

Let $M = \{\langle p, \mathcal{T}_M(p), \mathcal{I}_M(p), \mathcal{F}_M(p) \rangle | p \in E\}$ be the $\mathcal{NS}$ on E , then the complement of the set M ($R(M)$ clearly), maybe described as

$$(C_1) R(M) = \left\{ \left\langle p, \mathcal{F}_M(p), 1 - \mathcal{I}_M(p), 1 - \mathcal{T}_M(p) \right\rangle \mid p \in E \right\}$$

$$(C_2) R(M) = \left\{ \left\langle p, \mathcal{F}_M(p), (\mathcal{T}_M(p) + \mathcal{F}_M(p)) - \mathcal{I}_M(p), \mathcal{T}_M(p) \right\rangle \mid p \in E \right\}$$

$$(C_3) R(M) = \left\{ \left\langle p, 1 - \mathcal{T}_M(p), 1 - \mathcal{I}_M(p), 1 - \mathcal{F}_M(p) \right\rangle \mid p \in E \right\}$$

Proposition 2.4 (Smarandache, 2005).

If $M = \{\langle p, \mathcal{T}_M(p), \mathcal{I}_M(p), \mathcal{F}_M(p) \rangle | p \in E\}$ and $N = \{\langle x, \mathcal{T}_N(x), \mathcal{I}_N(x), \mathcal{F}_N(x) \rangle | x \in E\}$ are any two $\mathcal{NS}$ s of E then the proceedings are true

- $R(M \cap N) = R(M) \cup R(N)$
- $R(M \cup N) = R(M) \cap R(N)$

Definition 2.5 (Salama and Alblowi, 2012)

Let $\{M_i : i \in \eta\}$ is defined as a family of arbitrary set of $\mathcal{NS}$ in E . Then

- $\cap M_i = \left\{ \left\langle p, \wedge \mathcal{T}_{M_i}(p), \wedge \mathcal{I}_{M_i}(p), \vee \mathcal{F}_{M_i}(p) \right\rangle : p \in E \right\}$
- $\cup M_i = \left\{ \left\langle p, \vee \mathcal{T}_{M_i}(p), \vee \mathcal{I}_{M_i}(p), \wedge \mathcal{F}_{M_i}(p) \right\rangle : p \in E \right\}$

Definition 2.6 (Salama and Alblowi, 2012)

Let M and N are $\mathcal{NS}$ s in E , then $M|N$ may be described as

$$M|N = \left\{ \left\langle p, \mathcal{T}_M(p) \wedge \mathcal{F}_N(p), \mathcal{I}_M(p) \mathcal{I}_N(p), \mathcal{F}_M \vee \mathcal{T}_N(p) \right\rangle : p \in E \right\}.$$

Definition 2.7 (Salama and Alblowi, 2012)

The $\mathcal{NS}, 0_S$ and 1_S in E are described as follows:

0_S described as:

$$(0_1)_S = \{\langle p, 0, 0, 1 \rangle : p \in E\}$$

$$(0_2)_S = \{\langle p, 0, 1, 1 \rangle : p \in E\}$$

$$(0_3)_S = \{\langle p, 0, 1, 0 \rangle : p \in E\}$$

$$(0_4)_S = \{\langle p, 0, 0, 0 \rangle : p \in E\}$$

1_S described as:

$$(1_1)_S = \{\langle p, 1, 0, 0 \rangle : p \in E\}$$

$$(1_2)_S = \{\langle p, 1, 0, 1 \rangle : p \in E\}$$

$$(1_3)_S = \{\langle p, 1, 1, 0 \rangle : p \in E\}$$

$$(1_4)_S = \{\langle p, 1, 1, 1 \rangle : p \in E\}$$

where 1 and 0 indicates the maps of constant sending for element of E to 1 and 0 respectively.

Proposition 2.8 (Salama and Alblowi, 2012)

For any $\mathcal{NS}$ M the proceeding are satisfied

- $0_S \subseteq M, 0_S \subseteq 0_S$
- $M \subseteq 1_S, 1_S \subseteq 1_S$

Definition 2.9 (Smarandache, 2005).

Let $\bar{\alpha}, \bar{\beta}$ and $\bar{\gamma}$ be three real numbers in $[0,1]$ obtaining the inequality $\bar{\alpha} + \bar{\beta} + \bar{\gamma} \leq 3$. Then the triplet $\langle \alpha, \beta, \gamma \rangle$ is called a neutrosophic triplet. Let $\langle \bar{\alpha}_1, \bar{\beta}_1, \bar{\gamma}_1 \rangle$ and $\langle \bar{\alpha}_2, \bar{\beta}_2, \bar{\gamma}_2 \rangle$ be two neutrosophic triplet. Then define

- $\langle \bar{\alpha}_1, \bar{\beta}_1, \bar{\gamma}_1 \rangle \leq \langle \bar{\alpha}_2, \bar{\beta}_2, \bar{\gamma}_2 \rangle \Leftrightarrow \bar{\alpha}_1 \leq \bar{\alpha}_2, \quad \bar{\beta}_1 \leq \bar{\beta}_2$
and $\bar{\gamma}_1 \geq \bar{\gamma}_2$
- $\langle \bar{\alpha}_1, \bar{\beta}_1, \bar{\gamma}_1 \rangle = \langle \bar{\alpha}_2, \bar{\beta}_2, \bar{\gamma}_2 \rangle \Leftrightarrow \bar{\alpha}_1 = \bar{\alpha}_2, \quad \bar{\beta}_1 = \bar{\beta}_2$
and $\bar{\gamma}_1 = \bar{\gamma}_2$

- iii. If $\{\langle \bar{\alpha}_i, \bar{\beta}_i, \bar{\gamma}_i \rangle; i \in \eta\}$ is a family of neutrosophic triplets, then $\langle \bar{\alpha}_i, \bar{\beta}_i, \bar{\gamma}_i \rangle = \langle \vee \bar{\alpha}_i, \vee \bar{\beta}_i, \vee \bar{\gamma}_i \rangle$ and $\langle \bar{\alpha}_i, \bar{\beta}_i, \bar{\gamma}_i \rangle = \langle \wedge \bar{\alpha}_i, \wedge \bar{\beta}_i, \wedge \bar{\gamma}_i \rangle$
- iv. The complement of $\langle \bar{\alpha}, \bar{\beta}, \bar{\gamma} \rangle$ is defined by $\langle \bar{\alpha}, \bar{\beta}, \bar{\gamma} \rangle = \langle \bar{\gamma}, 1 - \bar{\beta}, \bar{\alpha} \rangle$
- v. $1^* = \langle 1, 1, 0 \rangle$ and $0^* = \langle 0, 0, 1 \rangle$

Definition 2.10 (Salama *et al.*, 2014)

A Neutrosophic Topology in Šostak's sense (clearly, $\mathcal{S}\text{-}\mathcal{NTS}$) on a set which is not empty set $\mathfrak{X}$ and ε is a $\mathcal{NTS}$ defined with $\varepsilon(M) = (\mathcal{T}_\varepsilon(M), \mathcal{I}_\varepsilon(M), \mathcal{F}_\varepsilon(M))$ on $\mathfrak{X}$ satisfying the following axioms:

- $\varepsilon(0_S) = 1^*$ and $\varepsilon(1_S) = 1^*$
- $\varepsilon(M_1 \cap M_2) \geq \varepsilon(M_1) \wedge \varepsilon(M_2)$ for any sets $M_1, M_2 \in \mathcal{NTS}(\mathfrak{X})$.
- $\varepsilon(\cup_{i \in \eta} M_i) \geq \bigwedge_{i \in \eta} (\varepsilon(M_i))$ for $\{M_i; i \in \eta\} \subseteq \mathcal{NTS}(\mathfrak{X})$.

The collection $(\mathfrak{X}, \varepsilon)$ is said to be an $\mathcal{S}\text{-}\mathcal{NTS}$. For any $M \in \mathcal{NTS}(\mathfrak{X})$, the number $\mathcal{T}_\varepsilon(M)$ is known to be the truth-membership function of M , $\mathcal{I}_\varepsilon(M)$ is called the indeterminacy-membership function and $\mathcal{F}_\varepsilon(M)$ is called the falsity-membership function of M .

Definition 2.11 (Smarandache, 2005).

Consider $\mathfrak{X}$ be a non-empty set and $\varepsilon \subset \mathcal{NTS}$. Then (τ, ε) is said to be a Neutrosophic Topological Space ($\mathcal{NTS}$) in the sense of change if it satisfies the below axioms:

- $0_S, 1_S \in \varepsilon$
- for any $M_1, M_2 \in \varepsilon$, implies $M_1 \cap M_2 \in \varepsilon$.
- for any $\{M_\theta | \theta \in \mathcal{I}\}$ implies $\cup_{\theta \in \mathcal{I}} M_\theta \in \varepsilon$.

Temporal Neutrosophic Set

Definition 3.1

Consider $\mathfrak{U}$ be a universe and $\mathcal{T}$ be a non-empty set. Then the elements of $\mathcal{T}$ "time periods". Based on the definition of $\mathcal{NTS}$, a $\mathcal{TNTS}$ is described as the corresponding:

$$M(\mathcal{T}) = \left\{ \begin{pmatrix} x, \mathcal{T}_M(x, \varsigma), \\ \mathcal{I}_M(x, \varsigma), \\ \mathcal{F}_M(x, \varsigma) \end{pmatrix} : \mathfrak{U} \times \mathcal{T} \right\}$$

where

- $M \subseteq \mathfrak{U}$ is a fixed set.
- $\mathcal{T}_M(x, \varsigma) + \mathcal{I}_M(x, \varsigma) + \mathcal{F}_M(x, \varsigma) \leq 3$ for every $(x, \varsigma) \in \mathfrak{U} \times \mathcal{T}$

$\mathcal{T}_M(x, \varsigma), \mathcal{I}_M(x, \varsigma)$ and $\mathcal{F}_M(x, \varsigma)$ are truth, indeterminacy and falsity membership function respectively, of the element $x \in \mathfrak{U}$ at the time period $\varsigma \in \mathcal{T}$.

Example 3.2

Let $\mathfrak{U} = \{\tilde{a}, \tilde{b}, \tilde{c}\}, \varsigma \in \mathcal{T}$

At time $\varsigma = 0$

$$\tilde{A}(0) = \left(\begin{pmatrix} \tilde{a}, (0.9, 0.6, 0.3), \\ \tilde{b}, (0.8, 0.8, 0.3), \\ \tilde{c}, (0.7, 0.8, 0.5) \end{pmatrix} \right) | \tilde{a}, \tilde{b}, \tilde{c} \in \mathfrak{U} \times \mathcal{T}$$

At time $\varsigma = 0$

$$\tilde{B}(0) = \left(\begin{pmatrix} \tilde{a}, (0.5, 0.7, 0.8), \\ \tilde{b}, (0.3, 0.7, 0.9), \\ \tilde{c}, (0.2, 0.8, 0.8) \end{pmatrix} \right) | \tilde{a}, \tilde{b}, \tilde{c} \in \mathfrak{U} \times \mathcal{T}$$

$$\tilde{A}(0) \cup \tilde{B}(0) = \left(\begin{pmatrix} \tilde{a}, (0.9, 0.7, 0.3), \\ \tilde{b}, (0.8, 0.8, 0.3), \\ \tilde{c}, (0.7, 0.8, 0.5) \end{pmatrix} \right) | \tilde{a}, \tilde{b}, \tilde{c} \in \mathfrak{U} \times \mathcal{T}$$

$$\tilde{A}(0) \cap \tilde{B}(0) = \left(\begin{pmatrix} \tilde{a}, (0.5, 0.6, 0.8), \\ \tilde{b}, (0.3, 0.7, 0.9), \\ \tilde{c}, (0.2, 0.8, 0.8) \end{pmatrix} \right) | \tilde{a}, \tilde{b}, \tilde{c} \in \mathfrak{U} \times \mathcal{T}$$

At time $\varsigma = 1$

$$\tilde{A}(1) = \left(\begin{pmatrix} \tilde{a}, (0.5, 0.6, 0.8), \\ \tilde{b}, (0.2, 0.8, 0.7), \\ \tilde{c}, (0.3, 0.7, 0.9) \end{pmatrix} \right) | \tilde{a}, \tilde{b}, \tilde{c} \in \mathfrak{U} \times \mathcal{T}$$

$$\tilde{A}(0) \cap \tilde{A}(1) = \left(\begin{pmatrix} \tilde{a}, (0.5, 0.6, 0.8), \\ \tilde{b}, (0.2, 0.8, 0.7), \\ \tilde{c}, (0.3, 0.7, 0.9) \end{pmatrix} \right) | \tilde{a}, \tilde{b}, \tilde{c} \in \mathfrak{U} \times \mathcal{T}$$

$$\tilde{A}(0) \cup \tilde{A}(1) = \left(\begin{pmatrix} \tilde{a}, (0.9, 0.6, 0.3), \\ \tilde{b}, (0.8, 0.8, 0.3), \\ \tilde{c}, (0.7, 0.8, 0.5) \end{pmatrix} \right) | \tilde{a}, \tilde{b}, \tilde{c} \in \mathfrak{U} \times \mathcal{T}$$

where $(0 \cup 1) \in \mathcal{T}$, that is time is 0 to 1

Definition 3.3

Assume that

$$M(\mathcal{T}') = \left\{ \begin{pmatrix} x, \mathcal{T}_M(x, \varsigma), \\ \mathcal{I}_M(x, \varsigma), \\ \mathcal{F}_M(x, \varsigma) \end{pmatrix} : (x, \varsigma) \in \mathfrak{X} \times \mathcal{T}' \right\}$$

and

$$N(\mathcal{T}'') = \left\{ \begin{pmatrix} x, \mathcal{T}_N(x, \varsigma), \\ \mathcal{I}_N(x, \varsigma), \\ \mathcal{F}_N(x, \varsigma) \end{pmatrix} : (x, \varsigma) \in \mathfrak{X} \times \mathcal{T}'' \right\}$$

where $\mathcal{T}'$ and $\mathcal{T}''$ have finite number of distinct time-elements or they are time intervals. Then,

$$\begin{aligned}
& M(\mathcal{T}') \cap N(\mathcal{T}'') \\
&= \left\{ \begin{pmatrix} x, \min(\bar{\mathcal{T}}_M(x, \varsigma), \bar{\mathcal{T}}_N(x, \varsigma)), \\ \min(\bar{\mathcal{T}}_M(x, \varsigma), \bar{\mathcal{T}}_N(x, \varsigma)), \\ \max(\bar{\mathcal{F}}_M(x, \varsigma), \bar{\mathcal{F}}_N(x, \varsigma)) \end{pmatrix} : (x, \varsigma) \right. \\
&\left. \in \mathfrak{X} \times (\mathcal{T}' \cup \mathcal{T}'') \right\} \\
& M(\mathcal{T}') \cup N(\mathcal{T}'') \\
&= \left\{ \begin{pmatrix} x, \max(\bar{\mathcal{T}}_M(x, \varsigma), \bar{\mathcal{T}}_N(x, \varsigma)), \\ \max(\bar{\mathcal{T}}_M(x, \varsigma), \bar{\mathcal{T}}_N(x, \varsigma)), \\ \min(\bar{\mathcal{F}}_M(x, \varsigma), \bar{\mathcal{F}}_N(x, \varsigma)) \end{pmatrix} : (x, \varsigma) \right. \\
&\left. \in \mathfrak{X} \times (\mathcal{T}' \cup \mathcal{T}'') \right\}
\end{aligned}$$

Subsets of $\mathcal{TNS}$ is described as:

$$\begin{aligned}
& M(\mathcal{T}') \subseteq N(\mathcal{T}'') \Leftrightarrow \bar{\mathcal{T}}_M(x, \varsigma) \leq \bar{\mathcal{T}}_N(x, \varsigma), \\
& \bar{\mathcal{T}}_M(x, \varsigma) \leq \bar{\mathcal{T}}_N(x, \varsigma) \text{ and } \bar{\mathcal{F}}_M(x, \varsigma) \geq \bar{\mathcal{F}}_N(x, \varsigma) \text{ for} \\
& \text{every } (x, \varsigma) \in \mathfrak{X} \times (\mathcal{T}' \cup \mathcal{T}'')
\end{aligned}$$

where

$$\begin{aligned}
\bar{\mathcal{T}}_M(x, \varsigma) &= \begin{cases} \mathcal{T}_M(x, \varsigma), & \text{if } \varsigma \in \mathcal{T}' \\ 0 & \text{if } \varsigma \in \mathcal{T}'' - \mathcal{T}' \end{cases} \\
\bar{\mathcal{T}}_N(x, \varsigma) &= \begin{cases} \mathcal{T}_N(x, \varsigma), & \text{if } \varsigma \in \mathcal{T}'' \\ 0 & \text{if } \varsigma \in \mathcal{T}' - \mathcal{T}'' \end{cases} \\
\bar{\mathcal{T}}_M(x, \varsigma) &= \begin{cases} \mathcal{T}_M(x, \varsigma), & \text{if } \varsigma \in \mathcal{T}' \\ 0.5 & \text{if } \varsigma \in \mathcal{T}'' - \mathcal{T}' \end{cases} \\
\bar{\mathcal{T}}_N(x, \varsigma) &= \begin{cases} \mathcal{T}_N(x, \varsigma), & \text{if } \varsigma \in \mathcal{T}'' \\ 0.5 & \text{if } \varsigma \in \mathcal{T}' - \mathcal{T}'' \end{cases} \\
\bar{\mathcal{F}}_M(x, \varsigma) &= \begin{cases} \mathcal{F}_M(x, \varsigma), & \text{if } \varsigma \in \mathcal{T}' \\ 1 & \text{if } \varsigma \in \mathcal{T}'' - \mathcal{T}' \end{cases} \\
\bar{\mathcal{F}}_N(x, \varsigma) &= \begin{cases} \mathcal{F}_N(x, \varsigma), & \text{if } \varsigma \in \mathcal{T}'' \\ 1 & \text{if } \varsigma \in \mathcal{T}' - \mathcal{T}'' \end{cases}
\end{aligned}$$

It is clear to observe that $\bar{\mathcal{T}}_M(x, \varsigma) = \mathcal{T}_M(x, \varsigma)$, $\bar{\mathcal{F}}_M(x, \varsigma) = \mathcal{F}_M(x, \varsigma)$, $\bar{\mathcal{T}}_N(x, \varsigma) = \mathcal{T}_N(x, \varsigma)$ when $\mathcal{T}' = \mathcal{T}''$.

Let η be the set which is arbitrary. Then $\mathcal{T} = \bigcup_{i \in \eta} \mathcal{T}_i$ where $\mathcal{T}_i$ is a time frame allocated to each $i \in \eta$. Then, the union and intersection of $\mathcal{TNS}$ s family

$$\begin{aligned}
\mathcal{F} &= \left\{ M_i(\mathcal{T}_i) = \begin{pmatrix} x, \mathcal{T}_{M_i}(x, \varsigma), \\ \mathcal{T}_{M_i}(x, \varsigma), \\ \mathcal{F}_{M_i}(x, \varsigma) \end{pmatrix} : (x, \varsigma) \in \mathfrak{X} \times \mathcal{T}_i, i \in \eta \right\} \text{ as:} \\
\bigcup_{i \in \eta} M(\mathcal{T}_i) &= \left\{ \begin{pmatrix} x, \max_{i \in \eta}(\bar{\mathcal{T}}_{M_i}(x, \varsigma)), \\ \max_{i \in \eta}(\bar{\mathcal{T}}_{M_i}(x, \varsigma)), \\ \min_{i \in \eta}(\bar{\mathcal{F}}_{M_i}(x, \varsigma)) \end{pmatrix} : (x, \eta) \in \mathfrak{X} \times \mathcal{T} \right\} \\
\bigcap_{i \in \eta} M(\mathcal{T}_i) &= \left\{ \begin{pmatrix} x, \min_{i \in \eta}(\bar{\mathcal{T}}_{M_i}(x, \varsigma)), \\ \min_{i \in \eta}(\bar{\mathcal{T}}_{M_i}(x, \varsigma)), \\ \max_{i \in \eta}(\bar{\mathcal{F}}_{M_i}(x, \varsigma)) \end{pmatrix} : (x, \eta) \in \mathfrak{X} \times \mathcal{T} \right\}
\end{aligned}$$

$$= \left\{ \begin{pmatrix} x, \min_{i \in \eta}(\bar{\mathcal{T}}_{M_i}(x, \varsigma)), \\ \min_{i \in \eta}(\bar{\mathcal{T}}_{M_i}(x, \varsigma)), \\ \max_{i \in \eta}(\bar{\mathcal{F}}_{M_i}(x, \varsigma)) \end{pmatrix} : (x, \varsigma) \in \mathfrak{X} \times \mathcal{T} \right\}$$

where

$$\begin{aligned}
\bar{\mathcal{T}}_{M_i}(x, \varsigma) &= \begin{cases} \mathcal{T}_{M_i}(x, \varsigma), & \text{if } \varsigma \in \mathcal{T}_i \\ 0 & \text{if } \varsigma \in \mathcal{T} - \mathcal{T}_i \end{cases} \\
\bar{\mathcal{T}}_{M_i}(x, \varsigma) &= \begin{cases} \mathcal{T}_{M_i}(x, \varsigma), & \text{if } \varsigma \in \mathcal{T}_i \\ 0.5 & \text{if } \varsigma \in \mathcal{T} - \mathcal{T}_i \end{cases} \\
\bar{\mathcal{F}}_{M_i}(x, \varsigma) &= \begin{cases} \mathcal{F}_{M_i}(x, \varsigma), & \text{if } \varsigma \in \mathcal{T}_i \\ 1 & \text{if } \varsigma \in \mathcal{T} - \mathcal{T}_i \end{cases}
\end{aligned}$$

The previously mentioned operations are defined over all of the time period.

The definition that follows will define these operations for a individual time period.

Definition 3.4

Let

$$\begin{aligned}
M(\mathcal{T}') &= \left\{ \begin{pmatrix} x, \mathcal{T}_M(x, \varsigma), \\ \mathcal{T}_M(x, \varsigma), \\ \mathcal{F}_M(x, \varsigma) \end{pmatrix} : (x, \varsigma) \in \mathfrak{X} \times \mathcal{T}' \right\} \\
N(\mathcal{T}'') &= \left\{ \begin{pmatrix} x, \mathcal{T}_N(x, \varsigma), \\ \mathcal{T}_N(x, \varsigma), \\ \mathcal{F}_N(x, \varsigma) \end{pmatrix} : (x, \varsigma) \in \mathfrak{X} \times \mathcal{T}'' \right\}
\end{aligned}$$

where $\mathcal{T}'$ and $\mathcal{T}''$ have finite number of distinct time-elements or they are time intervals. The definitions of instant union and instant intersection of $\mathcal{TNS}$ s are listed below:

$$\begin{aligned}
& M(\mathcal{T}') \cap_{\varsigma_0} N(\mathcal{T}'') \\
&= \left\{ \begin{pmatrix} x, \min(\bar{\mathcal{T}}_M(x, \varsigma_0), \bar{\mathcal{T}}_N(x, \varsigma_0)), \\ \min(\bar{\mathcal{T}}_M(x, \varsigma_0), \bar{\mathcal{T}}_N(x, \varsigma_0)), \\ \max(\bar{\mathcal{F}}_M(x, \varsigma_0), \bar{\mathcal{F}}_N(x, \varsigma_0)) \end{pmatrix} : (x, \varsigma_0) \right. \\
&\left. \in \mathfrak{X} \times (\mathcal{T}' \cup \mathcal{T}'') \right\}
\end{aligned}$$

$$\begin{aligned}
& M(\mathcal{T}') \cup_{\varsigma_0} N(\mathcal{T}'') \\
&= \left\{ \begin{pmatrix} x, \max(\bar{\mathcal{T}}_M(x, \varsigma_0), \bar{\mathcal{T}}_N(x, \varsigma_0)), \\ \max(\bar{\mathcal{T}}_M(x, \varsigma_0), \bar{\mathcal{T}}_N(x, \varsigma_0)), \\ \min(\bar{\mathcal{F}}_M(x, \varsigma_0), \bar{\mathcal{F}}_N(x, \varsigma_0)) \end{pmatrix} : (x, \varsigma_0) \right. \\
&\left. \in \mathfrak{X} \times (\mathcal{T}' \cup \mathcal{T}'') \right\}
\end{aligned}$$

Also, from subset definitions in $\mathcal{NS}$ theory, instant subsets of $\mathcal{TNS}$ described as:

$M(\mathcal{T}') \subseteq_{\varsigma_0} N(\mathcal{T}'') \Leftrightarrow \bar{\mathcal{T}}_M(x, \varsigma_0) \leq \bar{\mathcal{T}}_N(x, \varsigma_0)$,
 $\bar{\mathcal{T}}_M(x, \varsigma_0) \leq \bar{\mathcal{T}}_N(x, \varsigma_0)$, $\bar{\mathcal{F}}_M(x, \varsigma_0) \geq \bar{\mathcal{F}}_N(x, \varsigma_0)$ for
every $(x, \varsigma_0) \in \mathfrak{X} \times (\mathcal{T}' \cup \mathcal{T}'')$ where

$$\begin{aligned}\bar{\mathcal{T}}_M(x, \varsigma_0) &= \begin{cases} \mathcal{T}_M(x, \varsigma_0), & \text{if } \varsigma_0 \in \mathcal{T}' \\ 0 & \text{if } \varsigma_0 \in \mathcal{T}'' - \mathcal{T}' \end{cases} \\ \bar{\mathcal{T}}_N(x, \varsigma_0) &= \begin{cases} \mathcal{T}_N(x, \varsigma_0), & \text{if } \varsigma_0 \in \mathcal{T}'' \\ 0 & \text{if } \varsigma_0 \in \mathcal{T}' - \mathcal{T}'' \end{cases} \\ \bar{\mathcal{J}}_M(x, \varsigma_0) &= \begin{cases} \mathcal{J}_M(x, \varsigma_0), & \text{if } \varsigma_0 \in \mathcal{T}' \\ 0.5 & \text{if } \varsigma_0 \in \mathcal{T}'' - \mathcal{T}' \end{cases} \\ \bar{\mathcal{J}}_N(x, \varsigma_0) &= \begin{cases} \mathcal{J}_N(x, \varsigma_0), & \text{if } \varsigma_0 \in \mathcal{T}'' \\ 0.5 & \text{if } \varsigma_0 \in \mathcal{T}' - \mathcal{T}'' \end{cases} \\ \bar{\mathcal{F}}_M(x, \varsigma_0) &= \begin{cases} \mathcal{F}_M(x, \varsigma_0), & \text{if } \varsigma_0 \in \mathcal{T}' \\ 1 & \text{if } \varsigma_0 \in \mathcal{T}'' - \mathcal{T}' \end{cases} \\ \bar{\mathcal{F}}_N(x, \varsigma_0) &= \begin{cases} \mathcal{F}_N(x, \varsigma_0), & \text{if } \varsigma_0 \in \mathcal{T}'' \\ 1 & \text{if } \varsigma_0 \in \mathcal{T}' - \mathcal{T}'' \end{cases}\end{aligned}$$

Consider η be a set of arbitrary index. Then we
define that $\mathcal{T} = \bigcup_{i \in \eta} \mathcal{T}_i$ where $\mathcal{T}_i$ is a time set for
each $i \in \eta$. Then, the union and intersection of
 $\mathcal{TNS}$ s collection

$$\begin{aligned}\mathcal{F}_{\varsigma_0} &= \left\{ M_i(\mathcal{T}_i) = \begin{pmatrix} x, \mathcal{T}_{M_i}(x, \varsigma_0), \\ \mathcal{J}_{M_i}(x, \varsigma_0), \\ \mathcal{F}_{M_i}(x, \varsigma_0) \end{pmatrix} : (x, \varsigma_0) \in \mathfrak{X} \times \right. \\ &\left. \mathcal{T}_i, i \in \eta \right\} \\ \bigcup_{i \in \eta}^{\varsigma_0} M(\mathcal{T}_i) &= \left\{ \begin{pmatrix} x, \max_{i \in \eta} (\bar{\mathcal{T}}_{M_i}(x, \varsigma_0)), \\ \max_{i \in \eta} (\bar{\mathcal{J}}_{M_i}(x, \varsigma_0)), \\ \min_{i \in \eta} (\bar{\mathcal{F}}_{M_i}(x, \varsigma_0)) \end{pmatrix} : (x, \varsigma_0) \right. \\ &\left. \in \mathfrak{X} \times \mathcal{T} \right\} \\ \bigcap_{i \in \eta}^{\varsigma_0} M(\mathcal{T}_i) &= \left\{ \begin{pmatrix} x, \min_{i \in \eta} (\bar{\mathcal{T}}_{M_i}(x, \varsigma_0)), \\ \min_{i \in \eta} (\bar{\mathcal{J}}_{M_i}(x, \varsigma_0)), \\ \max_{i \in \eta} (\bar{\mathcal{F}}_{M_i}(x, \varsigma_0)) \end{pmatrix} : (x, \varsigma_0) \right. \\ &\left. \in \mathfrak{X} \times \mathcal{T} \right\}\end{aligned}$$

where

$$\begin{aligned}\bar{\mathcal{T}}_{M_i}(x, \varsigma_0) &= \begin{cases} \mathcal{T}_{M_i}(x, \varsigma_0), & \text{if } \varsigma_0 \in \mathcal{T}_i \\ 0 & \text{if } \varsigma_0 \in \mathcal{T} - \mathcal{T}_i \end{cases} \\ \bar{\mathcal{J}}_{M_i}(x, \varsigma_0) &= \begin{cases} \mathcal{J}_{M_i}(x, \varsigma_0), & \text{if } \varsigma_0 \in \mathcal{T}_i \\ 0.5 & \text{if } \varsigma_0 \in \mathcal{T} - \mathcal{T}_i \end{cases} \\ \bar{\mathcal{F}}_{M_i}(x, \varsigma_0) &= \begin{cases} \mathcal{F}_{M_i}(x, \varsigma_0), & \text{if } \varsigma_0 \in \mathcal{T}_i \\ 1 & \text{if } \varsigma_0 \in \mathcal{T} - \mathcal{T}_i \end{cases}\end{aligned}$$

these $\mathcal{TNS}$ operators can be seen as $\mathcal{NS}$
operators over $\mathcal{TNS}$ s. Since $\mathcal{TNS}$ are specified for
a single time period.

Definition 3.5

0_S^ς and $1_S^\varsigma \in \mathcal{TNS}(\mathfrak{X}, \mathcal{T})$ are defined as:
 $0_S^\varsigma = \{(x, 0, 0, 1) : (x, \varsigma) \in \mathfrak{X} \times \mathcal{T}\}$ and

$1_S^\varsigma = \{(x, 1, 1, 0) : (x, \varsigma) \in \mathfrak{X} \times \mathcal{T}\}$
for every instance of time ς , that is

$$\begin{aligned}\mathcal{T}_{0_S^\varsigma}(x, \varsigma) &= 0, \quad \mathcal{J}_{0_S^\varsigma}(x, \varsigma) = 0, \quad \mathcal{F}_{0_S^\varsigma}(x, \varsigma) = 1, \\ \mathcal{T}_{1_S^\varsigma}(x, \varsigma) &= 1, \quad \mathcal{J}_{1_S^\varsigma}(x, \varsigma) = 1, \quad \mathcal{F}_{1_S^\varsigma}(x, \varsigma) = 0 \quad \text{for} \\ &\text{each } (x, \varsigma) \in \mathfrak{X} \times \mathcal{T}.\end{aligned}$$

Definition 3.6

$0_S^{\varsigma_0}$ and $1_S^{\varsigma_0} \in \mathcal{TNS}(\mathfrak{X}, \mathcal{T})$ are defined as:

$$\begin{aligned}0_S^{\varsigma_0} &= \left\{ \begin{pmatrix} x, \mathcal{T}_{0_S^{\varsigma_0}}(x, \varsigma), \\ \mathcal{J}_{0_S^{\varsigma_0}}(x, \varsigma), \\ \mathcal{F}_{0_S^{\varsigma_0}}(x, \varsigma) \end{pmatrix} : (x, \varsigma) \in \mathfrak{X} \times \mathcal{T} \right\} \\ 1_S^{\varsigma_0} &= \left\{ \begin{pmatrix} x, \mathcal{T}_{1_S^{\varsigma_0}}(x, \varsigma), \\ \mathcal{J}_{1_S^{\varsigma_0}}(x, \varsigma), \\ \mathcal{F}_{1_S^{\varsigma_0}}(x, \varsigma) \end{pmatrix} : (x, \varsigma) \in \mathfrak{X} \times \mathcal{T} \right\}\end{aligned}$$

for individual time period $\varsigma_0 \in \mathcal{T}$, i.e.

$$\begin{aligned}\mathcal{T}_{0_S^{\varsigma_0}}(x, \varsigma_0) &= 0, \quad \mathcal{J}_{0_S^{\varsigma_0}}(x, \varsigma_0) = 0, \\ \mathcal{F}_{0_S^{\varsigma_0}}(x, \varsigma_0) &= 1 \\ \text{and} \\ \mathcal{T}_{1_S^{\varsigma_0}}(x, \varsigma_0) &= 1, \quad \mathcal{J}_{1_S^{\varsigma_0}}(x, \varsigma_0) = 1, \\ \mathcal{F}_{1_S^{\varsigma_0}}(x, \varsigma_0) &= 0 \text{ for each } (x, \varsigma_0) \in \mathfrak{X} \times \{\varsigma_0\}.\end{aligned}$$

Temporal Neutrosophic Topology in Sostak's sense ($\mathcal{ST-TNS}$)

Here we have defined the Temporal
Neutrosophic Topology in Sostak's sense. In this
Šostak's approach, grades are assigned to open sets.
When incorporating time moments, assigning
grades to open sets provides greater clarity in
addressing uncertainty in real world problem.

Definition 3.7

Temporal Neutrosophic Topology in Šostak's
sense (briefly, $\mathcal{ST-TNS}$) on a non-empty set $\tilde{T}$ is
a temporal neutrosophic set, ε_ς defined with

$\varepsilon_\varsigma(M) = (\mathcal{T}_{\varepsilon_\varsigma}(M), \mathcal{J}_{\varepsilon_\varsigma}(M), \mathcal{F}_{\varepsilon_\varsigma}(M))$ on $\tilde{T}$
satisfying the axioms listed below for every time
moment ς

- $\varepsilon_\varsigma(0_S^{\varsigma_0}) = 1^*$ and $\varepsilon_\varsigma(1_S^{\varsigma_0}) = 1^*$
- $\varepsilon_\varsigma(M_1 \cap M_2) \geq \varepsilon_\varsigma(M_1) \wedge \varepsilon_\varsigma(M_2)$ for any
sets $M_1, M_2 \in \mathcal{NS}(\tilde{T})$.
- $\varepsilon_\varsigma(\bigcup_{i \in \eta} M_i) \geq \bigwedge_{i \in \eta} (\varepsilon_\varsigma(M_i))$ for $\{M_i; i \in \eta\} \subseteq$
 $\mathcal{TNS}(\tilde{T})$.

the pair $(\tilde{T}, \varepsilon_\varsigma)$ refers to a $\mathcal{ST-TNS}$. For any
 $M \in \mathcal{ST-TNS}(\tilde{T}, \mathcal{T})$, and the number $\mathcal{T}_{\varepsilon_\varsigma}(M)$ is truth,
 $\mathcal{J}_{\varepsilon_\varsigma}(M)$ is indeterminacy and $\mathcal{F}_{\varepsilon_\varsigma}(M)$ is falsity
membership functions of M .

Proposition 3.8

Consider that $(\tilde{T}, \varepsilon_\zeta)$ be a $\mathcal{ST} - \mathcal{JNS}$ on $\tilde{T}$ and $\mathcal{T}$ be a set a time-moment. Then $(\tilde{T}, \wedge \varepsilon_\zeta)$ by expressed

$$\wedge \varepsilon_\zeta(M) = \begin{pmatrix} \min_{t \in \mathcal{T}'} \mathcal{J}_{\varepsilon_\zeta}(M), \\ \min_{t \in \mathcal{T}} \mathcal{J}_{\varepsilon_\zeta}(M), \\ \max_{t \in \mathcal{T}} \mathcal{J}_{\varepsilon_\zeta}(M) \end{pmatrix}$$

is a neutrosophic topology on $\mathcal{JNS}(\tilde{T}, \mathcal{T})$ in Sostak's sense.

Definition 3.9

Consider that $(\tilde{T}, \varepsilon_\zeta)$ be a $\mathcal{ST} - \mathcal{JNS}$ and $M \in \mathcal{JNS}(\tilde{T}, \mathcal{T})$. Then Establish the precise interior and closure of M at any given time ζ_0 based on ε_ζ correspondingly as:

$Cl^{\zeta_0}(M) = \cap \{ \xi \in \mathcal{JNS}(\tilde{T}); \varepsilon_{\zeta_0}^*(\xi) > 0_s, M \subseteq \xi \}$
 $int^{\zeta_0}(M) = \cup \{ \xi \in \mathcal{JNS}(\tilde{T}); \varepsilon_{\zeta_0}^*(\xi) > 0_s, \xi \subseteq M \}$
 As contracted to that, (σ, τ, ν) -instant closure and (σ, τ, ν) -instant interior of M are described by:

$$Cl_{(\sigma, \tau, \nu)}^{\zeta_0}(M) = \cap \left\{ \begin{array}{l} \xi \in \mathcal{JNS}(\tilde{T}); \\ \varepsilon_{\zeta_0}^*(\xi) > \langle \sigma, \tau, \nu \rangle, M \subseteq \xi \end{array} \right\}$$

$$int_{(\sigma, \tau, \nu)}^{\zeta_0}(M) = \cup \left\{ \begin{array}{l} \xi \in \mathcal{JNS}(\tilde{T}); \\ \varepsilon_{\zeta_0}^*(\xi) > \langle \sigma, \tau, \nu \rangle, \xi \subseteq M \end{array} \right\}$$

where

$\sigma \in [0, 1], \tau \in [0, 1], \nu \in [0, 1]$ with $\sigma + \tau + \nu \leq 3$.

Definition 3.10

Assume that $(\tilde{T}, \varepsilon_\zeta)$ and $(\tilde{U}, \pi_\zeta)$ be $\mathcal{ST} - \mathcal{JNS}$ s for non-empty sets $\tilde{T}, \tilde{U}$ and time sets $\mathcal{T}'$ and $\mathcal{T}''$. Let $f: \tilde{T} \rightarrow \tilde{U}$ be a function. Then,

- i. The preimage of $N \in \mathcal{JNS}(\tilde{U}, \mathcal{T}'')$ under f at time period ζ is described as

$$f^{-1}(M) = \left\{ \begin{pmatrix} u, f(\overline{\mathcal{T}}_M)(u, \zeta), \\ f(\overline{\mathcal{T}}_M)(u, \zeta), \\ f(\overline{\mathcal{F}}_M)(u, \zeta) \end{pmatrix} : u \in \tilde{U} \right\}$$

Where

$$\overline{\mathcal{T}}_N(f(t), \zeta) = \begin{cases} \mathcal{J}_N(f(t), \zeta), & \text{if } \zeta \in \mathcal{T}'' \\ 0, & \text{if } \zeta \in \mathcal{T}' - \mathcal{T}'' \end{cases}$$

$$\overline{\mathcal{J}}_N(f(t), \zeta) = \begin{cases} \mathcal{J}_N(f(t), \zeta), & \text{if } \zeta \in \mathcal{T}'' \\ 0, & \text{if } \zeta \in \mathcal{T}' - \mathcal{T}'' \end{cases}$$

$$\overline{\mathcal{F}}_N(f(t), \zeta) = \begin{cases} \mathcal{F}_N(f(t), \zeta), & \text{if } \zeta \in \mathcal{T}'' \\ 1, & \text{if } \zeta \in \mathcal{T}' - \mathcal{T}'' \end{cases}$$

- ii. The image of $M \in \mathcal{JNS}(\tilde{T}, \mathcal{T}')$ under f at time period ζ is described as

$$f^{-1}(M) = \left\{ \begin{pmatrix} \tilde{U}, f(\overline{\mathcal{T}}_M)(\tilde{U}, \zeta), \\ f(\overline{\mathcal{T}}_M)(\tilde{U}, \zeta), \\ f(\overline{\mathcal{F}}_M)(\tilde{U}, \zeta) \end{pmatrix} : u \in \tilde{U} \right\}$$

where

$$f(\overline{\mathcal{T}}_M)(u, \zeta) = \begin{cases} f(\mathcal{T}_M)(u, \zeta), & \text{if } \zeta \in \mathcal{T}' \\ 0, & \text{if } \zeta \in \mathcal{T}'' - \mathcal{T}' \end{cases}$$

$$f(\overline{\mathcal{J}}_M)(u, \zeta) = \begin{cases} f(\mathcal{J}_M)(u, \zeta), & \text{if } \zeta \in \mathcal{T}' \\ 0, & \text{if } \zeta \in \mathcal{T}'' - \mathcal{T}' \end{cases}$$

$$f(\overline{\mathcal{F}}_M)(u, \zeta) = \begin{cases} 1 - f(1 - \mathcal{T}_M)(u, \zeta), & \text{if } \zeta \in \mathcal{T}' \\ 0, & \text{if } \zeta \in \mathcal{T}'' - \mathcal{T}' \end{cases}$$

If $\mathcal{T}'' - \mathcal{T}'$, It is obvious that

$$f^{-1}(N) = \left\{ \begin{pmatrix} t, \mathcal{J}_N(f(t), \zeta), \mathcal{J}_N(f(t), \zeta), \mathcal{F}_N(f(t), \zeta) \end{pmatrix} : t \in \tilde{T} \right\}$$

$$f(M) = \left\{ \begin{pmatrix} u, f(\mathcal{T}_M)(u, \zeta), f(\mathcal{J}_M)(u, \zeta), f(\mathcal{F}_M)(u, \zeta) \end{pmatrix} : u \in \tilde{U} \right\}$$

Let $(\tilde{T}, \varepsilon_\zeta)$ and $(\tilde{U}, \pi_\zeta)$ $\mathcal{ST} - \mathcal{JNS}$ s for a non-empty set $\tilde{T}, \tilde{U}$ and time span $\mathcal{T}$.

If $\varepsilon_\zeta(f^{-1}(N)) \geq \pi_\zeta(N)$ for $\zeta \in \mathcal{T}$ and each $N \in \mathcal{JNS}(\tilde{U}, \mathcal{T})$, f is said to be Temporal Neutrosophic Continuous (TNCont) function at time period ζ .

If f is TNCont function at each time period, f is known as overall NCont function.

Instead of, if $\pi_\zeta(f(M)) \geq \varepsilon_\zeta(N)$ for $\zeta \in \mathcal{T}$ and each $M \in \mathcal{JNS}(\tilde{T}, \mathcal{T}')$, f is said to be temporal neutrosophic function which is open at the moment of time ζ .

If f is neutrosophic open function which is temporal at each time period, f is said to be neutrosophic and overall open function.

Temporal Neutrosophic Topology in Chang's sense ($\mathcal{CT} - \mathcal{JNS}$)

Definition 3.11

A $\mathcal{CT} - \mathcal{JNS}$ on a non-empty set $\tilde{T}$ is a family ε_{ζ_0} of $\mathcal{JFS}$ s meeting the following axioms for fixed time period ζ_0

- i. $0_s^{\zeta_0} \in \varepsilon_{\zeta_0}$ and $1_s^{\zeta_0} \in \varepsilon_{\zeta_0}$
- ii. For each $M_1, M_2 \in \varepsilon_{\zeta_0}, \exists \mathcal{F} \in \varepsilon_{\zeta_0}$ such that $\mathcal{J}_{\mathcal{F}}(x, \zeta_0) = \mathcal{J}_{M_1 \cap \zeta_0 M_2}(t, \zeta_0), \mathcal{J}_{\mathcal{F}}(t, \zeta_0) = \mathcal{J}_{M_1 \cap \zeta_0 M_2}(t, \zeta_0)$ and $\mathcal{F}_{\mathcal{F}}(t, \zeta_0) = \mathcal{F}_{M_1 \cap \zeta_0 M_2}(t, \zeta_0)$ for each $(t, \zeta_0) \in \tilde{T} \times \{\zeta_0\}$
- iii. For any family $\{M_i; i \in I\} \in \varepsilon_{\zeta_0}$, lies a $G \in \varepsilon_{\zeta_0}$ such as $\mathcal{J}_G(t, \zeta_0) = \mathcal{J}_{\cup_{i \in I} M_i}(t, \zeta_0), \mathcal{J}_G(t, \zeta_0) = \mathcal{J}_{\cup_{i \in I} M_i}(t, \zeta_0)$ and $\mathcal{F}_G(t, \zeta_0) = \mathcal{F}_{\cup_{i \in I} M_i}(t, \zeta_0)$ for each $(t, \zeta_0) \in \tilde{T} \times \{\zeta_0\}$.

The pair $((\tilde{T}, \mathcal{T}), \varepsilon_{\zeta_0})$ is said to be $\mathcal{CT} - \mathcal{JNS}$.

A member of ε_{ζ_0} is called Temporal Neutrosophic Open Set ($\mathcal{JNOs}$). The member of a complement ε_{ζ_0} is a Temporal Neutrosophic Closed Set ($\mathcal{JNCS}$).

Proposition 3.12

Let a non-empty set $\tilde{T}$ and consider ε_{ζ_0} is an $\mathcal{T}\text{-}\mathcal{TNS}$ and time period set $\mathcal{T}$, Then introduce $\mathcal{NS}$'s from each $M \in \varepsilon_{\zeta_0} \mathcal{TNS}$ s by following way:

$$\mathcal{T}_{\tilde{M}}(t) = \mathcal{T}_{M_i}(t, \zeta_0), \quad \mathcal{J}_{\tilde{M}}(t) = \mathcal{J}_{M_i}(t, \zeta_0) \quad \text{and} \quad \mathcal{F}_{\tilde{M}}(t) = \mathcal{F}_{M_i}(t, \zeta_0).$$

So that the new family $\varepsilon^{\zeta_0} = \{\tilde{M} : M \in \varepsilon_{\zeta_0}\}$ obtained from ε_{ζ_0} is an neutrosophic topology in Chang's sense.

Note:

In the topology the constant function described by chang's are noted as continuous, according to Lowen (Lowen, 1976). As a result, Lowen amended Chang's definition first axiom to address this flaw. These two definitions are treated equally in the literature.

Temporal Neutrosophic Topology in Lowen's sense ($\mathcal{LT}\text{-}\mathcal{TNS}$)

Definition 3.13

A $\mathcal{TNT}$ by Lowen's (briefly, $\mathcal{LT}\text{-}\mathcal{TNS}$) on a which is a nonempty set $\tilde{T}$ belongs to a family ε_{ζ_0} of $\mathcal{TNS}$ s considering these assumptions below for permenant time instant ζ_0

- i. For a $\mathcal{N}$ -pair (σ, τ, ν) , let define $\mathcal{TNS}$ s

$$M = (t, \mathcal{T}_M(t, \zeta), \mathcal{J}_M(t, \zeta), \mathcal{F}_M(t, \zeta) : (t, \zeta) \in \tilde{T} \times \mathcal{T})$$
such that $\mathcal{T}_M(t, \zeta_0) = \sigma$, $\mathcal{J}_M(t, \zeta_0) = \tau$, and $\mathcal{F}_M(t, \zeta_0) = \nu, \forall (t, \zeta_0) \in \tilde{T} \times \{\zeta_0\}$
 - ii. For every $M_1, M_2 \in \varepsilon_{\zeta_0}$, there exist a $\mathcal{F} \in \varepsilon_{\zeta_0}$ such that
 - iii. $\mathcal{T}_{\mathcal{F}}(t, \zeta_0) = \mathcal{T}_{M_1 \cap_{\zeta_0} M_2}(t, \zeta_0)$, $\mathcal{J}_{\mathcal{F}}(t, \zeta_0) = \mathcal{J}_{M_1 \cap_{\zeta_0} M_2}(t, \zeta_0)$ and $\mathcal{F}_{\mathcal{F}}(t, \zeta_0) = \mathcal{F}_{M_1 \cap_{\zeta_0} M_2}(t, \zeta_0) \forall (x, \zeta_0) \in \tilde{T} \times \{\zeta_0\}$.
 - iv. Any $\{M_i; i \in I\} \in \varepsilon_{\zeta_0}$, here $G \in \varepsilon_{\zeta_0}$ such that $\mathcal{T}_G(t, \zeta_0) = \mathcal{T}_{\bigcup_{i \in I} M_i}(t, \zeta_0)$, $\mathcal{J}_G(t, \zeta_0) = \mathcal{J}_{\bigcup_{i \in I} M_i}(t, \zeta_0)$ and $\mathcal{F}_G(t, \zeta_0) = \mathcal{F}_{\bigcup_{i \in I} M_i}(t, \zeta_0) \forall (t, \zeta_0) \in \tilde{T} \times \{\zeta_0\}$.
- $((\tilde{T}, \mathcal{T}), \varepsilon_{\zeta_0})$ is given to be $\mathcal{TNTS}$ in Lowen's sense.

Definition 3.14

A not empty set $\tilde{T}$ and consider ε_{ζ_0} is an $\mathcal{TNTS}$ by Chang' and time period set $\mathcal{T}$ and $M \in \varepsilon_{\zeta_0}$. Temporal neutrosophic interior and temporal neutrosophic closure of M defined as follows:

$$\begin{aligned} \text{int}_{\zeta_0}(M) &= \bigcup \{\lambda; \lambda \in \varepsilon_{\zeta_0}, \lambda \subseteq M\} \\ \text{Cl}_{\zeta_0}(A) &= \bigcap \{\mathcal{C}; \mathcal{C} \in \tau_{\zeta_0}, A \subseteq \mathcal{C}\} \end{aligned}$$

Definition 3.15

A not empty set $\tilde{T}$ and consider ε_{ζ_0} be a $\mathcal{TNTS}$ according to Chang's and time period set.

- i. M is a $\mathcal{TNCs}$ in $\varepsilon_{\zeta_0} \Leftrightarrow \text{Cl}_{\zeta_0}(M) = M$
- ii. M is a $\mathcal{TNOs}$ in $\varepsilon_{\zeta_0} \Leftrightarrow \text{int}_{\zeta_0}(M) = M$
- iii. $\text{Cl}_{\zeta_0}(M) = \overline{\text{int}_{\zeta_0}(M)}$ for any $M \in \mathcal{TNS}^{(\tilde{T}, \mathcal{T})}$
- iv. $\text{int}_{\zeta_0}(M) = \overline{\text{Cl}_{\zeta_0}(M)}$ for any $M \in \mathcal{TNS}^{(\tilde{T}, \mathcal{T})}$
- v. $\text{int}_{\zeta_0}(M) \subseteq M$ for any $M \in \mathcal{TNS}^{(\tilde{T}, \mathcal{T})}$
- vi. $M \subseteq \text{Cl}_{\zeta_0}(M)$ for any $M \in \mathcal{TNS}^{(\tilde{T}, \mathcal{T})}$
- vii. $M \subseteq N \Rightarrow \text{int}_{\zeta_0}(M) \subseteq \text{int}_{\zeta_0}(N)$ for any $M, N \in \mathcal{TNS}^{(\tilde{T}, \mathcal{T})}$
- viii. $M \subseteq N \Rightarrow \text{Cl}_{\zeta_0}(M) \subseteq \text{Cl}_{\zeta_0}(N)$ for any $M, N \in \mathcal{TNS}^{(\tilde{T}, \mathcal{T})}$
- ix. $\text{Cl}_{\zeta_0}(\text{Cl}_{\zeta_0}(M)) = \text{Cl}_{\zeta_0}(M)$ for any $M \in \mathcal{TNS}^{(\tilde{T}, \mathcal{T})}$
- x. $\text{int}_{\zeta_0}(\text{int}_{\zeta_0}(M)) = \text{int}_{\zeta_0}(M)$ for any $M \in \mathcal{TNS}^{(\tilde{T}, \mathcal{T})}$
- xi. $\text{int}_{\zeta_0}(M \cap N) = \text{int}_{\zeta_0}(M) \cap \text{int}_{\zeta_0}(N)$ for any $M, N \in \mathcal{TNS}^{(\tilde{T}, \mathcal{T})}$
- xii. $\text{Cl}_{\zeta_0}(M \cup N) = \text{Cl}_{\zeta_0}(M) \cup \text{Cl}_{\zeta_0}(N)$ for any $M, N \in \mathcal{TNS}^{(\tilde{T}, \mathcal{T})}$
- xiii. $\text{int}_{\zeta_0}(1_{\zeta_0}^{\zeta_0}) = 1_{\zeta_0}^{\zeta_0}$
- xiv. $\text{Cl}_{\zeta_0}(0_{\zeta_0}^{\zeta_0}) = 0_{\zeta_0}^{\zeta_0}$

Definition 3.16

Let $((\tilde{T}, \mathcal{T}'), \varepsilon_{\zeta_0})$ and $((\tilde{U}, \mathcal{T}''), \pi_{\zeta_0})$ be two $\mathcal{CT}\text{-}\mathcal{TNS}$ s and consider the function $f: \tilde{T} \rightarrow \tilde{U}$. Then f is known to be a $\mathcal{TNCont}$ at time period ζ_0 if and the temporal preimage of all $\mathcal{TNS}$ in π_{ζ_0} is an $\mathcal{TNS}$ in ε_{ζ_0} .

Definition 3.17

Let $((\tilde{T}, \mathcal{T}'), \varepsilon_{\zeta_0})$ and $((\tilde{U}, \mathcal{T}''), \pi_{\zeta_0})$ be two $\mathcal{CT}\text{-}\mathcal{TNS}$ s and consider the function $f: \tilde{T} \rightarrow \tilde{U}$. Then f be a temporal neutrosophic open at time period ζ_0 iff the temporal image of each $\mathcal{TNS}$ in ε_{ζ_0} is an $\mathcal{TNS}$ in π_{ζ_0} .

Proposition 3.18

$f: ((\tilde{T}, \mathcal{T}'), \varepsilon_{\zeta_0}) \rightarrow ((\tilde{U}, \mathcal{T}''), \pi_{\zeta_0})$ is $\mathcal{TNCont}$ iff the temporal preimage of every $\mathcal{TNCs}$ in π_{ζ_0} is an $\mathcal{TNCs}$ in ε_{ζ_0} .

Proposition 3.19

The proceeding equations are comparable to one another

- i. $f: ((\tilde{T}, \mathcal{T}'), \varepsilon_{\zeta_0}) \rightarrow ((\tilde{U}, \mathcal{T}''), \pi_{\zeta_0})$ is a $\mathcal{CT-TNS}$ on $\mathcal{TNS}$.
 TNCont,
 ii. $f^{-1}(int_{\zeta_0}(N)) \subseteq int_{\zeta_0} f^{-1}(N)$ for each $N \in \mathcal{TN}\mathcal{C}(\tilde{U}, \mathcal{T}'')$
 iii. $Cl_{\zeta_0}(f^{-1}(N)) \subseteq f^{-1}(Cl_{\zeta_0}(N))$ for each $N \in \mathcal{TN}\mathcal{C}(\tilde{U}, \mathcal{T}'')$

Proposition 3.20

Let ε_{ζ_0} is a $\mathcal{TNTS}$ in Chang's sense set of non-empty $\tilde{T}$ and time period set and $\langle \sigma_{\zeta_0}, \tau_{\zeta_0}, v_{\zeta_0} \rangle$ is a $\mathcal{N}$ eutrosophic-pair. Then, the $\mathcal{NF}_{\varepsilon_{\zeta_0}} \langle \sigma_{\zeta_0}, \tau_{\zeta_0}, v_{\zeta_0} \rangle$ which is described as follows:

$$\varepsilon_{\zeta_0}^{\langle \sigma_{\zeta_0}, \tau_{\zeta_0}, v_{\zeta_0} \rangle}(M) = \begin{cases} \langle \sigma_{\zeta_0}, \tau_{\zeta_0}, v_{\zeta_0} \rangle & \text{if } M \in \varepsilon_{\zeta_0} - \{0_S^{\zeta}, 0_S^{\zeta}, 1_S^{\zeta}\} \\ \langle 1, 1, 0 \rangle & \text{if } M \in \varepsilon\{0_S^{\zeta}, 0_S^{\zeta}, 1_S^{\zeta}\} \\ \langle 0, 0, 1 \rangle & \text{if } M \notin \varepsilon_{\zeta_0} - \{0_S^{\zeta}, 0_S^{\zeta}, 1_S^{\zeta}\} \end{cases}$$

is in $\mathcal{ST-TNS}$ is said to be a $\langle \sigma_{\zeta_0}, \tau_{\zeta_0}, v_{\zeta_0} \rangle$ - $\mathcal{ST-TNS}$.

On the other hand, Let $\varepsilon_{\zeta_0}^*$ is an $\mathcal{ST-TNS}$ on a set non-empty $\tilde{T}$ and time period set and $\langle \sigma_{\zeta_0}, \tau_{\zeta_0}, v_{\zeta_0} \rangle$ is a $\mathcal{N}$ -pair.

Then, the family of $\mathcal{TNS}$ s $\varepsilon_{\zeta_0}^* \langle \sigma_{\zeta_0}, \tau_{\zeta_0}, v_{\zeta_0} \rangle$ which is defined as

$M \in \varepsilon_{\zeta_0}^* \langle \sigma_{\zeta_0}, \tau_{\zeta_0}, v_{\zeta_0} \rangle \Leftrightarrow \varepsilon_{\zeta_0}^*(M) \geq \langle \sigma_{\zeta_0}, \tau_{\zeta_0}, v_{\zeta_0} \rangle$ is a $\mathcal{TNT}$ in Sostak's sense.

Definition 3.21

An Overall Neutrosophic Topology by Chang's (briely, $\mathcal{OCT-TNS}$) on a set of non-empty $\tilde{T}$ is a family ε_{ζ} of $\mathcal{TNS}$ s giving the axioms for each time period ζ .

- i. $0_S^{\zeta} \in \varepsilon_{\zeta}$ and $1_S^{\zeta} \in \varepsilon_{\zeta}$
 ii. For each $M_1, M_2 \in \varepsilon_{\zeta}$, there exist a $F \in \varepsilon_{\zeta}$ such that $\mathcal{J}_{\mathcal{F}}(x, \zeta) = \mathcal{J}_{M_1 \cap M_2}(x, \zeta)$, $\mathcal{J}_{\mathcal{F}}(t, \zeta) = \mathcal{J}_{M_1 \cap M_2}(t, \zeta)$ and $\mathcal{F}_{\mathcal{F}}(t, \zeta) = \mathcal{F}_{M_1 \cap M_2}(t, \zeta)$ for all $(t, \zeta) \in \tilde{T} \times \mathcal{T}$.
 iii. Consider some arbitrary pair $\{M_i; i \in I\} \in \varepsilon_{\zeta}$, $\exists G \in \varepsilon_{\zeta}$ such that $\mathcal{J}_G(t, \zeta) = \mathcal{J}_{\cup_{i \in I} M_i}(t, \zeta)$, $\mathcal{J}_G(t, \zeta) = \mathcal{J}_{\cup_{i \in I} M_i}(x, \zeta)$ and $\mathcal{F}_G(t, \zeta) = \mathcal{F}_{\cup_{i \in I} M_i}(t, \zeta)$ for each $(t, \zeta) \in \tilde{T} \times \mathcal{T}$.

Theorem 3.22

Consider, ε_{ζ} be an overall $\mathcal{NTS}$ by Chang's sense on set which is not empty $\tilde{T}$ and finite time period set $\mathcal{T}$. A family of $\mathcal{TNS}$ s ε which defined as:

$$M = \left\{ \left\langle t, \bigwedge_{\zeta \in \mathcal{T}} \mathcal{J}_M(t, \zeta), \bigwedge_{\zeta \in \mathcal{T}} \mathcal{J}_M(t, \zeta), \bigvee_{\zeta \in \mathcal{T}} \mathcal{F}_M(t, \zeta) \right\rangle \right\} \in \varepsilon$$

Proof:

It is important to demonstrate that the criteria specified in the theorem are true to a $\mathcal{CT-TNS}$

- i. Since $0_S^{\zeta_0} \in \varepsilon_{\zeta}$ for each $\zeta_0 \in \mathcal{T}$, $\bigwedge_{\zeta_0 \in \mathcal{T}} 0_S^{\zeta_0}(t, \zeta) = 0$ and $\bigvee_{\zeta_0 \in \mathcal{T}} 0_S^{\zeta_0}(t, \zeta) = 1$ then $\bigcap_{\zeta_0 \in \mathcal{T}} 0_S^{\zeta_0} = 0_S^{\zeta}$. Then it is understood that $0_S^{\zeta} \in \varepsilon_{\zeta}$.

Similarly to prove: $0_S^{\zeta} \in \varepsilon_{\zeta}$.

Since $0_S^{\zeta} \in \varepsilon_{\zeta}$ for each $\zeta \in \mathcal{T}$, $\bigwedge_{\zeta \in \mathcal{T}} 0_S^{\zeta}(t, \zeta) = 0$ and $\bigvee_{\zeta \in \mathcal{T}} 0_S^{\zeta}(t, \zeta) = 1$ then $\bigcap_{\zeta \in \mathcal{T}} 0_S^{\zeta} = 0_S^{\zeta}$. Then it is understood that $0_S^{\zeta} \in \varepsilon_{\zeta}$.

ii. Let

$$\hat{M}_i = \bigcap_{M_i \in \varepsilon_{\zeta}} M_i = \left\{ \left\langle t, \bigwedge_{t \in \mathcal{T}} \mathcal{J}_{M_i}(t, \zeta), \bigwedge_{t \in \mathcal{T}} \mathcal{J}_{M_i}(t, \zeta), \bigvee_{t \in \mathcal{T}} \mathcal{F}_{M_i}(t, \zeta) \right\rangle \right\}$$

and

$$\hat{M}_j = \bigcap_{M_j \in \varepsilon_{\zeta}} M_j = \left\{ \left\langle t, \bigwedge_{t \in \mathcal{T}} \mathcal{J}_{M_j}(t, \zeta), \bigwedge_{t \in \mathcal{T}} \mathcal{J}_{M_j}(t, \zeta), \bigvee_{t \in \mathcal{T}} \mathcal{F}_{M_j}(t, \zeta) \right\rangle \right\}.$$

Then

$$\begin{aligned} \hat{M}_i \cap \hat{M}_j &= \left(\bigcap_{M_i \in \varepsilon_{\zeta}} M_i \right) \cap \left(\bigcap_{M_j \in \varepsilon_{\zeta}} M_j \right) \\ &= \left\{ \left\langle t, \left(\bigwedge_{\zeta \in \mathcal{T}} \mathcal{J}_{M_i}(t, \zeta) \right) \wedge \left(\bigwedge_{\zeta \in \mathcal{T}} \mathcal{J}_{M_j}(t, \zeta) \right), \left(\bigwedge_{\zeta \in \mathcal{T}} \mathcal{J}_{M_i}(t, \zeta) \right) \wedge \left(\bigwedge_{\zeta \in \mathcal{T}} \mathcal{J}_{M_j}(t, \zeta) \right), \left(\bigvee_{\zeta \in \mathcal{T}} \mathcal{F}_{M_i}(t, \zeta) \right) \vee \left(\bigvee_{\zeta \in \mathcal{T}} \mathcal{F}_{M_j}(t, \zeta) \right) \right\rangle \right\} \\ &= \left\{ \left\langle t, \left(\bigwedge_{\zeta \in \mathcal{T}} \left(\mathcal{J}_{M_i}(t, \zeta) \wedge \mathcal{J}_{M_j}(t, \zeta) \right) \right), \left(\bigwedge_{\zeta \in \mathcal{T}} \left(\mathcal{J}_{M_i}(t, \zeta) \wedge \mathcal{J}_{M_j}(t, \zeta) \right) \right), \left(\bigvee_{\zeta \in \mathcal{T}} \left(\mathcal{F}_{M_i}(t, \zeta) \vee \mathcal{F}_{M_j}(t, \zeta) \right) \right) \right\rangle \right\} \\ &= \bigcap_{\zeta \in \mathcal{T}} M_i \cap M_j. \end{aligned}$$

Since ε_{ζ} is $\mathcal{OCT-TNS}$. It is understood that

$$\hat{M}_i \cap \hat{M}_j \in \varepsilon.$$

- iii. Let

$$\begin{aligned}\hat{M}_i &= \bigcup_{M_i \in \mathcal{E}_\zeta} M_i \\ &= \left\{ \left(t, \bigvee_{\zeta \in \mathcal{T}} \mathcal{T}_{M_i}(t, \zeta), \bigvee_{\zeta \in \mathcal{T}} \mathcal{J}_{M_i}(t, \zeta), \bigwedge_{\zeta \in \mathcal{T}} \mathcal{F}_{M_i}(t, \zeta) \right) \right\}\end{aligned}$$

and

$$\begin{aligned}\hat{M}_j &= \bigcup_{M_j \in \mathcal{E}_\zeta} M_j \\ &= \left\{ \left(t, \bigvee_{\zeta \in \mathcal{T}} \mathcal{T}_{M_j}(t, \zeta), \bigvee_{\zeta \in \mathcal{T}} \mathcal{J}_{M_j}(t, \zeta), \bigwedge_{\zeta \in \mathcal{T}} \mathcal{F}_{M_j}(t, \zeta) \right) \right\}.\end{aligned}$$

Then

$$\begin{aligned}\hat{M}_i \cup \hat{M}_j &= \left(\bigcup_{M_i \in \mathcal{E}_\zeta} M_i \right) \cup \left(\bigcup_{M_j \in \mathcal{E}_\zeta} M_j \right) \\ &= \left\{ \left(t, \left(\bigvee_{\zeta \in \mathcal{T}} \mathcal{T}_{M_i}(t, \zeta) \right) \vee \left(\bigvee_{\zeta \in \mathcal{T}} \mathcal{T}_{M_j}(t, \zeta) \right), \left(\bigvee_{\zeta \in \mathcal{T}} \mathcal{J}_{M_i}(t, \zeta) \right) \vee \left(\bigvee_{\zeta \in \mathcal{T}} \mathcal{J}_{M_j}(t, \zeta) \right), \left(\bigwedge_{\zeta \in \mathcal{T}} \mathcal{F}_{M_i}(t, \zeta) \right) \wedge \left(\bigwedge_{\zeta \in \mathcal{T}} \mathcal{F}_{M_j}(t, \zeta) \right) \right\} \\ &= \left\{ \left(t, \left(\bigvee_{\zeta \in \mathcal{T}} \left(\mathcal{T}_{M_i}(t, \zeta) \vee \mathcal{T}_{M_j}(t, \zeta) \right) \right), \left(\bigvee_{\zeta \in \mathcal{T}} \left(\mathcal{J}_{M_i}(t, \zeta) \vee \mathcal{J}_{M_j}(t, \zeta) \right) \right), \left(\bigwedge_{\zeta \in \mathcal{T}} \left(\mathcal{F}_{M_i}(t, \zeta) \wedge \mathcal{F}_{M_j}(t, \zeta) \right) \right) \right\}\end{aligned}$$

$$= \bigcup_{\zeta \in \mathcal{T}} M_i \cap M_j.$$

Since $\mathcal{E}_\zeta$ is $\mathcal{OCT-TNS}$.

It is understood that $\hat{M}_i \cup \hat{M}_j \in \mathcal{E}_\zeta$.

Conclusions

This work introduces the concept of Temporal Neutrosophic Sets and examines their properties. It further develops the notion of Temporal Neutrosophic Topology in the sense of Chang, Lowen, and Šostak, and outlines some fundamental properties and descriptions of these topological structures.

The future research directions based on this study are outlined as follows:

- Exploring lower and higher separation axioms, convergence properties, and compactness to enhance research in functional analysis and topology-driven solutions.

- Also, extend this concept to Pythagorean, Spherical, and Fermatean neutrosophic sets to create a more general framework.

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