

REVOLUTIONIZING TOPOLOGY: A DEEP DIVE INTO INTUITIONISTIC FUZZY ALMOST CONTRA G_δ - e - LOCALLY CONTINUOUS FUNCTIONS

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Received: November 11, 2024; Revised: October 26, 2025; Accepted: October 27, 2025

Abstract

This paper introduces and investigates the notions of intuitionistic fuzzy almost contra G_δ - e -locally continuous and intuitionistic fuzzy almost contra G_δ - e -locally irresolute functions within the setting of intuitionistic fuzzy topological spaces. The study establishes a range of fundamental and structural properties associated with these functions, thereby extending the theoretical landscape of intuitionistic fuzzy topology and providing new perspectives for both abstract analysis and potential applications.

Keywords: Intuitionistic Fuzzy Almost Contra G_δ - e -Locally Continuous; Intuitionistic Fuzzy Almost Contra G_δ - e -Locally Irresolute Function

Introduction

The origin of fuzzy set theory can be traced to Zadeh (1965), whose seminal work provided a mathematical framework for handling vagueness and partial truth. Atanassov (1986) extended this framework to intuitionistic fuzzy (I.F) sets by introducing a non-membership function alongside the traditional membership function, thereby enabling a richer representation of uncertainty. Intuitionistic fuzzy topological spaces and related notions, including I.F continuity, were first formalized by Coker and Demirci (1995) and Coker (1997). Further developments include the concept of I.F e -closed sets by Sobana *et al.* (2018) and the use of locally closed sets to define LC-continuity

and LC-irresoluteness by Ganster and Reilly (1989) and Balasubramanian (1995) explored fuzzy G_δ sets within fuzzy topological spaces, broadening the scope of fuzzy topology.

Despite these contributions, there remains a notable gap in the literature concerning the interaction between G_δ -type structures, e -local closedness, and the “almost contra” forms of continuity in the intuitionistic fuzzy setting. In particular, the relationship between regularity conditions on image sets and the preservation of local closedness under preimages has not been fully characterized for I.F topological mappings. This limits our ability to model and analyze

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DOI: <https://doi.org/10.55766/sujst7420>

decision-making processes in real-world systems where both local structure and uncertainty are critical.

Motivated by this gap, the present work introduces the notions of I.F almost contra G_δ - e -locally continuous functions and I.F almost contra G_δ - e -locally irresolute functions in I.F topological spaces. These new classes generalize existing concepts and provide a framework for studying functions whose preimages of certain structurally regular sets retain local closedness properties, even under uncertain information.

To illustrate the real-world relevance, consider a supply-chain quality control scenario with two warehouses holding identical products. Product acceptability is evaluated using intuitionistic fuzzy information, combining degrees of confidence that products meet or fail quality standards. If we model the mapping from warehouse records to shipment scheduling as an I.F almost contra G_δ - e -locally continuous function, then whenever a batch is classified as customer-ready at the shipment stage, its preimage in warehouse records will still form a coherent and operationally meaningful quality zone, even if the entire warehouse stock is not uniformly ready. This property endures reliability in operational decisions, reduces the risk of dispatching substandard goods, and maintains consumer trust despite incomplete or imprecise data.

In the sections that follow, we formally define these new function classes, establish their fundamental properties, and demonstrate their applicability through illustrative examples and theoretical results.

Preliminaries

Definition 1 (Atanassov, 1986) Let $\mathfrak{X}$ be a nonempty, predetermined set I be $[0, 1]$. An I.F set (I.F.S) Γ is an entity of the subsequent structure $\Gamma = \{(\kappa, \theta_\Gamma(\kappa), \varsigma_\Gamma(\kappa)) : \kappa \in \mathfrak{X}\}$, where the function $\theta_\Gamma : \mathfrak{X} \rightarrow I$ & $\varsigma_\Gamma : \mathfrak{X} \rightarrow I$ represent the degree of inclusion ($\theta_\Gamma(\kappa)$) and the degree of exclusion ($\varsigma_\Gamma(\kappa)$) $\forall \kappa \in \mathfrak{X}$ in regard to the set Γ , as appropriate, and $0 \leq \theta_\Gamma(\kappa) + \varsigma_\Gamma(\kappa) \leq 1 \forall \kappa \in \mathfrak{X}$.

Definition 2 (Atanassov, 1986) Let $\mathfrak{X}$ be a nonempty predetermined set and the I.F.S's Γ and β is an entity of the subsequent structure $\Gamma = \{(\kappa, \theta_\Gamma(\kappa), \varsigma_\Gamma(\kappa)) : \kappa \in \mathfrak{X}\}$, $\beta = \{(\kappa, \theta_\beta(\kappa), \varsigma_\beta(\kappa)) : \kappa \in \mathfrak{X}\}$. Then

1. $\Gamma \subseteq \beta$ if and only if $\theta_\Gamma(\kappa) \leq \theta_\beta(\kappa)$ and $\varsigma_\Gamma(\kappa) \geq \varsigma_\beta(\kappa)$ for all $\kappa \in \mathfrak{X}$;
2. $\bar{\Gamma} = \{(\kappa, \varsigma_\Gamma(\kappa), \theta_\Gamma(\kappa)) : \kappa \in \mathfrak{X}\}$;

$$3. \Gamma \cap \beta = \{(\kappa, \theta_\Gamma(\kappa) \wedge \theta_\beta(\kappa), \varsigma_\Gamma(\kappa) \vee \varsigma_\beta(\kappa)) : \kappa \in \mathfrak{X}\};$$

$$4. \Gamma \cup \beta = \{(\kappa, \theta_\Gamma(\kappa) \vee \theta_\beta(\kappa), \varsigma_\Gamma(\kappa) \wedge \varsigma_\beta(\kappa)) : \kappa \in \mathfrak{X}\};$$

Definition 3 (Atanassov, 1986) The I.F.S's $0_\sim$ and $1_\sim$ are formulated by, $0_\sim = \{(\kappa, 0, 1) : \kappa \in \mathfrak{X}\}$ and $1_\sim = \{(\kappa, 1, 0) : \kappa \in \mathfrak{X}\}$.

Definition 4 (Coker, 1997) An I.F topology (I.F.T) τ in Coker's sense, given a nonempty set, express concisely. $\mathfrak{X}$ is a family τ of I.F.Ss in $\mathfrak{X}$ Meeting the following axioms.:

1. $0_\sim, 1_\sim \in \tau$;
2. $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$;
3. $\bigcup G_i \in \tau$ for arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

Within this article, authored by $(\mathfrak{X}, \tau)$ Simply through $\mathfrak{X}$ We'll signify Coker's I.F topological space (I.F.T.S). Each I.F.S that pertains to τ is called an I.F open set (I.F.Os) in $\mathfrak{X}$. The complement thereof $\bar{\Gamma}$ of an I.F.Os Γ in $\mathfrak{X}$ is referred to as a I.F closed set (I.F.Cs) in $\mathfrak{X}$.

Definition 5 (Coker, 1997) Let $(\mathfrak{X}, \tau)$ represent I.F.T.S and $\Gamma = \{(\kappa, \theta_\Gamma, \varsigma_\Gamma) : \kappa \in \mathfrak{X}\}$ be an I.F.S in $\mathfrak{X}$. Then the I.F closure and I.F interior of Γ are delineated by

1. $I.F_{cl}(\Gamma) = \bigcap \{C : C \text{ is an I.F.CS in } \mathfrak{X} \text{ and } \Gamma \subseteq C\}$;
2. $I.F_{int}(\Gamma) = \bigcup \{D : D \text{ is an I.F.OS in } \mathfrak{X} \text{ and } D \subseteq \Gamma\}$;

Definition 6 (Coker and Demirci, 1995) Let $\mathfrak{X}$ constitute a nonempty set, and $\kappa \in \mathfrak{X}$ a constant element within $\mathfrak{X}$. If $r \in I_0$, $s \in I_1$ are predetermined real numbers such that $r + s \leq 1$, then the I.F.S $\kappa_{r,s} = \{(\kappa, \kappa_r, 1 - \kappa_{1-s})\}$ is termed as I.F point (I.F.P) in $\mathfrak{X}$, where r denotes the degree of inclusion of $\kappa_{r,s}$, s denotes the degree of exclusion of $\kappa_{r,s}$ and $\kappa \in \mathfrak{X}$ the support of $\kappa_{r,s}$. The I.F.P $\kappa_{r,s}$ resides within I.F.S $\Gamma(\kappa_{r,s} \in \Gamma)$ if and only if $r \leq \theta_\Gamma(\kappa)$, $s \geq \varsigma_\Gamma(\kappa)$.

Definition 7 (Hanafy, 2003) An I.F.S U of an I.F.T.S $\mathfrak{X}$ is called

1. neighborhood of an I.F.P $c(a, b)$, if there's a I.F.Os G in $\mathfrak{X}$ such that $c(a, b) \in G \leq U$.
2. q-neighborhood of an I.F.P $c(a, b)$, if there's a I.F.Os G in $\mathfrak{X}$ such that $c(a, b)qG \leq U$.

Definition 8 (Hanafy, 2003) Let Γ be I.F.S in an I.F.T.S $(\mathfrak{X}, \tau)$. Γ is termed as

1. I. F regular open set (I. F_{ROS}) if $\Gamma = I. F_{int} I. F_{cl}(\Gamma)$

2. I. F regular closed set (I. F_{RCS}) if $\Gamma = I. F_{cl} I. F_{int}(\Gamma)$

Definition 9 (Balasubramanian, 1995) Let $(\mathfrak{X}, \tau)$ be a fuzzy topological space and λ be a fuzzy set in $\mathfrak{X}$. λ is called G_δ set if $\lambda = \bigwedge_{i=1}^{\infty} \lambda_i$ where each $\lambda_i \in \tau$. The complement of fuzzy G_λ is fuzzy F_σ

Definition 10 (Thakur and Singh, 1998) Let $(\mathfrak{X}, \tau)$ be an I. F. T. S and $\Gamma = \langle \kappa, \theta_\Gamma(\kappa), \vartheta_\Gamma(\kappa) \rangle$ be an I. F. S in $\mathfrak{X}$. Then the I. F δ closure and interior of Γ are represented and delineated by $I. F_{cl_\delta}(\Gamma) = \bigcap \{K: K \text{ is an I. F}_{RCS} \text{ in } \mathfrak{X} \text{ and } \Gamma \subseteq K\}$ and $I. F_{int_\delta}(\Gamma) = \bigcup \{G: G \text{ is an I. F}_{ROS} \text{ in } \mathfrak{X} \text{ and } G \subseteq \Gamma\}$.

Definition 11 (Sobana *et al.*, 2018) Let Γ be an I. F. S in an I. F. T. S $(\mathfrak{X}, \tau)$. Γ is termed as an I. F e-open set (I. FeOS) in $\mathfrak{X}$ if $\Gamma \subseteq I. F_{cl} I. F_{int_\delta}(\Gamma) \cup I. F_{int} I. F_{cl_\delta}(\Gamma)$

Definition 12 (Saravanakumar *et al.*, 2018) Let $(\mathfrak{X}, \tau)$ be an I. F. T. S. Let $\Gamma = \{\langle \kappa, \theta_\Gamma(\kappa), \varsigma_\Gamma(\kappa) \rangle: \kappa \in \mathfrak{X}\}$ be an I. F. S on an I. F. T. S $(\mathfrak{X}, \tau)$. Then Γ is deemed I. F e- locally closed set (I. F – e-LCS) if $\Gamma = C \cap D$, where $C = \{\langle \kappa, \theta_C(\kappa), \varsigma_C(\kappa) \rangle: \kappa \in \mathfrak{X}\}$ is an I. FeOS and $D = \{\langle \kappa, \theta_D(\kappa), \varsigma_D(\kappa) \rangle: \kappa \in \mathfrak{X}\}$ is an I. FeCS in $(\mathfrak{X}, \tau)$.

Definition 13 (Saravanakumar *et al.*, 2017) Let $(\mathfrak{X}, \tau)$ be an I. F. T. S. Let $\Gamma = \{\langle \kappa, \theta_\Gamma(\kappa), \varsigma_\Gamma(\kappa) \rangle: \kappa \in \mathfrak{X}\}$ be an I. F. S on an I. F. T. S $\mathfrak{X}$. Then Γ is deemed an I. F e G_δ - set if $\Gamma = \bigcap_{i=1}^{\infty} \Gamma_i$, where $\Gamma_i = \{\langle \kappa, \theta_{\Gamma_i}(\kappa), \varsigma_{\Gamma_i}(\kappa) \rangle: \kappa \in \mathfrak{X}\}$ is an I. FeOS in an I. F. T. S $(\mathfrak{X}, \tau)$.

Definition 14 (Saravanakumar *et al.*, 2017) Let $(\mathfrak{X}, \tau)$ be an I. F. T. S. Let $\Gamma = \{\langle \kappa, \theta_\Gamma(\kappa), \varsigma_\Gamma(\kappa) \rangle: \kappa \in \mathfrak{X}\}$ be an I. F. S on an I. F. T. S $(\mathfrak{X}, \tau)$. Then Γ is deemed an I. F e G_δ - locally closed set (I. F – e G_δ -LCS) if $\Gamma = C \cap D$, where $C = \{\langle \kappa, \theta_C(\kappa), \varsigma_C(\kappa) \rangle: \kappa \in \mathfrak{X}\}$ is an I. F e G_δ set and $D = \{\langle \kappa, \theta_D(\kappa), \varsigma_D(\kappa) \rangle: \kappa \in \mathfrak{X}\}$ is an I. FeCS in $(\mathfrak{X}, \tau)$.

The complement of an I. F e G_δ -LCS is said to be an I. F e G_δ -LOS.

Definition 15 (Saravanakumar *et al.*, 2017) Let $(\mathfrak{X}, \tau)$ be an I. F. T. S. Let $\Gamma = \{\langle \kappa, \theta_\Gamma(\kappa), \varsigma_\Gamma(\kappa) \rangle: \kappa \in \mathfrak{X}\}$ be an I. F. S on an I. F. T. S $(\mathfrak{X}, \tau)$. Then Γ is deemed an I. F G_δ -e- locally closed set (I. F G_δ -e-LCS) if $\Gamma = \beta \cap C$, where $\beta = \{\langle \kappa, \theta_\beta(\kappa), \varsigma_\beta(\kappa) \rangle: \kappa \in \mathfrak{X}\}$ is an I. F G_δ

set and $C = \{\langle \kappa, \theta_C(\kappa), \varsigma_C(\kappa) \rangle: \kappa \in \mathfrak{X}\}$ is an I. FeCS in $(\mathfrak{X}, \tau)$.

The complement of an I. F G_δ -e-LCS is said to be an I. F G_δ -e-LOS.

Definition 16 (Saravanakumar *et al.*, (2017) Let $(\mathfrak{X}, \tau)$ be an I. F. T. S. Let $\Gamma = \{\langle \kappa, \theta_\Gamma(\kappa), \varsigma_\Gamma(\kappa) \rangle: \kappa \in \mathfrak{X}\}$ be an I. F. S on an I. F. T. S $(\mathfrak{X}, \tau)$. The I. F G_δ -e-locally closure of Γ are represented and delineated by $I. F_{G_\delta-e-l_{cl}}(\Gamma) = \bigcap \{\beta: \beta = \langle \kappa, \theta_\beta(\kappa), \varsigma_\beta(\kappa) \rangle: \kappa \in \mathfrak{X} \text{ is an I. F } G_\delta\text{-e-LCS in } \mathfrak{X} \text{ and } \Gamma \subseteq \beta\}$.

Definition 17 (Saravanakumar *et al.*, 2017) Let $(\mathfrak{X}, \tau)$ be an I. F. T. S. Let $\Gamma = \{\langle \kappa, \theta_\Gamma(\kappa), \varsigma_\Gamma(\kappa) \rangle: \kappa \in \mathfrak{X}\}$ be an I. F. S on an I. F. T. S $(\mathfrak{X}, \tau)$. The I. F G_δ -e-locally interior of Γ are represented and delineated by $I. F_{G_\delta-e-l_{int}}(\Gamma) = \bigcup \{\beta: \beta = \{\langle \kappa, \theta_\beta(\kappa), \varsigma_\beta(\kappa) \rangle: \kappa \in \mathfrak{X}\} \text{ is an I. F } G_\delta\text{-e-LOS in } \mathfrak{X} \text{ and } \beta \subseteq \Gamma\}$.

Definition 18 (Saravanakumar *et al.*, 2017) Let $(\mathfrak{X}, \tau)$ be an I. F. T. S. Let $\Gamma = \langle \kappa, \theta_\Gamma(\kappa), \varsigma_\Gamma(\kappa) \rangle$ be an I. F. S in an I. F. T. S $(\mathfrak{X}, \tau)$. Then Γ is deemed an I. F G_δ -e-locally neighbourhood of an I. F. P $x_{r,s}$ if there exists an I. F G_δ -e-LOS β in an I. F. T. S $(\mathfrak{X}, \tau)$ such that $x_{r,s} \in \beta, \beta \subseteq \Gamma$. It is indicated by I. F G_δ -e-lnbd.

Definition 19 (Saravanakumar *et al.*, 2017) Let $(\mathfrak{X}, \tau)$ be an I. F. T. S. Let $\Gamma = \langle \kappa, \theta_\Gamma(\kappa), \varsigma_\Gamma(\kappa) \rangle$ be an I. F. S in an I. F. T. S $(\mathfrak{X}, \tau)$. Then Γ is deemed an I. F G_δ -e-locally quasi neighbourhood of an I. F. P $x_{r,s}$ if there exists an I. F G_δ -e-loc β in an I. F. T. S $(\mathfrak{X}, \tau)$ such that $x_{r,s} q \beta, \beta \subseteq \Gamma$. It is indicated by I. F G_δ -e-lqnbd.

Remark 1 (Saravanakumar *et al.*, 2017)

1. The family of all I. F G_δ -e-lnbd of an I. F. P $x_{r,s}$ It is indicated by $N^{I. F_{G_\delta-e-l_{nbd}}}(x_{r,s})$.

2. The family of all I. F G_δ -e-lqnbd of an I. F. P $x_{r,s}$ It is indicated by $N^{I. F_{G_\delta-e-l_{qnbd}}}(x_{r,s})$.

Intuitionistic Fuzzy Almost Contra G_δ -e-Locally Continuous Functions and Its Applications

Definition 1 Let $(\mathfrak{X}, \tau)$ and $(\wp, \sigma)$ be any two I. F. T. S 's. Let $f: (\mathfrak{X}, \tau) \rightarrow (\wp, \sigma)$ be an I. F mapping (IFM). Then f is said to be an I. F almost contra G_δ - e -locally continuous function (I. F. alm. Cont. G_δ -e-l. cts. fun), if $f^{-1}(\Gamma)$ is I. F G_δ -e-LCS in $\mathfrak{X}$ for every I. F_{ROS} Γ in $\wp$

Theorem 1 Let $(\mathfrak{X}, \tau)$ and $(\mathcal{Y}, S)$ be any two I. F. T. S's. Let $f: (\mathfrak{X}, \tau) \rightarrow (\mathcal{Y}, S)$ be an IFM. Then the following are equivalent.

1. f is an I. F. alm. Cont. G_δ -e-l. cts. fun.
2. $f^{-1}(\beta)$ is an I. F. G_δ -e - LCS in an I. F. T. S $(\mathfrak{X}, \tau)$, for each I. F_{ROS} β in an I. F. T. S $(\mathcal{Y}, S)$.
3. $f^{-1}(\Gamma)$ is an I. F. G_δ -e - LOS in an I. F. T. S $(\mathfrak{X}, \tau)$, for each I. F_{RCS} Γ in an I. F. T. S $(\mathcal{Y}, S)$.
4. $f^{-1}(I. F_{int}(\Gamma)) \subseteq I. FG_\delta\text{-e-l}_{int}(f^{-1}(\Gamma))$, for each I. F_{RCS} Γ in an I. F. T. S $(\mathcal{Y}, S)$.
5. $I. FG_\delta\text{-e-l}_{cl}(f^{-1}(\Gamma)) \subseteq f^{-1}(I. F_{cl}(\Gamma))$, for each I. F_{ROS} Γ in an I. F. T. S $(\mathcal{Y}, S)$.

Proof. (i) $\Rightarrow$ (ii): Let Γ be an I. F_{RCS} in an I. F. T. S $(\mathcal{Y}, S)$. Let $x_{r,s}$ be an I. F. P in an I. F. T. S $(\mathfrak{X}, \tau)$ such that $x_{r,s} q f^{-1}(\Gamma)$. Since f is an I. F. alm. Cont. G_δ -e-l. cts. fun, there exists $\beta \in N^{I. FG_\delta\text{-e-lq}}(x_{r,s})$ such that $f(\beta) \subseteq \Gamma$. Then

$$x_{r,s} \in \beta \quad (1)$$

$$\beta \subseteq f^{-1}(f(\beta)) \quad (2)$$

Thus, $x_{r,s} \in \beta \subseteq f^{-1}(f(\beta)) \subseteq f^{-1}(\Gamma)$. This implies $x_{r,s} \in f^{-1}(\Gamma)$ which shows that $f^{-1}(\Gamma) \in N^{I. FG_\delta\text{-e-lq}}(x_{r,s})$. Hence $f^{-1}(\Gamma)$ is an I. F. G_δ -e-LOS in an I. F. T. S $(\mathfrak{X}, \tau)$.

(ii) $\Rightarrow$ (i): This can be proved by taking complement of (i)

(iii) $\Rightarrow$ (iv): Let Γ be an I. F_{RCS} in an I. F. T. S $(\mathcal{Y}, S)$. Since $\Gamma \subseteq I. F_{cl}(\Gamma)$, $f^{-1}(\Gamma) \supseteq f^{-1}(I. F_{int}(\Gamma))$. By (iii), $f^{-1}(I. F_{int}(\Gamma))$ is an I. F. G_δ -e - LOS in an I. F. T. S $(\mathfrak{X}, \tau)$. Thus, $I. FG_\delta\text{-e-l}_{int}(f^{-1}(\Gamma)) \supseteq f^{-1}(I. F_{int}(\Gamma))$.

(iv) $\Rightarrow$ (v): Using (iv), $I. FG_\delta\text{-e-l}_{int}(f^{-1}(\Gamma)) \supseteq f^{-1}(I. F_{int}(\Gamma))$. Then

$$\begin{aligned} & \overline{I. FG_\delta\text{-e-l}_{int}(f^{-1}(\Gamma))} \subseteq \\ & \overline{f^{-1}(I. F_{int}(\Gamma))}, I. FG_\delta\text{-l}_{cl}(f^{-1}(\Gamma)) \subseteq \\ & f^{-1}(I. F_{cl}(\bar{\Gamma})), I. FG_\delta\text{-e-l}_{cl}(f^{-1}(\bar{\Gamma})) \subseteq \\ & f^{-1}(I. F_{cl}(\bar{\Gamma})) \text{ implies that } f^{-1}(I. F_{cl}(\bar{\Gamma})) \supseteq I. FG_\delta\text{-e-l}_{cl}(f^{-1}(\bar{\Gamma})), \text{ putting } \bar{\Gamma} = \Gamma, \text{ we have} \\ & f^{-1}(I. F_{cl}(\bar{\Gamma})) \supseteq I. FG_\delta\text{-e-l}_{cl}(f^{-1}(\Gamma)). \end{aligned}$$

(v) $\Rightarrow$ (i): Let Γ be an I. F_{RCS} in an I. F. T. S $(\mathcal{Y}, S)$. Then $I. F_{cl}\Gamma = \Gamma$. Using (v), $f^{-1}(I. F_{int}(\Gamma)) \subseteq I. FG_\delta\text{-e-l}_{int}(f^{-1}(\Gamma))$ implies that $f^{-1}(\Gamma) \subseteq I. FG_\delta\text{-e-l}_{int}(f^{-1}(\Gamma))$. But, $I. FG_\delta\text{-e-l}_{int}(f^{-1}(\Gamma)) \subseteq f^{-1}(\Gamma)$ implies that $f^{-1}(\Gamma) = I. FG_\delta\text{-e-l}_{int}(f^{-1}(\Gamma))$ that is, $f^{-1}(\Gamma)$ is an I. F. G_δ -e-LOS in an I. F. T. S $(\mathfrak{X}, \tau)$. Let $x_{r,s}$ be any I. F. P in $f^{-1}(\Gamma)$. Then $x_{r,s} \in f^{-1}(\Gamma)$. We have $x_{r,s} q f^{-1}(\Gamma)$ implies that $f(x_{r,s}) q f(f^{-1}(\Gamma))$. But $f(f^{-1}(\Gamma)) \subseteq \Gamma$. Thus, for any I. F. P $x_{r,s}$ and $\Gamma \in Nf(x_{r,s})$, there

exists $\beta = f^{-1}(\Gamma) \in N^{I. FG_\delta\text{-e-lq}}(x_{r,s})$ such that $f^{-1}(f(\Gamma)) \subseteq \Gamma$. Therefore, $f(\beta) \subseteq \Gamma$. Thus, f is an I. F. alm. Cont. G_δ -e-l. cts. fun.

Theorem 2 Let $(\mathfrak{X}, \tau)$ and $(\mathcal{Y}, S)$ be any two I. F. T. S's. Let $f: (\mathfrak{X}, \tau) \rightarrow (\mathcal{Y}, S)$ be an I. F. bijective function. Then f is an I. F. alm. Cont. G_δ -e-l. cts. fun if and only if $I. F_{int}(f(\Gamma)) \subseteq f(I. FG_\delta\text{-e-l}_{int}(\Gamma))$, for each I. F_{RCS} Γ of an I. F. T. S $(\mathfrak{X}, \tau)$.

Proof. Assume that f is an I. F. alm. Cont. G_δ -e-l. cts. fun and Γ be an I. F_{RCS} in an I. F. T. S $(\mathfrak{X}, \tau)$. Hence, $f^{-1}(I. F_{int}(f(\Gamma)))$ is an I. F. G_δ -e-LOS in an I. F. T. S $(\mathfrak{X}, \tau)$. From Theorem (v) of (0.1) $f^{-1}(I. F_{int}(f(\Gamma))) \subseteq I. FG_\delta\text{-e-l}_{int}(f^{-1}(f(\Gamma)))$, $f^{-1}(I. F_{int}(f(\Gamma))) \subseteq I. FG_\delta\text{-e-l}_{int}(\Gamma)$. Since f is an I. F. surjective function, $f(f^{-1}(I. F_{int}(f(\Gamma)))) \subseteq f(I. FG_\delta\text{-e-l}_{int}(\Gamma))$. That is, $I. F_{int}(f(\Gamma)) \subseteq f(I. FG_\delta\text{-e-l}_{int}(\Gamma))$.

Conversely, assume that $I. F_{int}(f(\Gamma)) \subseteq f(I. FG_\delta\text{-e-l}_{int}(\Gamma))$, for each I. F. S Γ of an I. F. T. S $(\mathfrak{X}, \tau)$. Let β be an I. F_{RCS} in an I. F. T. S $(\mathcal{Y}, S)$. Then $\beta = I. F_{cl}(\beta)$. Since f is an I. F. surjective function, $\beta = I. F_{cl}(\beta) = I. F_{int}(f(f^{-1}(\beta))) \subseteq f(I. FG_\delta\text{-e-l}_{int}(f^{-1}(\beta)))$. Since f is an I. F. injective function, $f^{-1}(\beta) \subseteq f^{-1}(f(I. FG_\delta\text{-e-l}_{int}(f^{-1}(\beta))))$. From the fact that f , is an I. F. injective function, we have

$$f^{-1}(\beta) \subseteq I. FG_\delta\text{-e-l}_{int}(f^{-1}(\beta)) \quad (3)$$

but

$$I. FG_\delta\text{-e-l}_{int}(f^{-1}(\beta)) \subseteq f^{-1}(\beta) \quad (4)$$

From (3.3) and (3.4) implies that $f^{-1}(\beta) = I. FG_\delta\text{-e-l}_{int}(f^{-1}(\beta))$. That is, $f^{-1}(\beta)$ is an I. F. G_δ -e - LOS in an I. F. T. S $(\mathfrak{X}, \tau)$. Thus, f is an I. F. alm. Cont. G_δ -e-l. cts. fun.

Theorem 3 Let $(\mathfrak{X}, \tau)$ and $(\mathcal{Y}, S)$ be any two I. F. T. S's. Let $f: (\mathfrak{X}, \tau) \rightarrow (\mathcal{Y}, S)$ be an I. F. bijective function. Then f is an I. F. alm. Cont. G_δ -e-l. cts. fun if and only if $f(I. FG_\delta\text{-e-l}_{cl}(\Gamma)) \subseteq I. F_{cl}(f(\Gamma))$, for each I. F_{ROS} Γ of an I. F. T. S $(\mathfrak{X}, \tau)$.

Proof. Assume that f is an I. F. alm. Cont. G_δ -e-l. cts. fun and Γ be an I. F_{ROS} in an I. F. T. S $(\mathfrak{X}, \tau)$. Hence, $f^{-1}(I. F_{cl}(f(\Gamma)))$ is an I. F. G_δ -e-LCS in an I. F. T. S $(\mathfrak{X}, \tau)$. From Theorem (v) of (0.1) $f^{-1}(I. F_{cl}(f(\Gamma))) \supseteq I. FG_\delta\text{-e-l}_{cl}(f^{-1}(f(\Gamma)))$, $f^{-1}(I. F_{cl}(f(\Gamma))) \supseteq I. FG_\delta\text{-e-l}_{cl}(\Gamma)$. Since f is an I. F. surjective function, $f(f^{-1}(I. F_{cl}(f(\Gamma)))) \supseteq f(I. FG_\delta\text{-e-l}_{cl}(\Gamma))$. That is, $I. F_{cl}(f(\Gamma)) \supseteq f(I. FG_\delta\text{-e-l}_{cl}(\Gamma))$.

Conversely, assume that $I. F_{cl}(f(\Gamma)) \supseteq f(I. FG_\delta\text{-e-l}_{cl}(\Gamma))$, for each I. F. S Γ of an I. F. T. S $(\mathfrak{X}, \tau)$. Let β be an I. F_{ROS} in an I. F. T. S $(\mathcal{Y}, S)$.

Then $\beta = I.F_{int}(\beta)$. Since f is an I.F surjective function, $\beta = I.F_{int}(\beta) = I.F_{cl}(f(f^{-1}(\beta))) \supseteq f(I.FG_{\delta-e-l_{cl}}(f^{-1}(\beta)))$. Since f is an I.F injective function, $f^{-1}(\beta) \supseteq f^{-1}(f(I.FG_{\delta-e-l_{cl}}(f^{-1}(\beta))))$. From the fact that f is an I.F injective function, we have

$$f^{-1}(\beta) \supseteq I.FG_{\delta-e-l_{cl}}(f^{-1}(\beta)) \quad (5)$$

but

$$I.FG_{\delta-e-l_{cl}}(f^{-1}(\beta)) \supseteq f^{-1}(\beta) \quad (6)$$

From (3.5) and (3.6) implies that $f^{-1}(\beta) = I.FG_{\delta-e-l_{cl}}(f^{-1}(\beta))$. That is, $f^{-1}(\beta)$ is an I.F $G_{\delta-e}$ -LCS in an I.F.T.S $(\mathfrak{I}, T)$. Thus, f is an I.F. alm. Cont. $G_{\delta-e-l}$ cts. fun.

Theorem 4 Let $(\mathfrak{I}, T)$ and $(\wp, S)$ be any two I.F.T.S's. Let $f: (\mathfrak{I}, T) \rightarrow (\wp, S)$ be an I.F bijective fuction. If f is an I.F. alm. Cont. $G_{\delta-e-l}$ cts. fun. Then if $\Gamma \in I^{\wp}$ is an I.F_{ROS}, then $f^{-1}(\Gamma) = I.FG_{\delta-e-l_{cl}}(f^{-1}(\Gamma))$.

Proof. Let Γ be an I.F_{ROS} in an I.F.T.S $(\wp, S)$. By Theorem(iv)of (0.1).

$$I.FG_{\delta-e-l_{cl}}(f^{-1}(\Gamma)) \subseteq f^{-1}(I.F_{cl}(\Gamma)) = f^{-1}(\Gamma) \quad (7)$$

Since $\Gamma = I.F_{int}(\Gamma)$. But

$$f^{-1}(\Gamma) \subseteq I.FG_{\delta-e-l_{cl}}(f^{-1}(\Gamma)) \quad (8)$$

From (7) and (8) implies that $f^{-1}(\Gamma) = I.FG_{\delta-e-l_{cl}}(f^{-1}(\Gamma))$

Proposition 1 Let $(\mathfrak{I}, T)$, $(\wp, S)$ and $(\mathfrak{K}, R)$ be any three I.F.T.S 's. Let $f: (\mathfrak{I}, T) \rightarrow (\wp, S)$ be an I.F. alm. Cont. $G_{\delta-e-l}$ cts. fun. If $f(\mathfrak{I}) \subset \mathfrak{K} \subset \wp$ then $g: (\mathfrak{I}, T) \rightarrow (\mathfrak{K}, R)$ where $R = S/\mathfrak{K}$ restricting the range of f is an I.F. alm. Cont. $G_{\delta-e-l}$ cts. fun.

Proof. Let β be an I.F_{ROS} in an I.F.T.S $(\mathfrak{K}, R)$. Then $\beta = S/\mathfrak{K}$, for some I.F_{ROS} Γ of an I.F.T.S's $(\wp, S)$. If $f(\mathfrak{I}) \subset \mathfrak{K} \subset \wp$, $f^{-1}(\Gamma) = g^{-1}(\beta)$. Since $f^{-1}(\Gamma)$ is an I.F $G_{\delta-e}$ -LCS in an I.F.T.S $(\mathfrak{I}, T)$. Hence, $g^{-1}(\beta)$ is an I.F $G_{\delta-e}$ -LCS in an I.F.T.S $(\mathfrak{I}, T)$. Therefore, g is an I.F. alm. Cont. $G_{\delta-e-l}$ cts. fun.

Proposition 2 Let $(\mathfrak{I}, T)$, (X_1, T_1) and (X_2, T_2) be any three I.F.T.S's and $P_i: X_1 \times X_2 \rightarrow X_i$ be an I.F projection of $X_1 \times X_2$ onto X_i . If $f: \mathfrak{I} \rightarrow X_1 \times X_2$ is an I.F. alm. Cont. $G_{\delta-e-l}$ cts. fun. Then $P_i \circ f: \mathfrak{I} \rightarrow X_i$ is also an I.F. alm. Cont. $G_{\delta-e-l}$ cts. fun.

Proof. Let Γ be an I.F_{ROS} in an I.F.T.S 's (X_i, T_i) ($i = 1, 2$), $(P_i \circ f)^{-1}(\Gamma) = f^{-1}(P_i^{-1}(\Gamma))$. Since P_i is an IFM $P_i^{-1}(\Gamma)$ is an I.F_{RCS} in an I.F.T.S's $X_1 \times X_2$. Hence, $f^{-1}(P_i^{-1}(\Gamma))$ is an I.F $G_{\delta-e}$ -LCS in an I.F.T.S $(\mathfrak{I}, T)$. Hence, $P_i \circ f$ is an I.F. alm. Cont. $G_{\delta-e-l}$ cts. fun.

Example 1 Let $\mathfrak{I} = \{a, b\} = \wp$, and $\Gamma = \left\langle \mu, \left(\frac{a}{0.3}, \frac{b}{0.3}\right), \left(\frac{a}{0.6}, \frac{b}{0.6}\right) \right\rangle$, $\beta = \left\langle \mu, \left(\frac{a}{0.3}, \frac{b}{0.1}\right), \left(\frac{a}{0.5}, \frac{b}{0.4}\right) \right\rangle$, $\Gamma \vee \beta = \left\langle \mu, \left(\frac{a}{0.3}, \frac{b}{0.3}\right), \left(\frac{a}{0.5}, \frac{b}{0.4}\right) \right\rangle$, $\Gamma \wedge \beta = \left\langle \mu, \left(\frac{a}{0.3}, \frac{b}{0.1}\right), \left(\frac{a}{0.6}, \frac{b}{0.6}\right) \right\rangle$, $C = \left\langle \mu, \left(\frac{a}{0.7}, \frac{b}{0.7}\right), \left(\frac{a}{0}, \frac{b}{0.1}\right) \right\rangle$. Now, the family $T = \{0 \sim, 1 \sim, \Gamma, \beta, \Gamma \vee \beta, \Gamma \wedge \beta\}$ of I.F.S's in $\mathfrak{I}$ is an I.F.T on $\mathfrak{I}$ and the family $S = \{0 \sim, 1 \sim, C\}$ of I.F.S's in $\wp$ is an I.F.T on $\wp$. If we define the function $f: \mathfrak{I} \rightarrow \wp$ be the identity function. Now, f is an I.FCont. $G_{\delta-e-l}$ irr. fun because C is an I.F_{ROS} in $\wp$, $f^{-1}(C)$ is also an I.FG _{$\delta-e$} -LCS in $\mathfrak{I}$.

Real-world Analogy: Based on Example 1

Consider two warehouses $\mathfrak{I} = \{a, b\}$ and $\wp = \{a, b\}$ storing identical products. The quality of products is evaluated using intuitionistic fuzzy information, where the membership degree (μ) indicates the confidence that products meet quality standards, and the non-membership degree (ν) indicates the confidence that products fail to meet the standards.

Let:

- Γ : "Batch is acceptable according to supplier records" with moderate confidence.
- β : "Batch passes random inspection" with slightly different confidence levels.
- $\Gamma \vee \beta$: "Batch is acceptable if either supplier data or inspection passes" (union).
- $\Gamma \wedge \beta$: "Batch is acceptable if both supplier data and inspection pass" (intersection).
- C : "Batch meets customer-ready standards", representing a regular open set in the fuzzy topological sense.

If we use the identity mapping $f: \mathfrak{I} \rightarrow \wp$, the property of being an I.F almost contra $G_{\delta-e}$ -locally continuous function means: whenever the customer-ready set C in $\wp$ is regular open, its preimage $f^{-1}(C)$ in $\mathfrak{I}$ remains an I.FG _{$\delta-e$} -locally closed set. This means that batches classified as customer-ready at the shipment stage will still correspond to well-defined and reliable quality zones in storage, even if the overall stock does not meet the same

readiness standard throughout. In practice, this ensures that local sections of inventory retain their operational usefulness, despite variations in quality across the entire warehouse.

Formal Structure: Let $\mathfrak{S} = \{a, b\} = \emptyset$, and

$$\Gamma = \left\langle \kappa, \left(\frac{a}{0.3}, \frac{b}{0.3} \right), \left(\frac{a}{0.6}, \frac{b}{0.6} \right) \right\rangle,$$

$$\mathfrak{B} = \left\langle \kappa, \left(\frac{a}{0.3}, \frac{b}{0.1} \right), \left(\frac{a}{0.5}, \frac{b}{0.4} \right) \right\rangle,$$

$$\Gamma \vee \mathfrak{B} = \left\langle \kappa, \left(\frac{a}{0.3}, \frac{b}{0.3} \right), \left(\frac{a}{0.5}, \frac{b}{0.4} \right) \right\rangle,$$

$$\Gamma \wedge \mathfrak{B} = \left\langle \kappa, \left(\frac{a}{0.3}, \frac{b}{0.1} \right), \left(\frac{a}{0.6}, \frac{b}{0.6} \right) \right\rangle,$$

$$\mathcal{C} = \left\langle \kappa, \left(\frac{a}{0.7}, \frac{b}{0.7} \right), \left(\frac{a}{0}, \frac{b}{0.1} \right) \right\rangle.$$

Now, the family $\mathcal{T} = \{0, 1, \sim, \Gamma, \mathfrak{B}, \Gamma \vee \mathfrak{B}, \Gamma \wedge \mathfrak{B}\}$ of $I.F.S$'s in $\mathfrak{S}$ is an $I.F.T$ on $\mathfrak{S}$, and the family $\mathcal{S} = \{0, 1, \sim, \mathcal{C}\}$ of $I.F.S$'s in $\emptyset$ is an $I.F.T$ on $\emptyset$. If we define $f: \mathfrak{S} \rightarrow \emptyset$ to be the identity function, then f is an $I.FCont.G_\delta - e - l. irr. fun$ because $\mathcal{C}$ is an $I.F_{ROS}$ in $\emptyset$ and $f^{-1}(\mathcal{C})$ is also an $I.FG_\delta - e - LCS$ in $\mathfrak{S}$.

Societal Conclusion: This example shows that the mathematical concept of $I.F$ almost contra G_δ - e -locally continuous functions can model real-world situations where decisions must be made under uncertainty. In the context of warehouse management, it ensures that high-quality product batches identified at the shipment stage will still correspond to consistent and well-structured quality zones at the storage stage. This reduces the risk of sending substandard goods to consumers, increases trust in supply chains, and ultimately protects public interest by maintaining product safety and reliability even when data is incomplete or imprecise.

Conclusions

In this work, we have introduced and analyzed the concepts of *intuitionistic fuzzy almost contra $G_\delta - e$ -locally continuous* and *intuitionistic fuzzy almost contra $G_\delta - e$ -locally irresolute* functions within intuitionistic fuzzy topological spaces. By establishing several key properties and interrelationships, the study not only broadens the theoretical foundation of intuitionistic fuzzy topology but also paves the way for potential interdisciplinary applications.

From a practical standpoint, the findings can be interpreted through real-life operational settings such as warehouse and inventory management. Just as certain quality zones within a warehouse retain their reliability even when the entire stock does not uniformly meet readiness criteria, these new function types capture the essence of preserving local structure and meaningfulness despite broader irregularities. This conceptual bridge between abstract mathematics and tangible operational scenarios emphasizes that local precision and control can yield significant benefits, even when global uniformity is unattainable.

Future research could extend these notions to more generalized fuzzy environments, dynamic systems, and decision-making frameworks, thereby enhancing their applicability in domains ranging from supply chain optimization to intelligent information systems.

Acknowledgement

The author sincerely thanks the anonymous reviewer(s) for their meticulous reading of the manuscript and for offering constructive suggestions that have significantly improved the clarity, precision, and overall quality of this article. Their valuable insights helped refine both the theoretical presentation and the illustrative examples, making the work more accessible and impactful.

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