

SLENDERNESS EFFECTS IN SFRC LONG BEAMS

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Abstract

This study presents the findings from nine laboratory tests on rectangular long beams made of Reinforced Concrete (RC) and Steel Fiber Reinforced Concrete (SFRC). The experimental investigation shows that the long beams with SFRC show different behaviour in terms of load-deflection curve as compared to the RC long/slender beams. This study examines hooked-end steel fibers with two different aspect ratios i.e. 63.63 and 77.78. Three different grades of concrete (M25, M40 and M55) are considered to study as to how the variation in grade of concrete impacts the slenderness of the steel fibre RC long beams. The experimental study however shows that the deflection control of steel fibres with aspect ratio 63.63 is better than the steel fibres with aspect ratio 77.78. Also, it has been observed that the beams with grade of concrete M55 has a lower load carrying capacity as compared to other two grades as it fails due to instability (bending/twisting) at lower values of load. Finite Element (FE) analysis using ATENA confirms the findings from laboratory tests.

Keywords: ATENA; Hooked End; Long; Slender; Slenderness; Steel Fibres

Introduction

Slender RC beams exhibit different structural behavior compared to regular-sized beams. The abrupt destabilization of these differentially long beams is identified to be the cause of failure. A comparatively wide beam would indeed be sensitive to the consequences of slenderness, failing in bending at a load equal to the bending capacity, signifying structural collapse. Regardless of the concept that engineering guidelines (British Standards Institution, 1997; Bureau of Indian Standards, 2000; Council of Standards Australia, 2001; European Committee for Standardization, 2003; American Concrete Institute (ACI) Committee 318, 2008) specified slenderness with respect to beam specifications, research has shown

that the slenderness of RC beams is influenced by a variety of design elements. In BIS building codes, there is presently no method for analyzing M_{br} (moment corresponding to critical buckling), for RC slender beams and M_{uf} (moment at ultimate flexure) for steel fiber RC long beams. The conduct of thin columns has also been the subject of extensive research across the world. Internationally, much research on the performance of slender pillars has been done and shows that in terms have been incorporated into industry standards (Cortes *et al.*, 2011; Pillai and Menon, 2009). According to Hansell and Winter (1959), Marshall was the pioneer to examine the endurance of reinforced cement concrete (RCC) beams, longitudinally, in the 1940.

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They confirmed this with scientific research and concluded that, contrasting to common perception, the ratio of length-to-width of the beam is not a true gauge for slenderness in case of long/slender beams, as stated in the ACI (American Concrete Institute) code of practice. Siev (1960) conducted laboratory studies to investigate the impact of reinforcement amount and the effect of concrete grade used in the design mix on the performance of slender reinforced beams and concluded that RCC beams cast with higher-grade design mixes are more prone to collapse due to instability. Sant and Bletzacker (1961) investigated several over-reinforced RC long beams, developed a quantitative method for predicting M_{bcr} , and highlighted the importance of the depth-to-width (d/b) ratio. To compensate for the failure, Massey (1967) emphasized the role of reinforcements, including longitudinal rebars and lateral stirrups, along with the quality of concrete. Massey and Walter (1969) further developed this concept by investigating RCC beams simply supported at both ends under point loads.

Revathi and Menon (2006) conducted seven experiments, Sant and Bletzacker (1961) developed three tests, and Massey (1967) planned eleven tests to verify the proposed M_{bcr} formula. Because flexural strength in most cases was reduced owing to slenderness, as reported by Girija and Menon (2011), Revathi and Menon (2007) examined eight beams that deformed by buckling, with failure occurring due to flexural tension. Further tests on RCC beams with rectangular cross sections, as well as the effects of variations in horizontal and vertical reinforcement proportions, are needed to corroborate the recommended M_{bcr} and moment reduction factor formulations. Girija and Menon (2011) also performed 15 laboratory experiments on thin rectangular cross section RC beams and amended the formulation for moment due to critical buckling, as proposed by Revathi and Menon (2007), to accommodate the non-linear response of concrete leading to cracking. In particular, an improved system of equations for estimating the slenderness ratio, λ , was proposed (Girija and Menon, 2011).

$$\lambda = \sqrt{\frac{M_{uf}}{M_{bcr}}}$$

Concrete loses its tensile strength and collapses in a delicate way due to its lower ductility (Chalioris *et al.*, 2019). By incorporating additional fibres as volumetric reinforcement, reinforced cement concrete systems may withstand shearing as well as solve faults in concrete, minimizing and postponing

crack development and enhancing overall functionality (Chalioris *et al.*, 2019). Owing to the fibres' ability to convey tensile stresses across fractured surfaces, concrete strengthened with irregularly shaped tiny steel fibres distributed in a random fashion is regarded as a composite material with good fracture toughness (crack-bridging) (Simoes *et al.*, 2017). In compression, SFRC exhibits improved endurance only in composites with a large number of fibres and an adequate aspect ratio (Shon *et al.*, 2019); nonetheless, in most cases analyzed, the fibres have only a minor impact on compressive behaviour (Wang and Gao, 2016; Caggiano *et al.*, 2017). The deformation response of steel fibrous concrete reinforced with steel bars was investigated using four-point bending tests on comparatively small samples. According to a series of studies, adding steel fibres to the post-fracture protocol increased the response (Bencardino *et al.*, 2013).

Long steel fibres outperform shorter ones in managing bending response at considerable deflections, which was used to estimate robustness, failure tolerance, toughness, and fracturing efficiency (Yoo *et al.*, 2018). Deformed hooked-end steel fibres with a higher length-to-diameter ratio have been shown to effectively enhance fracture resistance, energy loss, and ductility indices (Barros and Figueiras, 1999; Campione and Mangiavillano, 2008). In addition, the study found that industrially cut short steel fibres with a low aspect ratio (≈ 40), embedded at a volume fraction of approximately 2%, were able to improve segmental flexural toughness and tensile stress equivalent to the crosswise deformations of RC columns under eccentric compressive force (Li *et al.*, 2019). Numerous research journals, international conferences, and academic panels (ACI Committee 544, 1988; Vandewalle, 2000; fib Bulletin 57, 2010) have revealed that the construction market has been exploring the use of steel fibres as a stirrup alternative in RC structural systems for the past 40 years. However, due to a lack of technical requirements in existing legislation, the role of steel fibres in the industry has been limited. Since SFRC finds major applications in pavements, runways, and other structures, this study further suggests that its use in pavements can be enhanced (Hoy *et al.*, 2024; Kantatham *et al.*, 2024). The purpose of this study is to determine how adding steel fibres to concrete long beams impacts deflection regulation and to demonstrate that steel fibres further enhance deflection control. Additionally, this would improve understanding of how the slenderness of SFRC long beams is influenced by the strength of the concrete. Furthermore, it also investigates the impact of changing the aspect ratio of these steel fibres on deflection and load-bearing capacity.

Experimentation Method

A total of nine under-reinforced RC beams with rectangular cross-sections were tested experimentally at the Heavy Laboratory Facility of Guru Nanak Dev Engineering College in Ludhiana. The beams were modelled and casted with specifications as in Table 1. Steel fibres were not present in three samples RCC beams for the purpose of drawing comparison. On the other hand, the remaining 6 samples were casted with 1% steel fibres.

To analyze the effect of steel fiber aspect ratio on load-bearing capacity, hooked-end steel fibers with two different aspect ratios were utilized. Owing to the market availability, the steel fibres with hooked end shape and aspect ratios 63.63 and 77.78 were used in the study. In the longitudinal and horizontal planes, the beams were simply supported but were torsionally restricted and permitted to rotate. The test setup as in the laboratory study is as

shown in Figure 1(a) and corresponding schematic diagram is as shown in Figure 1(b). This test set up was led by Hansell and Winter (1959) investigation, and it was kept expressly the same as in prior experiments. It was made up of two load assembly units that utilised rollers between two steel plates to transfer the applied point loads with minimum lateral constraints. A preliminary test was performed utilizing the load arrangement to check that the imposed loading only restricts the beam's lateral displacement minutely.

Specimens

The ratio of limiting slenderness developed by Revathi and Menon (2007) was used to determine the beam size. The tensile reinforcement in all the test samples was kept constant at 1 percent. In all beams, an initial irregularity of 1-8 mm was noted. The test beam data is as shown in Table 1.

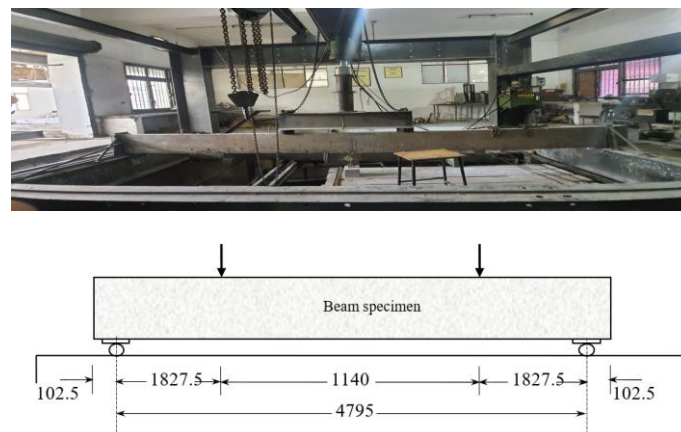


Figure 1. (a) Test Setup for experimental study and (b) Schematic diagram of the test Setup for experimental study

Table 1. Test beam Data

S.No.	Beam Sticker	Dim. $b \times D \times L$ (mm)	Fibre type	Aspect Ratio	f_{ck}	E_c	Initial Imperfection (mm)	Remarks
1	B1S0	$80 \times 360 \times 5000$	-	-	31.00	27838.82	4	Control beam (without fibres-grade C1)
2	B1S1	$80 \times 360 \times 5000$	Hooked end	63.63	33.40	28896.37	1	Steel fibre RC beam with C1 grade of concrete
3	B1S2	$80 \times 360 \times 5000$	Hooked- end	77.78	34.40	29240.38	3	Steel fibre RC beam with C1 grade of concrete
4	B2S0	$80 \times 360 \times 5000$	-	-	42.20	32480.76	3	Control beam (without fibres-grade C2)
5	B2S1	$80 \times 360 \times 5000$	Hooked end	63.63	43.00	32787.19	1	Steel fibre RC beam with C2 grade of concrete
6	B2S2	$80 \times 360 \times 5000$	Hooked end	77.78	42.00	32403.70	1	Steel fibre RC beam with C2 grade of concrete
7	B3S0	$80 \times 360 \times 5000$	-	-	55.0	37080.99	3	Control beam (without fibres-grade C3)
8	B3S1	$80 \times 360 \times 5000$	Hooked end	63.63	57.4	37881.39	4	Steel fibre RC beam with C3 grade of concrete
9	B3S2	$80 \times 360 \times 5000$	Hooked- end	77.78	58.7	38373.17	3	Steel fibre RC beam with C3 grade of concrete

Table 2. Materials

Fine Aggregates	Zone II, Fineness Modulus = 4.96, Sp. Gr. = 2.6
Coarse Aggregates	Crushed granite stone-size 16mm(max.), Fineness Modulus = 1.98, Sp. Gr. = 2.97
Cement	PPC, OPC 43[Ultratech]
Admixture (Adm.)	FOSROC - Auramix 400
Concrete (Cement: Fine Aggregate: Coarse Aggregate: Water), Adm.	• C1-1:2.16:2.66::0.47(with PPC), Adm: 0.4% [Avg. grade of concrete-M25] • C2-1:1.62:2.24::0.39(with PPC), Adm: 0.42% [Avg. grade of concrete-M40] • C3-1:1.37:2.84::0.34(with cement as OPC 43 grade, Silica Fumes 0.0625 and Fly ash 0.18), Adm: 0.5% [Avg. grade of concrete-M55]
Steel	Reinforcing bars-Fe 500

Materials

The materials used during the experimental investigation are as shown in Table 2.

Arrangement for the specimen and the test technique

The lateral and longitudinal deflections at the effective span's midpoint were measured using dial gauges with a precision of 0.01 mm. It was determined that the load was correctly applied. All the beams were progressively stressed, with the predicted collapse occurring after 10 to 20 load steps. The beams were loaded to failure, as predicted by Revathi and Menon's prior experimental results (Revathi and Menon, 2006; Revathi and Menon, 2007), and dial gauges were used to measure the full range of deflection for each load step. The peak load and the path of collapse were also reported. Table 2 also shows the experimental and finite element analysis (FEA) test beam failure loads, as well as the various failure modes observed throughout both studies employing design codes (British Standards Institution, 1997; Bureau of Indian Standards, 2000; Council of Standards Australia, 2001; European Committee for Standardization, 2003; American Concrete Institute (ACI) Committee 318, 2008).

Mechanism of Collapse

Case 1: Failure due to flexural tension (i.e. negligible transverse deflection)

When rupture occurs owing to bending, lateral displacement is minimal or negligible, and it is most seen in the soffit zone, expanding progressively to both sides under load. These fissures develop higher, eventually resulting to final failure owing to concrete breaking in the compression side.

Case 2: Tensile collapse but with increased flexural capacity (i.e. with lateral deflection)

When fractures due to flexure, form because of a force applied with a regulated lateral displacement. Under increased loading, cracks were commonly seen in the soffit area, expanding progressively to both sides. These breaks spread higher in the same way as Case 1 did, however in

this type of collapse the beam can bear more load than expected, resulting to eventual collapse owing to higher vertical displacement due to bending without indication.

Case 3: Instability leading to failure (i.e. large lateral deflections)

Due to substantial horizontal deflection or else increasing twisting with load and fractures, the beam is unable to support any additional weight. The beam fails rapidly when it moves out of the loading arrangement owing to considerable lateral displacement, and the magnitude of load was less than the estimated flexural load bearing capacity of the beam.

Numerical Validation (FEA Analysis)

All the beams which were experimentally tested were modelled in ATENA for Finite Element Analysis (FEA). ATENA 3D v5 was used to perform the evaluation. To get the accurate results in ATENA, in line with results from experimental study, it is mandatory to replicate the same boundary conditions for the beam. The geometry of the beam and corresponding input for the material must be same as in the experimental testing. For material definition, concrete was represented using 3D Non-Linear Cementitious 2 material and for each case, the value of compressive strength of concrete (f_{ck}) required as an input in ATENA is same as in the experimental study, whereas rebars and stirrups were modelled using Reinforcement material. Steel plates with 3D Elastic isotropic material were employed to apply load and provide support to the beam. The beams were modelled in ATENA with the identical support conditions and loaded in the same pattern as in experimental testing, and a four-point bending test was performed as shown in Figure 2(a). The end conditions of the beam were applied such that, one end of the beam was confined in the X and Z pathways, while the other end had to be restricted in the Y and Z orientations. As in a four-point bending test, two steel plates were used

to deliver point loads at the middle third span points. The beams in ATENA were modeled in accordance with the experimental investigation to obtain the most precise outcomes. The deflected shape of the beam in ATENA on the application of load and the corresponding flexural cracks in the beam after failure as visible in ATENA is as shown in Figure 2(b) and Figure 2(c) respectively.

The outcomes of both qualitative and quantitative research are as shown in Table 3, and it can be stated that experimental and analytical results do not differ considerably. The numerical values variate to almost 10% owing to the environmental

factors, constraints on measurement accuracy, and testing variability contributing to fluctuations in load-bearing capacity. The numerical analysis also shows a similar load-deflection curve profile to the experimental results as shown in Figures 3-11. In ATENA, a load deflection curve was obtained, and the reading of the failure load was noted, as well as the manner of failure. Further in Section 4.2, the LD Curves for different sets of concrete mixes are as shown in Figures 12-14 and fibre type are shown in Figures 15-16 were contrasted to note the conclusions of the research.

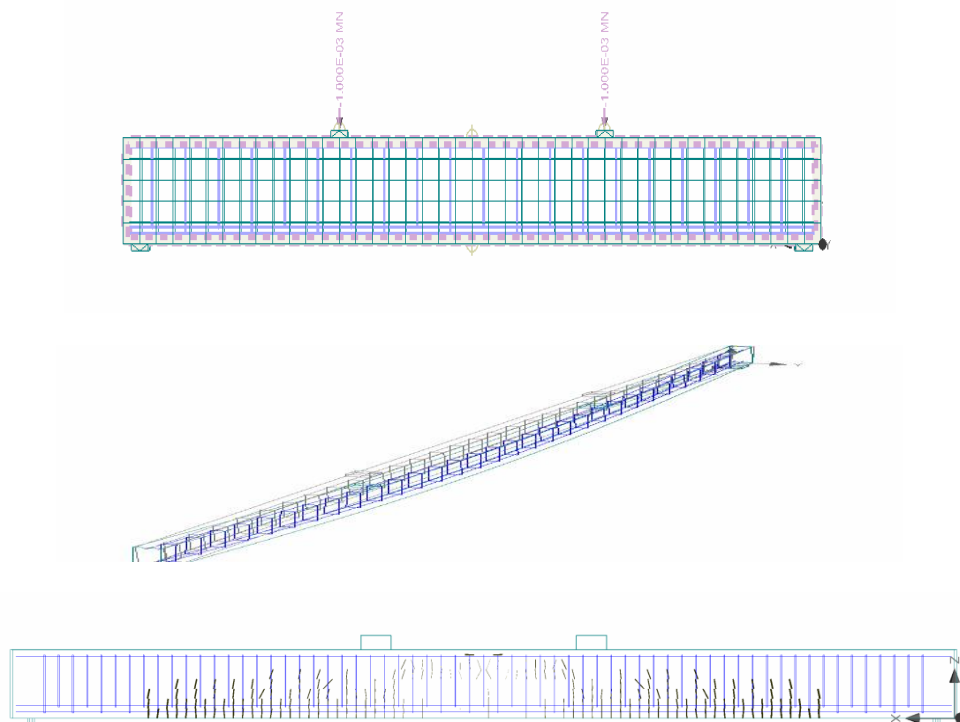


Figure 2. (a) shows a beam model as in ATENA, (b) shows the deflected beam in ATENA on the application of load and (c) shows the flexural cracks in the beam as visible in ATENA

Table 3. Comparison of results from experimental and FEA analysis

S.No.	Beam Label	Experimental Peak Load (kN)	Peak load from FEA(ATENA) (kN)	Type of Collapse from Experimental test Results	Type of Collapse from ATENA (Failure with warning or Instability Failure)
1	B1S0	43.200	46.000	Flexural tension failure (with negligible lateral deflection)	Failure with warning
2	B1S1	48.500	47.920	Flexural collapse with improved tensile performance but with horizontal deflection	Failure with warning
3	B1S2	48.400	48.540		
4	B2S0	52.500	47.480		
5	B2S1	50.200	49.400		
6	B2S2	52.900	49.200	Collapse because of sudden instability due to large lateral deflections	Instability Failure
7	B3S0	44.80	49.4		
8	B3S1	50.80	48.6		
9	B3S2	46.20	48.2		

Results and Discussions

Based on the experimental and Numerical investigations the results are as shown in Table 3.

Load vs deflection (LD) curves

Below are the Load vs deflection (LD) curves presented from Figures 3-11, for all the experimentally tested beams, along with comparison to the LD curves as obtained from ATENA analysis for the same beam.

- Load deflection curves for C1 grade:

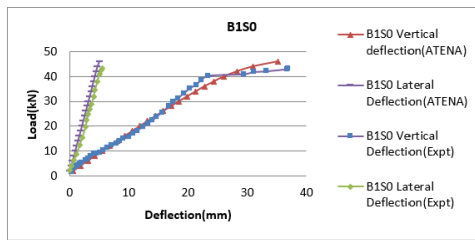


Figure 3. Load vs deflection curve for B1S0

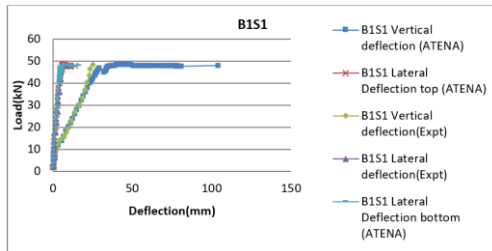


Figure 4. Load vs deflection curve for B1S1

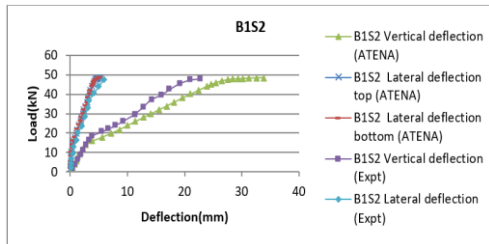


Figure 5. Load vs deflection curve for B1S2

- Load deflection curves for C2 grade:

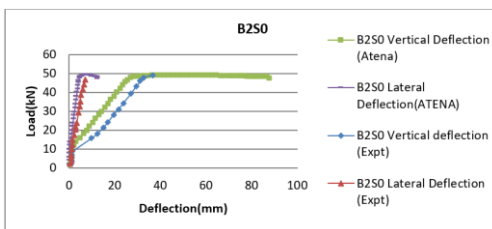


Figure 6. Load vs deflection curve for B2S0

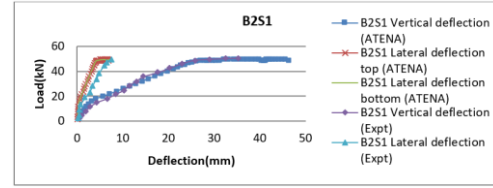


Figure 7. Load vs deflection curve for B2S1

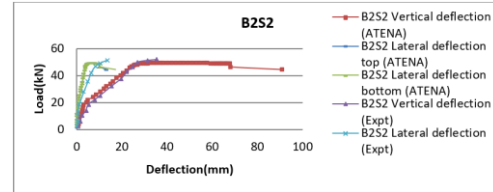


Figure 8. Load vs deflection curve for B2S2

- Load deflection curves for C3 grade:

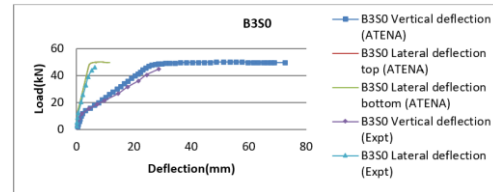


Figure 9. Load vs deflection curve for B3S0

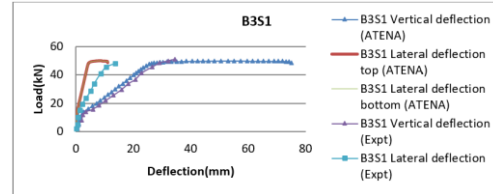


Figure 10. Load vs deflection curve for B3S1

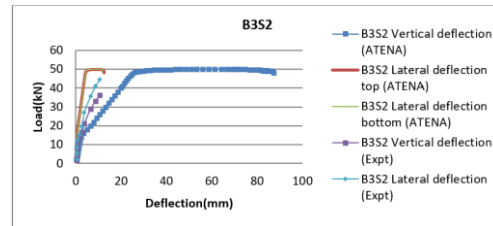


Figure 11. Load vs deflection curve for B3S2

From the above presented LD curves, it has been inferred that performance of SFRC slender beams is attributed to the concrete grade, aspect ratio of steel fibres. The load deflection curve obtained from laboratory test in comparison to the load deflection curve obtained from ATENA analysis are as shown in Figures 3-11. The vertical and lateral deflections of both laboratory and ATENA analysis are contrasted and have found to agree. It has been concluded that the ATENA

analysis gives the larger values of deflection as compared to the laboratory test results. The load setup in laboratory tests allowed a lateral deflection 20mm and hence due to the practical difficulties, the tests had to be terminated in these cases.

LD curves (Figure 12-16)

Comparing Load vs deflection curve obtained from Experimental investigation for Grade of concrete C1, C2 and C3- (Figures 12-14).

There is significant improvement seen in the load carrying capacity of lower grade concrete C1 with the incorporation of steel fibres in the RC long beams as shown in Figure 12. The flexural strength enhanced upon adding steel fibres. Beams with hooked end fibres having aspect ratio 63.63 shows better response as compared to the beams having hooked end fibres with higher aspect ratio i.e. 77.78.

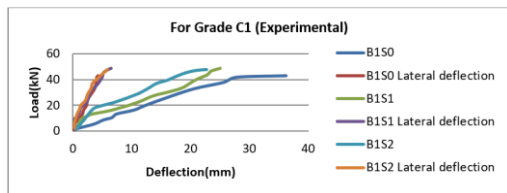


Figure 12. Load vs deflection curves for Grade C1

From the above L-D curve as shown in Figure 13, it has been inferred that the control beam shows higher load carrying capacity. There is no significant increase in load carrying capacity upon addition of steel fibres, but a good ductile response is exhibited by the SFRC beams as compared to control beams. The beams with hooked end steel fibres with higher aspect ratio show a minimal improvement in load carrying capacity but better ductility was shown as compared to SFRC beams with lower aspect ratio.

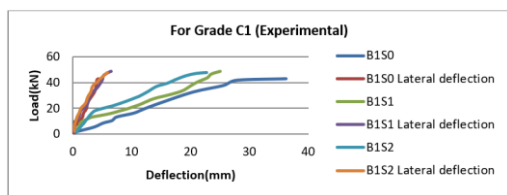


Figure 13. Load vs deflection curve for Grade C2

It was seen that the higher strength control beam had shown significantly reduced load carrying capacity because of manifestation of slenderness effects in terms of lateral deflections. SFRC beams of grade C3 as shown in Figure 14, failed due to sudden instability and hence showed significant reduction in flexural strength as compared to beams with grade C1 and C2. Large lateral deflections were

observed such that the beams moved out of the load assembly and sudden instability was observed. This has been attributed to the due to the loss of lateral flexural rigidity in case of higher grade of concrete beams and the decrease in effective torsional rigidity which indicates the higher resistance to lateral instability (Massey 1967). The steel fibres enhanced the ductile response and there was a slight increase in the load carrying capacity of these beams in comparison to the control beams. Therefore, the SFRC beams with hooked end steel fibres with aspect ratio 63.63 show better ductile response and increased flexural strength but failed due to instability.

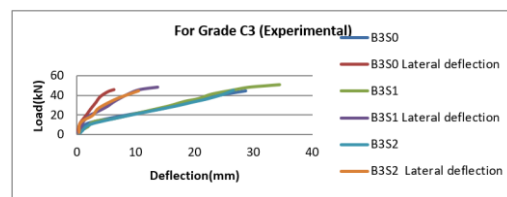


Figure 14. Load vs deflection curve for Grade C3

Load vs deflection curve for fibre types S1 and S2 (Figures 15-16).

It has been observed from L-D curve as shown in Figure 15, the fibre type S1 (i.e. hooked end steel fibre with aspect ratio 63.63) tends to show better ductile response and increased flexural strength for both low- and high-grade concretes i.e. C1 and C3 but when used in high grade concrete beams it cannot avoid instability failure as it could not help in controlling the lateral deflections.

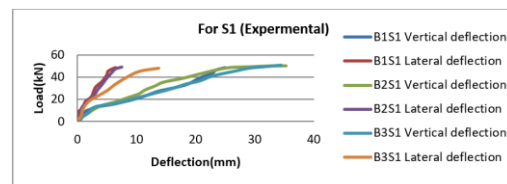


Figure 15. Load vs deflection curve (Experimental) for Fibre type S1

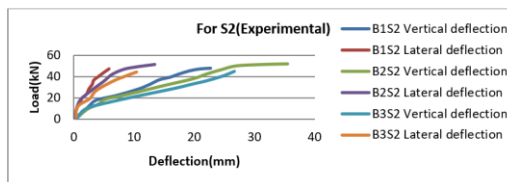


Figure 16. Load vs deflection curve for Fibre type S2

L-D curve for Fibre type S2 (i.e. hooked end fibre with aspect ratio 77.78) shows, as shown in Figure 16, that the S2 fibre type exhibits much better flexural response and ductility when used in C2

grade of concrete. Therefore, it can be inferred that the higher aspect ratio hooked end fibres shows increased flexural capacity and ductile response when used in C2 grade of concrete.

Conclusions

In this paper, the following has been concluded on the effect on slenderness with the inclusion of steel fibres in reinforced concrete.

1. High strength SFRC beams exhibit sudden instability collapse in the form of bending and twisting, resulting in reduced flexural strength as compared to C1 and C2 grades of concrete. This has been attributed to the due to the loss of lateral flexural rigidity in case of higher grade of concrete beams and the decrease in effective torsional rigidity which indicates the higher resistance to lateral instability. Since SFRC beams with C1 grade of concrete exhibits a lower flexural load carrying capacity as compared to the SFRC beams with C2 grade of concrete. Consequently, the load bearing capacity of SFRC beams with C2 grade of concrete is higher than that of beams with concrete grades C1 and C3.

2. There were no shear fractures seen, and all beams collapsed in flexure, supporting the behavior of long/slender beams. This behavior can be attributed to the Four point bending test prescribed for the study, as the region enclosed in between the two load points has a constant bending moment, such that no shear stresses will occur in that section.

3. The results of experimental testing are validated against numerical analysis with ATENA as the load deflection curve follows same profile.

4. The failure mechanism observed in the experimental investigation is consistent with the results obtained from ATENA analysis. As a result, ATENA successfully predicts beam behavior and can thus be used to evaluate the performance of SFRC long beams.

5. For lower grade of concrete C1, hooked end steel fibres with aspect ratio 63.63 shows slightly better load-bearing capacity than hooked end fibres with higher aspect ratio i.e., 77.78. This trend is observed due to the lower aspect ratio steel fibres having larger area of cross section, in comparison to the higher aspect ratio steel fibres because of which it resists higher load. On the contrary the SFRC beams with hooked end steel fibres with higher aspect ratio exhibits a better deflection control in both vertical and lateral directions than hooked end fibres with lower aspect ratio. This behavior can be attributed to the higher aspect ratio steel fibres having larger length in

comparison to the lower aspect ratio steel fibres. Owing to larger length of these fibres it shows better deflection.

6. For higher grade of concrete C3, hooked-end steel fibres with lower aspect ratio 63.63 demonstrated better flexural load carrying capacity than the hooked end steel fibres with higher aspect ratio 77.78, which failed at lower loads. These high strength concrete SFRC beams, fails by sudden instability because of twisting of beams, and hence exhibiting large lateral deflections leading to the beams moving out of load assembly.

Acknowledgments

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