

PERCEPTIONIZATION OF M/D/1 QUEUING MODEL UNDER TRAPEZOIDAL NEUTROSOPHIC NUMBERS

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Abstract

In this paper, we provide an NM/ND/1 queuing model with an infinite limit based on the neutrosophic environment. Both the entry and utility rates are represented in terms of single valued neutrosophic trapezoidal numbers (SVTNN). The performance measures of the proposed model are evaluated using the DSW approach and (α, β, γ) -cut approach. A numerical example pertaining to a single valued trapezoidal neutrosophic number is obtained for different values of α, β, γ .

Keywords: DSW algorithm; Neutrosophic set; Single valued neutrosophic set; Single valued neutrosophic number Queuing model; (α, β, γ) -cut

Introduction

Queuing models may be used in queuing theory to approximate the actual waiting scenario or system. A queue is a regular event in everyday life. A queuing model allows different relevant metrics of performance and stable conditions to be derived. To disseminate the queuing theory, the variables entry rate (λ) and utility rate (μ) are necessary. Gross *et al.* presented the principles of queuing theory in 1985. Al-Kridi *et al.* (2018) who introduced the new approach to FM/FM/1 queues and their performance measures. Intuitionistic fuzzy sets are developed by Atanassov in 1986. Kalpana and Anusheela (2018) analysed a single server non-preemptive fuzzy priority queue using LR Method. The idea of fuzzy queue is broadly discussed by scientists such as Kanufmann (1975), Li and Lee (1989) and the basics of fuzzy subset theory have been outlined.

Klir and Yuan (2009), proposed a theory and applications of Fuzzy Sets and Fuzzy Logic. Shortle *et al.* (2018) are given an overview of the queuing theory. Using the DSW method, Srinivasa (2014) presents a novel approach to fuzzy sets. Shanmugasundaram and Venkatesh (2015) presented the Study of fuzzy multi server queuing model using DSW algorithm. Usha Prameela and Kumar (2019a; 2019b) who introduced the FM/F Ek/1 queuing model with Erlang service under different types of fuzzy numbers, as well as the execution proportions of the multi-server queuing model with pentagonal fuzzy number: DSW algorithm approach, in 2019 Varadharajan and Susmitha (2018) introduced the evaluation of performance measures of priority queues with fuzzy parameters using α cut approach.

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Kumuthavalli and Sangeetha (2017) focused on developing a neutrosophic probability for solving queue operation in the real standard domain. Mohamed Bisher Zeina (2020a; 2020b; 2020c) provided neutrosophic little's formulas which is a main tool in queuing systems problems under neutrosophic environment. Also, he discussed about Erlang service queuing model under neutrosophic environment. Heba Rashad and Mohamed (2021) discussed the performance measurements of NM/NM/1, NM/NM/s, and NM/NM/1/b queuing systems.

Sumathi and Antony Crispin Sweetey (2019) who suggested using the trapezoidal neutrosophic number to solve differential equations. Florentin Smarandache has explored various concepts such as neutrosophic measure, neutrosophic logic, neutrosophic probability etc. Wang *et al.* (2010) discussed about operations and attributes of single valued neutrosophic set (SVNS).

An NM/ND/1 model, where the utility period is fixed and it has several advantages in quality control, communications systems, and other disciplines. The basic concept of this study is to extract precise crisp values from fuzzy values, which are used to type of enrolment capacity trapezoidal in queuing performance equations. The purpose of this research is to look at a single server queuing model with fixed utility period that uses SVNS.

The Neutrosophic Queuing Model (NM/ND/1)

In this section, we are going to introduce a single server queue with deterministic service time in a neutrosophic environment.

Presumptions

Corresponding assumptions are used in the current model

1. NM/ND/1/FCFS queuing system with single server has an unbounded limit.
2. Entry period that are distributed exponentially
3. Deterministic utility period, i.e. fixed
4. Entry rate and utility rates are neutrosophic trapezoidal fuzzy numbers

Suggested Queuing Miniature

In this paper, we prefer (FCFS) first come first serve single server queuing model. In Kendall's notation, it is denoted as (NM/ND/1): (/FCFS). Here, ND denotes neutrosophic steady (constant) distribution with utility rate and NM denotes neutrosophic exponential distribution with entry rate. This stochastic process, which includes any

entity currently engaged in administration, has a sequence with the values 0, 1, 2, 3 where the numbers denotes how many peoples currently utilising the server. Its size is infinite, therefore there is no limitation to the no of clients it can accommodate. The performance measurements of this suggested model are described as below.

Expected no of consumer count in the system:

$$N_{L_s} = \frac{\lambda}{\mu} + \frac{\lambda^2}{2\mu(\mu - \lambda)}$$

Expected no of consumers waiting in line:

$$N_{L_q} = \frac{\lambda^2}{2\mu(\mu - \lambda)}$$

Expected time of an average consumer waits in queue:

$$N_{W_q} = \frac{\lambda}{2\mu(\mu - \lambda)}$$

System's average client hold time:

$$N_{W_s} = \frac{1}{\mu} + \frac{\lambda}{2\mu(\mu - \lambda)}$$

Solution Procedure

A simple method called DSW (Dong, Shah, and Wong) uses intervals at different α , β , γ -cut dimensions to describe execution proportions. The segregation extending on the neutrosophic variable area prevents variance from the output membership function. Each ongoing involvement capacity can be addressed by an endless range of cuts with terms ranging from 0 to 1. Let $\mu_A(\tilde{a})$ & $\mu_S(\tilde{s})$ be the membership performs of the utility period and the inter entry period. Intervals between entry period and utility period are shown as neutrosophic sets as follows:

$$A^1 = \{(\tilde{a}, T_{A^1}(\tilde{a}), I_{A^1}(\tilde{a}), F_{A^1}(\tilde{a}), \tilde{a} \in w)\} \& \\ S^1 = \{(\tilde{s}, T_{S^1}(\tilde{s}), I_{S^1}(\tilde{s}), F_{S^1}(\tilde{s}), \tilde{s} \in Z)\}$$

The α , β , γ -cuts of entry period, utility period are represented as:

$$A^1(\alpha) = \{(\tilde{a}, T_{A^1}(\tilde{a}), I_{A^1}(\tilde{a}), F_{A^1}(\tilde{a}), \tilde{a} \in w)\} \& \\ S^1(\alpha) = \{(\tilde{s}, T_{S^1}(\tilde{s}), I_{S^1}(\tilde{s}), F_{S^1}(\tilde{s}), \tilde{s} \in Z)\}$$

The DSW calculation contains the following steps:

Step 1: Define (α, β, γ) -cut value where $0 \leq \alpha, \beta, \gamma \leq 1$.

Step 2: Next to compare with this α, β, γ find the membership performs intervals.

Step 3: The interval for the output membership perform is processed for the selected (α, β, γ) - cut measurements using common binary interval operation.

Step 4: Repeat steps 1-3 for various estimates of α, β, γ .

Formulation of NM/ND/1 Queuing Model

Consider a single server NM/ND/1 queuing model with deterministic service time. The neutrosophic entry period $\bar{A}$, neutrosophic utility period $\bar{S}$ are approximately known and are described as follows;

$$\bar{A} = \{(a, T_{\bar{A}}(a), I_{\bar{A}}(a), F_{\bar{A}}(a)) / a \in W\}$$

$$\bar{S} = \{(s, T_{\bar{S}}(s), I_{\bar{S}}(s), F_{\bar{S}}(s)) / s \in Z\}$$

where W and Z represents crisp sets of the neutrosophic entry period and neutrosophic utility period and $\mu_{\bar{A}}(a), \mu_{\bar{S}}(s)$ are the corresponding membership performs. The (α, β, γ) -cut of $\bar{A}$ and $\bar{S}$ are $A(\alpha, \beta, \gamma) = \{a \in X / T_{\bar{A}}(a) \geq \alpha, I_{\bar{A}}(a) \leq \beta, F_{\bar{A}}(a) \leq \gamma\}$ and $S(\alpha, \beta, \gamma) = \{s \in X / T_{\bar{S}}(s) \geq \alpha, I_{\bar{S}}(s) \leq \beta, F_{\bar{S}}(s) \leq \gamma\}$ where the $A(\alpha, \beta, \gamma)$ and $S(\alpha, \beta, \gamma)$ are the crisp subsets of W and Z respectively. Here we can represent neutrosophic inter entry time and neutrosophic overhauled time by the different levels of intervals using (α, β, γ) -cuts.

Consequently, a neutrosophic queue can be reduced to a family of crisp queues with different (α, β, γ) -cuts

$$\{A(\alpha, \beta, \gamma) : 0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1\}$$

$$\{S(\alpha, \beta, \gamma) : 0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1\}.$$

In this paper, we proposed a queuing model with both inter entry time $\bar{A}$ and overhauled time $\bar{S}$ are defined as SVTNN and the intervals of $\bar{A}$ and $\bar{S}$ are denoted by $[l_{\bar{A}(\alpha, \beta, \gamma)}, u_{\bar{A}(\alpha, \beta, \gamma)}]$ and $[l_{\bar{S}(\alpha, \beta, \gamma)}, u_{\bar{S}(\alpha, \beta, \gamma)}]$. Let us denote the performance measure by $P(\bar{A}, \bar{S})$ then using zadeh (1965) extension principle we can define the truth membership function, indeterminacy membership function and falsity membership function of $P(\bar{A}, \bar{S})$ is followed by,

$$T_{P(\bar{A}, \bar{S})}(w) = \sup\{\min_{a \in X, a' \in Y}(\mu_{\bar{A}}(a), \mu_{\bar{S}}(a')) : w = p(a, a')\}$$

$$I_{P(\bar{A}, \bar{S})}(w) = \inf\{\min_{a \in X, a' \in Y}(\mu_{\bar{A}}(a), \mu_{\bar{S}}(a')) : w = p(a, a')\}$$

$$F_{P(\bar{A}, \bar{S})}(w) = \inf\{\min_{a \in X, a' \in Y}(\mu_{\bar{A}}(a), \mu_{\bar{S}}(a')) : w = p(a, a')\}$$

We can find the lower and upper bounds of the (α, β, γ) cuts of $(\bar{A}, \bar{S})$ as follows;

$$l_{P(\alpha, \beta, \gamma)} = \min p(a, a') \quad \exists l_{\bar{A}(\alpha, \beta, \gamma)} \leq a \leq u_{\bar{A}(\alpha, \beta, \gamma)}, l_{\bar{S}(\alpha, \beta, \gamma)} \leq a' \leq u_{\bar{S}(\alpha, \beta, \gamma)} \quad (1)$$

$$u_{P(\alpha, \beta, \gamma)} = \max p(a, a') \quad \exists l_{\bar{A}(\alpha, \beta, \gamma)} \leq a \leq u_{\bar{A}(\alpha, \beta, \gamma)}, l_{\bar{S}(\alpha, \beta, \gamma)} \leq a' \leq u_{\bar{S}(\alpha, \beta, \gamma)} \quad (2)$$

Provided $a \in \bar{A}(\alpha, \beta, \gamma)$ and $a' \in \bar{S}(\alpha, \beta, \gamma)$. If both $l_{P(\alpha, \beta, \gamma)}$ and $u_{P(\alpha, \beta, \gamma)}$ are invertible with respect to (α, β, γ) then the left $L_T(w) = (l_{P(\alpha, \beta, \gamma)})^{-1}$ and right shape functions $R_T(w) = (u_{P(\alpha, \beta, \gamma)})^{-1}$ may be constructed and the truth membership function $\mu_{P(\bar{A}, \bar{S})}(w)$ is obtained by

$$T_{P(\bar{A}, \bar{S})}(w) = \begin{cases} L_T(w); w_1^T \leq w \leq w_2^T \\ R_T(w); w_3^T \leq w \leq w_4^T \\ 0; otherwise \end{cases}$$

where $w_1^T \leq w \leq w_4^T$ and $L_T(w_1^T) = R_T(w_4^T) = 0$ for the SVTNN.

Similarly, the indeterminacy membership function $\eta_{P(\bar{A}, \bar{S})}(w)$ and the falsity membership function $\zeta_{P(\bar{A}, \bar{S})}(w)$, are derived in the same way,

$$I_{P(\bar{A}, \bar{S})}(w) = \begin{cases} L_T(w); w_1^T \leq w \leq w_2^T \\ R_T(w); w_3^T \leq w \leq w_4^T \\ 0; otherwise \end{cases}$$

where $w_1^I \leq w \leq w_4^I$ and $L_I(w_1^I) = R_I(w_4^I) = 0$ for the SVTNN.

$$F_{P(\bar{A}, \bar{S})}(w) = \begin{cases} L_F(w); w_1^F \leq w \leq w_2^F \\ R_F(w); w_3^F \leq w \leq w_4^F \\ 0; otherwise \end{cases}$$

where $w_1^F \leq w \leq w_4^F$ and $L_F(w_1^F) = R_F(w_4^F) = 0$ for the SVTNN.

The proposed NM/ND/1 queue with deterministic service time can be reduced it to classical M/D/1 queue by using the concept of (α, β, γ) - cut approach.

Numerical illustration

Consider an NM/ND/1/ ∞ /FIFO model in which both the entry rate and utility rate are represented by single valued trapezoidal neutrosophic number. And from Equations (1) and (2) the membership functions of $\tilde{N}_{L_q}$, $\tilde{N}_{L_s}$ and $\tilde{N}_{w_q}$, $\tilde{N}_{w_s}$ are listed below.

$$\tilde{A} = [(2,3,4,5), (5,6,10,11), (3,4,7,8)]$$

$$\tilde{S} = [(7,8,9,10), (16,17,18,19), (14,15,16,17)]$$

The (α, β, γ) -cut of $\tilde{A}$ and $\tilde{S}$ are

$$\begin{aligned}\tilde{A} &= \langle [2 + \alpha, 5 - \alpha], [6 - \beta, 10 + \beta], [4 - \gamma, 7 + \gamma] \rangle \tilde{S} \\ &= \langle [7 + \alpha, 10 - \alpha], [17 - \beta, 18 + \beta], [15 - \gamma, 16 + \gamma] \rangle\end{aligned}$$

$$l_{L_q}(\alpha) = \min \left\{ \frac{e_1^2}{2e_2(e_2 - e_1)} \right\} \quad (3)$$

and

$$u_{L_q}(\alpha) = \max \left\{ \frac{e_1^2}{2e_2(e_2 - e_1)} \right\} \quad (4)$$

$$\text{Such that } \begin{aligned} 2 + \alpha &< e_1 < 5 - \alpha \\ 7 + \alpha &< e_2 < 10 - \alpha \end{aligned}$$

where $0 < \alpha \leq 1$, $l_{L_q}(\alpha)$ is discovered while e_1 approaching their lower bound (l.b) and e_2 approaching its upper bound (u.b) and as well as $u_{L_q}(\alpha)$ is discovered while e_1 approaching their u.b and e_2 approaching its l.b. As a result, the best solution for (3) and (4) is

$$\begin{aligned}l_{L_q}(\alpha) &= \frac{\alpha^2 + 4\alpha + 4}{4\alpha^2 - 56\alpha + 160}; \\ u_{L_q}(\alpha) &= \frac{\alpha^2 - 10\alpha + 25}{4\alpha^2 + 32\alpha + 28}\end{aligned}$$

The truth membership function is

$$T_{\tilde{N}_{L_q}}(w) = \begin{cases} L_T(w); & 0.0125 \leq w \leq 0.0833 \\ R_T(w); & 0.25 \leq w \leq 0.8928 \\ 0; & \text{otherwise} \end{cases}$$

$$l_{L_q}(\beta) = \min \left\{ \frac{e_1^2}{2e_2(e_2 - e_1)} \right\} \quad (5)$$

and

$$u_{L_q}(\beta) = \max \left\{ \frac{e_1^2}{2e_2(e_2 - e_1)} \right\} \quad (6)$$

$$\text{Such that } \begin{aligned} 6 - \beta &< e_1 < 10 + \beta \\ 17 - \beta &< e_2 < 18 + \beta \end{aligned}$$

where $0 < \alpha \leq 1$, $l_{L_q}(\beta)$ is discovered while e_1 approaching their lower bound (l.b) and e_2 approaching its upper bound (u.b) and as well as $u_{L_q}(\beta)$ is discovered while e_1 approaching their u.b and e_2 approaching its l.b. As a result, the best solution for (5) and (6) is

$$\begin{aligned}l_{L_q}(\beta) &= \frac{\beta^2 - 12\beta + 36}{4\beta^2 + 96\beta + 432}; \\ u_{L_q}(\beta) &= \frac{\beta^2 + 20\beta + 100}{4\beta^2 - 82\beta + 238}\end{aligned}$$

The indeterminacy membership function

$$I_{\tilde{N}_{L_q}}(w) = \begin{cases} L_I(w); & 0.0469 \leq zw \leq 0.0833 \\ R_I(w); & 0.4201 \leq w \leq 0.7562 \\ 0; & \text{otherwise} \end{cases}$$

$$l_{L_q}(\gamma) = \min \left\{ \frac{e_1^2}{2e_2(e_2 - e_1)} \right\} \quad (7)$$

and

$$u_{L_q}(\gamma) = \max \left\{ \frac{e_1^2}{2e_2(e_2 - e_1)} \right\} \quad (8)$$

$$\text{Such that } \begin{aligned} 4 - \gamma &< e_1 < 7 + \gamma \\ 15 - \gamma &< e_2 < 16 + \gamma \end{aligned}$$

where $0 < \alpha \leq 1$, $l_{L_q}(\gamma)$ is discovered while e_1 approaching their lower bound (l.b) and e_2 approaching its upper bound (u.b) and as well as $u_{L_q}(\gamma)$ is discovered while e_1 approaching their u.b and e_2 approaching its l.b. As a result, the best solution for (7) and (8) is

$$\begin{aligned}l_{L_q}(\gamma) &= \frac{\gamma^2 - 8\gamma + 16}{4\gamma^2 + 88\gamma + 384}; \\ u_{L_q}(\gamma) &= \frac{\gamma^2 + 14\gamma + 49}{4\gamma^2 - 76\gamma + 240}\end{aligned}$$

The falsity membership function is

$$F_{\tilde{N}_{L_q}}(w) = \begin{cases} L_F(w); & 0.0189 \leq w \leq 0.0416 \\ R_F(w); & 0.2041 \leq w \leq 0.38095 \\ 0; & \text{otherwise} \end{cases}$$

For different values of $\alpha, \beta, \gamma \in [0,1]$, expected no of consumers waiting in line $\tilde{N}_{L_q}$ is calculated and given in Table (1). Also, a graphical analysis of truth, indeterminacy and falsity of $\tilde{N}_{L_q}$ is shown in Figures 1, 2, and 3.

Table 1. α, β, γ cuts for expected no of consumers in the queue

α	$L_q(\alpha)$	$u_{L_q}(\alpha)$	β	$L_q(\beta)$	$u_{L_q}(\beta)$	γ	$L_q(\gamma)$	$u_{L_q}(\gamma)$
0	0.025	0.89285	0	0.08333	0.42016	0	0.04166	0.20416
0.1	0.02855	0.76856	0.1	0.07882	0.44383	0.1	0.03871	0.21687
0.2	0.03249	0.66666	0.2	0.07453	0.46915	0.2	0.03594	0.23044
0.3	0.03684	0.58192	0.3	0.07045	0.49630	0.3	0.03332	0.24494
0.4	0.04166	0.51061	0.4	0.06657	0.52545	0.4	0.03086	0.26046
0.5	0.04699	0.45	0.5	0.06289	0.55681	0.5	0.02855	0.27709
0.6	0.05287	0.39802	0.6	0.05938	0.59062	0.6	0.02637	0.29493
0.7	0.05938	0.35313	0.7	0.05605	0.62713	0.7	0.02433	0.31410
0.8	0.06657	0.31410	0.8	0.05287	0.66666	0.8	0.02240	0.33472
0.9	0.07453	0.27998	0.9	0.04986	0.70956	0.9	0.02060	0.35695
1	0.08333	0.25	1	0.04699	0.75625	1	0.01890	0.38095

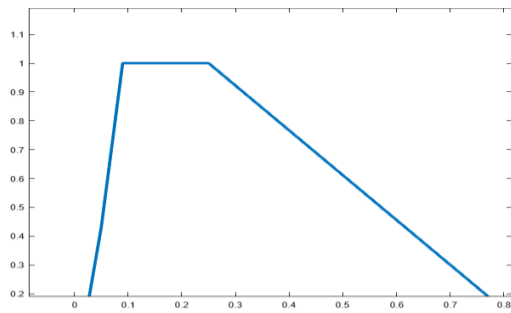


Figure 1. Truth membership value for in the queue $\tilde{N}_{L_q}$

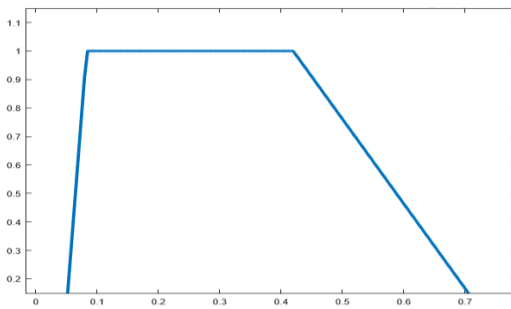


Figure 2. Indeterminacy membership value for $\tilde{N}_{L_q}$

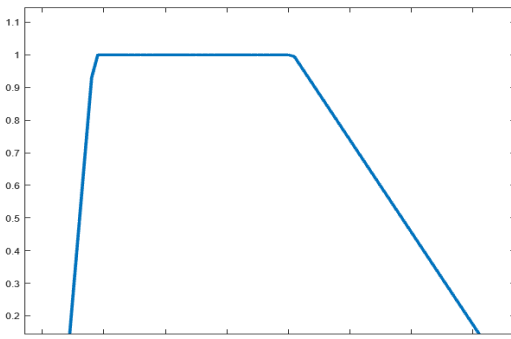


Figure 3. Falsity membership value for $\tilde{N}_{L_q}$

similarly the performance measure of $\tilde{N}_{L_S}$ for α -cut is given below,

$$l_{L_S}(\alpha) = \min \left\{ \frac{e_1}{e_2} + \frac{e_1^2}{2e_2(e_2 - e_1)} \right\} \quad (9)$$

The objective functions for $\tilde{N}_{L_S}$ is given through the Equations (9) and (10)

$$u_{L_S}(\alpha) = \max \left\{ \frac{e_1}{e_2} + \frac{e_1^2}{2e_2(e_2 - e_1)} \right\} \quad (10)$$

$$l_{L_S}(\alpha) = \frac{3\alpha^3 - 42\alpha^2 + 74\alpha + 360}{-4\alpha^3 + 96\alpha^2 - 720\alpha + 1600};$$

$$u_{L_S}(\alpha) = \frac{-3\alpha^3 - 15\alpha^2 + 87\alpha + 315}{4\alpha^3 + 60\alpha^2 + 252\alpha + 196}$$

$$T_{\tilde{N}_{L_S}}(w) = \begin{cases} L_T(w); & 0.225 \leq w \leq 0.4166 \\ R_T(w); & 0.75 \leq w \leq 1.6071 \\ 0; & \text{otherwise} \end{cases}$$

Similarly the performance measure of $\tilde{N}_{L_S}$ for β -cut is given below,

$$l_{L_S}(\beta) = \min \left\{ \frac{e_1}{e_2} + \frac{e_1^2}{2e_2(e_2 - e_1)} \right\} \quad (11)$$

$$u_{L_S}(\beta) = \max \left\{ \frac{e_1}{e_2} + \frac{e_1^2}{2e_2(e_2 - e_1)} \right\} \quad (12)$$

The objective functions for $\tilde{N}_{L_S}$ is given through the Equations (11) and (12)

$$l_{L_S}(\beta) = \frac{3\beta^3 - 66\beta^2 - 36\beta + 3240}{-4\beta^3 + 168\beta^2 + 2160\beta + 7776};$$

$$u_{L_S}(\beta) = \frac{3\beta^3 - 45\beta^2 - 342\beta + 4080}{-4\beta^3 + 150\beta^2 - 1632\beta + 4046}$$

$$I_{\tilde{N}_{L_S}}(w) = \begin{cases} L_I(w); & 0.3105 \leq w \leq 0.4166 \\ R_I(w); & 1.0084 \leq w \leq 1.4437 \\ 0; & \text{otherwise} \end{cases}$$

Similarly the performance measure of $\tilde{N}_{L_S}$ for γ -cut is given below,

$$l_{L_S}(\gamma) = \max \left\{ \frac{e_1}{e_2} + \frac{e_1^2}{2e_2(e_2 - e_1)} \right\} \quad (13)$$

$$u_{L_S}(\gamma) = \max \left\{ \frac{e_1}{e_2} + \frac{e_1^2}{2e_2(e_2 - e_1)} \right\} \quad (14)$$

The objective functions for $\tilde{N}_{L_S}$ is given through the Equations (13) and (14)

$$l_{L_S}(\gamma) = \frac{-3\gamma^3 - 74\gamma^2 - 64\gamma + 1632}{4\gamma^3 + 112\gamma^2 + 912\gamma + 2304},$$

$$u_{L_S}(\gamma) = \frac{3\gamma^3 - 47\gamma^2 - 131\gamma + 2415}{-4\gamma^3 + 136\gamma^2 - 1380\gamma + 3600}$$

$$F_{\tilde{N}_{L_S}}(w) = \begin{cases} L_F(w); & 0.4474 \leq w \leq 0.7083 \\ R_F(w); & 0.6708 \leq w \leq 0.9523 \\ 0; & \text{otherwise} \end{cases}$$

For different values of $\alpha, \beta, \gamma \in [0, 1]$, expected no of consumer count in the system $\tilde{N}_{L_S}$ is calculated and given in Table 2. Also, a graphical analysis of truth, indeterminacy and falsity of $\tilde{N}_{L_S}$ is shown in Figures 4, 5, and 6.

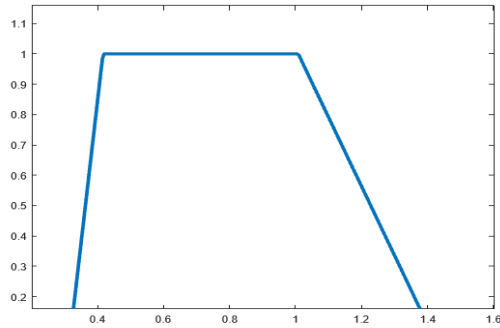


Figure 4. Truth membership value for $\tilde{N}_{L_S}$

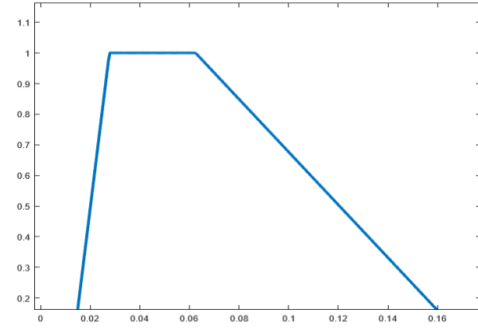


Figure 5. Indeterminacy membership value for $\tilde{N}_{L_S}$

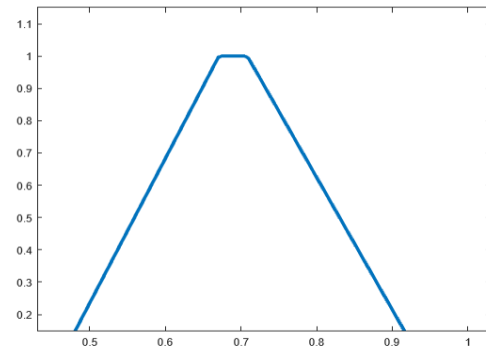


Figure 6. Falsity membership value for $\tilde{N}_{L_S}$

The objective functions for $\tilde{N}_{W_q}$ is given through the Equations (15) and (16)

$$l_{W_q}(\alpha) = \frac{2 + \alpha}{4\alpha^2 - 56\alpha + 160},$$

$$u_{W_q}(\alpha) = \frac{5 - \alpha}{4\alpha^2 + 32\alpha + 28} T_{\tilde{N}_{W_q}}(w) = \begin{cases} L_T(w); & 0.0125 \leq w \leq 0.0277 \\ R_T(w); & 0.0625 \leq w \leq 0.1785 \\ 0; & \text{otherwise} \end{cases}$$

Table 2. α, β, γ cuts for expected no of consumers in the system

α	$l_{L_S}(\alpha)$	$u_{L_S}(\alpha)$	β	$l_{L_S}(\beta)$	$u_{L_S}(\beta)$	γ	$l_{L_S}(\gamma)$	$u_{L_S}(\gamma)$
0	0.225	1.60714	0	0.416666	1.00840	0	0.70833	0.67083
0.1	0.24067	1.45870	0.1	0.40478	1.04146	0.1	0.67806	0.69338
0.2	0.25698	1.33333	0.2	0.393211	1.07629	0.2	0.64884	0.71692
0.3	0.27396	1.22576	0.3	0.381928	1.11307	0.3	0.62063	0.74154
0.4	0.29166	1.13223	0.4	0.370923	1.15196	0.4	0.59336	0.76731
0.5	0.31015	1.05	0.5	0.360187	1.19318	0.5	0.56701	0.79433
0.6	0.32947	0.97697	0.6	0.34970	1.23696	0.6	0.54153	0.82271
0.7	0.34970	0.91157	0.7	0.33947	1.28357	0.7	0.51686	0.85256
0.8	0.37092	0.85256	0.8	0.329474	1.33333	0	0.49299	0.88402
0.9	0.39321	0.79896	0.9	0.31970	1.38658	0.9	0.46987	0.91723
1	0.41666	0.75	1	0.310150	1.44375	1	0.44747	0.95238

similarly the performance measure of $\tilde{N}_{wq}$ for α -cut is given below,

$$l_{wq}(\alpha) = \min \left\{ \frac{e_1}{2e_2(e_2 - e_1)} \right\} \quad (15)$$

and

$$u_{wq}(\alpha) = \max \left\{ \frac{e_1}{2e_2(e_2 - e_1)} \right\} \quad (16)$$

similarly the performance measure of $\tilde{N}_{wq}$ for β -cut is given below,

$$l_{wq}(\beta) = \min \left\{ \frac{e_1}{2e_2(e_2 - e_1)} \right\} \quad (17)$$

and

$$u_{wq}(\beta) = \max \left\{ \frac{e_1}{2e_2(e_2 - e_1)} \right\} \quad (18)$$

The objective functions for $\tilde{N}_{wq}$ is given through the Equations (17) and (18)

$$l_{wq}(\beta) = \frac{6 - \beta}{4\beta^2 + 96\beta + 432}; u_{wq}(\beta) = \frac{10 + \beta}{4\beta^2 - 82\beta + 238}$$

$$I_{\tilde{N}_{wq}}(w) = \begin{cases} L_I(w); 0.0093 \leq w \leq 0.01388 \\ R_I(w); 0.04201 \leq w \leq 0.06875 \\ 0; \text{otherwise} \end{cases}$$

performance measure of $\tilde{N}_{wq}$ for γ -cut is given below,

$$l_{wq}(\gamma) = \min \left\{ \frac{e_1}{2e_2(e_2 - e_1)} \right\} \quad (19)$$

and

$$u_{wq}(\gamma) = \max \left\{ \frac{e_1}{2e_2(e_2 - e_1)} \right\} \quad (20)$$

The objective functions for $\tilde{N}_{wq}$ is given through the equations (19) and (20)

$$l_{wq}(\gamma) = \frac{4 - \gamma}{4\gamma^2 + 88\gamma + 384};$$

$$u_{wq}(\gamma) = \frac{7 + \gamma}{4\gamma^2 - 76\gamma + 240}$$

$$F_{\tilde{N}_{wq}}(w) = \begin{cases} L_F(w); 0.0063 \leq w \leq 0.01041 \\ R_F(w); 0.02916 \leq w \leq 0.0476 \\ 0; \text{otherwise} \end{cases}$$

For different values of $\alpha, \beta, \gamma \in [0,1]$, expected time of an average customer waits in queue $\tilde{N}_{wq}$ is calculated and given in Table (3). Also, a graphical analysis of truth, indeterminacy and falsity of $\tilde{N}_{wq}$ is shown in Figures 7, 8, and 9.

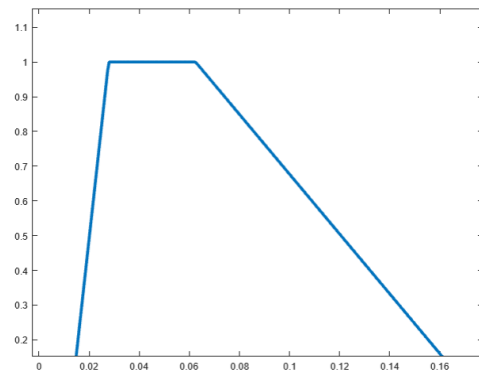


Figure 7. Truth membership value for $\tilde{N}_{wq}$

Table 3. α, β, γ cuts for expected no of consumers waiting in queue

α	$l_{wq}(\alpha)$	$u_{wq}(\alpha)$	β	$l_{wq}(\beta)$	$u_{wq}(\beta)$	γ	$l_{wq}(\gamma)$	$u_{wq}(\gamma)$
0	0.0125	0.17857	0	0.01388	0.04201	0	0.01041	0.02916
0.1	0.01359	0.15685	0.1	0.01335	0.04394	0.1	0.00992	0.03054
0.2	0.01476	0.13888	0.2	0.01285	0.04599	0.2	0.00945	0.03200
0.3	0.01602	0.12381	0.3	0.01236	0.04818	0.3	0.00900	0.03355
0.4	0.01736	0.11100	0.4	0.01188	0.05052	0.4	0.00857	0.03519
0.5	0.01879	0.1	0.5	0.01143	0.05303	0.5	0.00815	0.03694
0.6	0.02033	0.09046	0.6	0.01099	0.05571	0.6	0.00775	0.03880
0.7	0.02199	0.08212	0.7	0.01057	0.05861	0.7	0.00737	0.04079
0.8	0.02377	0.07478	0.8	0.01016	0.06172	0.8	0.00700	0.04291
0.9	0.0257	0.06828	0.9	0.00977	0.06509	0.9	0.00664	0.04518
1	0.02777	0.0625	1	0.00939	0.06875	1	0.00630	0.04761

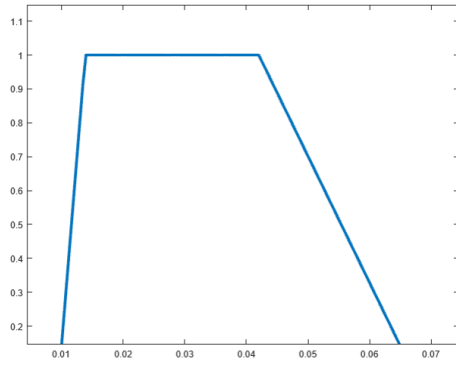


Figure 8. Indeterminacy membership value for $\tilde{N}_{W_q}$

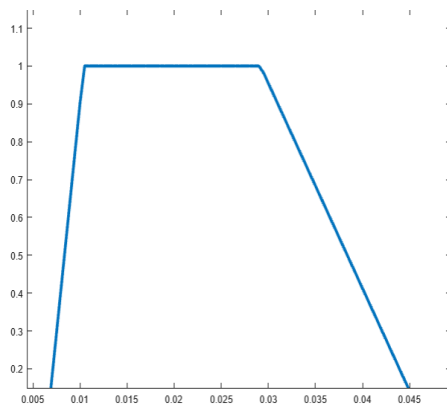


Figure 9. Falsity membership value for $\tilde{N}_{W_q}$

$$T_{\tilde{N}_{W_s}}(w) = \begin{cases} L_T(w); 0.1125 \leq w \leq 0.1388 \\ R_T(w); 0.3213 \leq w \leq 0.1875 \\ 0; \text{otherwise} \end{cases}$$

similarly the performance measure of $\tilde{N}_{W_s}$ for α -cut is given below,

$$l_{W_s}(\alpha) = \min \left\{ \frac{1}{e_2} + \frac{e_1}{2e_2(e_2 - e_1)} \right\} \quad (21)$$

and

$$u_{W_s}(\alpha) = \max \left\{ \frac{1}{e_2} + \frac{e_1}{2e_2(e_2 - e_1)} \right\} \quad (22)$$

The objective functions for $\tilde{N}_{W_s}$ is given through the equations (21) and (22)

$$l_{W_s}(\alpha) = \frac{3\alpha^2 - 48\alpha + 180}{-4\alpha^3 + 96\alpha^2 - 720\alpha + 1600};$$

$$u_{W_s}(\alpha) = \frac{3\alpha^2 + 30\alpha + 63}{4\alpha^3 + 60\alpha^2 + 252\alpha + 196}$$

similarly the performance measure of $\tilde{N}_{W_s}$ for β -cut is given below,

$$l_{W_s}(\beta) = \min \left\{ \frac{1}{e_2} + \frac{e_1}{2e_2(e_2 - e_1)} \right\} \quad (23)$$

and

$$u_{W_s}(\beta) = \max \left\{ \frac{1}{e_2} + \frac{e_1}{2e_2(e_2 - e_1)} \right\} \quad (24)$$

The objective functions for $\tilde{N}_{W_s}$ is given through the Equations (23) and (24)

$$l_{W_s}(\beta) = \frac{3\beta^2 + 84\beta + 540}{4\beta^3 + 168\beta^2 + 2160\beta + 7776};$$

$$u_{W_s}(\beta) = \frac{3\beta^2 - 75\beta + 408}{-4\beta^3 + 150\beta^2 - 1632\beta + 4046}$$

$$I_{\tilde{N}_{W_s}}(w) = \begin{cases} L_I(w); 0.0619 \leq w \leq 0.0693 \\ R_I(w); 0.1008 \leq w \leq 0.1312 \\ 0; \text{otherwise} \end{cases}$$

similarly the performance measure of $\tilde{N}_{W_s}$ for γ -cut is given below,

$$l_{W_s}(\gamma) = \min \left\{ \frac{1}{e_2} + \frac{e_1}{2e_2(e_2 - e_1)} \right\} \quad (25)$$

and

$$u_{W_s}(\gamma) = \max \left\{ \frac{1}{e_2} + \frac{e_1}{2e_2(e_2 - e_1)} \right\} \quad (26)$$

The objective functions for $\tilde{N}_{W_s}$ is given through the equations (25) and (26)

$$l_{W_s}(\gamma) = \frac{3\gamma^2 + 76\gamma + 448}{4\gamma^3 + 152\gamma^2 + 1792\gamma + 6144};$$

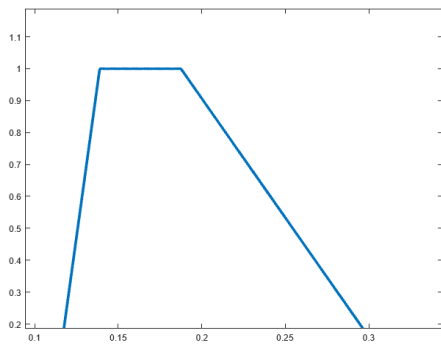
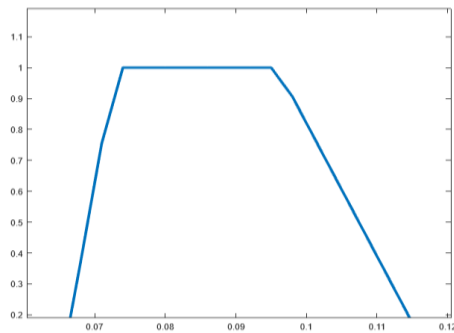
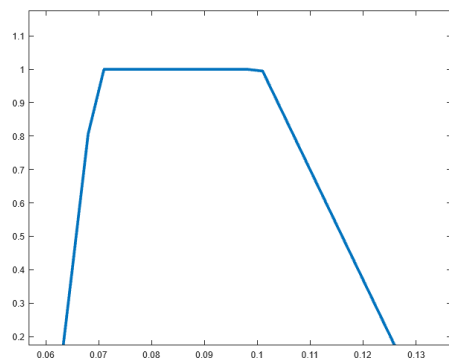
$$u_{W_s}(\gamma) = \frac{3\gamma^2 - 68\gamma + 345}{-4\gamma^3 + 136\gamma^2 - 1380\gamma + 3600}$$

$$F_{\tilde{N}_{W_s}}(w) = \begin{cases} L_F(w); 0.0651 \leq w \leq 0.0729 \\ R_F(w); 0.0957 \leq w \leq 0.11904 \\ 0; \text{otherwise} \end{cases}$$

For different values of $\alpha, \beta, \gamma \in [0,1]$, system's average client hold time $\tilde{N}_{W_s}$ is calculated and given in Table (4). Also, a graphical analysis of truth, indeterminacy and falsity of $\tilde{N}_{W_s}$ is shown in Figures 10, 11, and 12.

Table 4. α, β, γ cuts system's average client hold time

α	$l_{ws}(\alpha)$	$u_{ws}(\alpha)$	β	$l_{ws}(\beta)$	$u_{ws}(\beta)$	γ	$l_{ws}(\gamma)$	$u_{ws}(\gamma)$
0	0.1125	0.32142	0	0.06944	0.10084	0	0.07291	0.09583
0.1	0.11460	0.29769	0.1	0.06860	0.10311	0.1	0.07204	0.09766
0.2	0.11681	0.27777	0.2	0.06779	0.10551	0.1	0.07204	0.09766
0.3	0.11911	0.26080	0.3	0.06700	0.10806	0.3	0.07035	0.10158
0.4	0.12152	0.24613	0.4	0.06623	0.11076	0.4	0.06955	0.10369
0.5	0.12406	0.23333	0.5	0.06548	0.11363	0.5	0.06876	0.10591
0.6	0.12672	0.22203	0.6	0.06476	0.11669	0.6	0.06799	0.10825
0.7	0.12952	0.21199	0.7	0.06405	0.11996	0.7	0.06725	0.11072
0.8	0.13247	0.20299	0.8	0.06336	0.12345	0.8	0.06652	0.11333
0.9	0.13559	0.19487	0.9	0.06268	0.12721	0.9	0.06581	0.11610
1	0.13888	0.1875	1	0.06203	0.13125	1	0.06512	0.11904

**Table 10.** Truth membership value for $\tilde{N}_{ws}$ **Table 11.** Indeterminacy membership value for $\tilde{N}_{ws}$ **Table 12.** Falsity membership value for $\tilde{N}_{ws}$

Conclusions

Here, we hypothesis that neutrosophic set hypothesis and lining hypothesis have a stronger relationship than before the landing and during administration times. The execution proportions, such as system size, line size, system period, line period, and so on, are also ambiguous in general. The accuracy of

DSW computation is shown by numerical illustration and the performance measures of the system can be handled in a neutrosophic environment. The suggested model's entry period and utility period are both stated as single valued neutrosophic numbers. Here, it should be noted that adding more variables can improve the performance of the queuing model. There are other angles from which this work could be expanded. Some of those is to consider random variables, to utility rate and entry rate. This study might be utilised to study a new dimension of neutrosophic set developments like Pythagorean and Fermatean neutrosophic sets.

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