

COMPREHENSIVE DISCUSSION OF THE REPAIRABLE SINGLE SERVER CATASTROPHE AND MULTIPLE VACATION QUEUEING MODEL

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Abstract

The queueing system with two types of vacation policies and disasters occurring in every state are examined in this study. As soon as the system is empty, the server goes on a short-term vacation. If there are no active clients in the system when the server returns from a short-term vacation, then the server goes on a long-term vacation. Upon seeing at least one client in the queue during a short or long-term vacation, the server will turn on right away. Catastrophes can happen during short or long vacations and also when the server is active. All current clients are removed from the system when a disaster occurs. When the system has been fixed, it starts to work properly once again. After repair, the server immediately switches to working mode(model 1) or takes a short-term vacation(model 2). A steady-state solution is determined by using the PGF (Probability-Generating Function). The above two categories of after-repair will be examined in this paper. Additionally, numerical examples related to performance measures, cost models, and the steady analysis of the provided models are discussed. Management for networking disaster recovery gives suggestions for restarting regular operations and network services following a disaster.

Keywords: Cost analysis; differentiated vacation; disaster; markovian arrival process; repair

Introduction

Queueing theory is often thought to be a subfield of applied probability theory. It has various uses, including in computer systems, manufacturing facilities, and communication networks, and it is also used in many other areas. Servers are always available under the traditional queueing approach. However, the server could be accessible in many real-world scenarios for many reasons. The Central Processing Unit (CPU) continually complies with

the demands of computer programmes that direct it regarding which data to handle and in what manner. The system, or Central Processing Unit, is unavailable for the primary job when doing system updates or virus scans. Queueing systems with server vacations have attracted the interest of various researchers ever since the idea was first introduced in an article by Levy and Yechiali (1975). Doshi (1986), Doshi (1990), and Ke *et al.*

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(2010) have conducted a number of great surveys on various vacation models, while Tian (1990), Takagi (1991), and Tian and Zhang (2006) have written books specifically on the topic.

When a server implements the multiple vacation technique, the server immediately begins another break if the server returns from the previous one and the waiting room is empty. The server starts serving clients in accordance with the current service rules if there is at least one person waiting. In this context, breaks of various lengths may occur, called differentiated vacations. Differentiated vacation queueing models were newly introduced by Ibe and Isijola (2014). He derived the steady-state analysis and presented some numerical illustrations. Afterwards, Ibe *et al.* (2015) presented a multiple vacation queueing system with differentiated vacations, each vacation of which can be terminated when the system's client count reaches distinct predefined criteria. The M/M/1 queueing system with differentiated server vacations and an impatient tendency was later described by Sampath and Liu (2020). They also discussed explicit formulations of time-dependent probabilities using modified Bessel functions of the first type, Laplace transforms, and probability-generating function approaches.

Afterwards, Vijayasree *et al.* (2021) demonstrated how explicit analytical formulations of transient state probabilities are produced using the differentiated vacation-generating function approach.

Servers can become damaged in many real-world applications. For instance, in a computer system, files or some operations linked to files may be compared to those of regular users, whereas those affected by viruses may be compared to catastrophes. All files and certain connected works are flushed out in this situation, and only fresh clients are started when repairs have been made. These models were first presented by Chao (1995). Kalidass *et al.* (2012) discussed a single-server Markovian queueing system with a finite buyer and described steady-state analysis with numerical illustrations. Following this, Kalidass *et al.* (2014) took into account an M/M/1 queue with a repairable server and several vacations, developed a transient analysis of the vacation model theory, and examined practical performance assessment. The M/M/1 queueing model with the potential for disasters at the service station has a transitory solution, according to Kumar and Arivudainambi (2000). Later, M/M/2 queues with potential catastrophes were described by Kumar and Madheswari (2003). The transient system size in the M/M/1 queueing system with the probability of disasters and server failures is obtained by Kumar and Madheswari

(2005), who derived steady-state probabilities of the system size. Then, Kumar *et al.* (2007) looked at disasters, state-dependent arrivals, and service rates. Yechiali (2007) examined the model to derive several quality-of-service metrics, including the average sojourn length of a served client, the proportion of customers serviced, the rate of lost consumers due to catastrophes, and the rate of abandonment due to impatience. Kumar *et al.* (2014) discussed Sudhesh's (2010) paper rework and found a correct transient solution and some performance measures as well. In order to calculate the time-dependent probability of the system size, Vadivukarasi and Kalidass (2022) examined a single-server Markovian queue with the potential for disasters at the service station.

In this paper, we discussed two kinds of vacations, known as short and long-term vacations. In Section 2 of this study, we go into a detailed description of the model formulation. We analyse the steady-state analysis of models 1 in Section 2.1 and 2 in Section 2.2. Section 2.3 presents a few special cases. Performance metrics for the models are developed in Section 3.1. Section 3.2 of this paper presents the cost analysis of the given models, and Section 3.3 discusses some numerical results. Section 3.4 provides a practical illustration. Finally, section 4 has a conclusion.

Materials and Methods

This section provides model formulation, steady state analysis of model 1, steady state analysis of model 2, and some special cases of the given model.

Model Formulation

Consider an M/M/1 single server queue with two types of vacations, where new clients come using a Poisson process with rate λ . It is assumed that client service times have an exponential distribution with a mean of $\frac{1}{\mu}$. We suppose there are two different kinds of vacations: short-term and long-term. After a busy period, servers go on short-term vacations, while long-term vacations are taken when no clients are waiting for the server to return from a short-term vacation. We assume that short-term vacations are exponentially distributed with rate $1/\gamma_1$, and independent of the busy time. Long-term vacations are similarly considered to have an exponential distribution with a mean of $1/\gamma_2$. If there are no clients in the system after the conclusion of the short-term vacation, the server switches to the long-term vacation. The system suffers disastrous breakdowns (failures) with Poisson rate η , occurring when the server is in

a functioning or vacation state. All of the clients using the system are deleted (ejected) and lost forever once the system or server crashes due to catastrophes. A repair procedure begins instantly and quickly with rate ζ . Model 1 illustrates how the system server returns to its busy state right away after repair, whereas Model 2 depicts the system server taking a short-term vacation right after repair. Figure 1 represents the state-transition-rate diagram of model 1, and Figure 2 represents the state-transition-rate diagram of model 2.

Steady State Analysis: Model 1

We examine the steady-state discussion of model 1 in Section 3. The rate-in, rate-out approach is used first to develop the system's governing equations. Let $J(t)$ represents the server or system's current state and $X(t)$ represents the total number of clients in the system, including the one being serviced. In the event that $J(t) = B$ the system is operating normally and serving clients; in the event that $J(t) = h_1$ the system is on short-term vacation; in the event that $J(t) = h_2$ the system is on long-term vacation; and in the event that $J(t) = Q$ the system is down and undergoing maintenance. The system states at time t are evidently characterized by a two-dimensional continuous-time Markov process $\{(X(t); J(t)); t \geq 0\}$ with state space $S = \{(n, j); n = 0, 1, 2, \dots, j = B, h_1, h_2, Q\}$.

Let's consider, $p_{i,j} = \lim_{t \rightarrow \infty} p_{i,j}$ where $p_{i,j}$ be the likelihood that there will be i clients in the system and that the service provider will be in j^{th} state. The suggested model's steady-state balance equations, according to Markov theory, are as follows:

$$(\lambda + \mu + \eta)p_{1,B} = \mu p_{2,B} + \gamma_1 p_{1,h_1} + \gamma_2 p_{1,h_2} + \zeta Q \quad (1)$$

$$(\lambda + \mu + \eta)p_{n,B} = \mu p_{n+1,B} + \gamma_1 p_{n,h_1} + \gamma_2 p_{n,h_2} + \lambda p_{n-1,B} \quad n \geq 2 \quad (2)$$

$$(\lambda + \gamma_1 + \eta)p_{0,h_1} = \mu p_{1,B} \quad (3)$$

$$(\lambda + \gamma_1 + \eta)p_{n,h_1} = \lambda p_{n-1,h_1} \quad n \geq 1 \quad (4)$$

$$(\lambda + \eta)p_{0,h_2} = \gamma_1 p_{0,h_1} \quad (5)$$

$$(\lambda + \gamma_2 + \eta)p_{n,h_2} = \lambda p_{n-1,h_2} \quad n \geq 1 \quad (6)$$

$$\zeta Q = \eta(1 - Q) \quad (7)$$

Define the partial probability generating function (PGF)

$$P_B(z) = \sum_{i=1}^{\infty} p_{i,B} z^i, P_{h_1}(z) = \sum_{i=1}^{\infty} p_{i,h_1} z^i \text{ and}$$

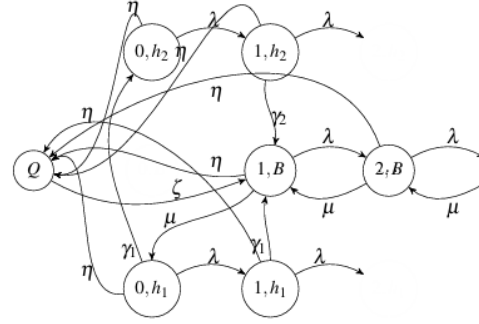


Figure 1. State-transition-rate diagram for model 1

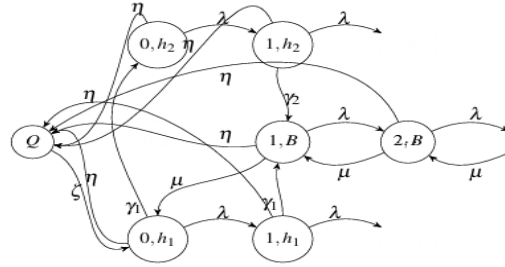


Figure 2. State-transition-rate diagram for model 2

$$P_{h_2}(z) = \sum_{i=1}^{\infty} p_{i,h_2} z^i$$

After some mathematical calculation of Equations (1) and (2), we get the PGF of the server state at busy.

$$P_B(z) = \frac{\mu z p_{1,B} - \gamma_1 z P_{h_1}(z) - \gamma_2 z P_{h_2}(z) - \zeta z^2 Q}{\lambda z^2 - (\lambda + \mu + \eta)z + \mu} \quad (8)$$

Again, with some simplification of Equations (4), (6), and (7), we obtain the PGF of the state at short length vacation, long-length vacation, and disaster.

$$P_{h_1}(z) = \frac{\lambda z}{\lambda + \gamma_1 + \eta - \lambda z} p_{0,h_1} \quad (9)$$

$$P_{h_2}(z) = \frac{\lambda z}{\lambda + \gamma_2 + \eta - \lambda z} p_{0,h_2} \quad (10)$$

$$Q = \frac{\eta}{\eta + \zeta} \quad (11)$$

$$\text{As } \lim_{z \rightarrow 1} P_B(z) = P_B(1) = P_B,$$

$$\lim_{z \rightarrow 1} P_{h_1}(z) = P_{h_1}(1) = P_{h_1} \text{ and}$$

$$\lim_{z \rightarrow 1} P_{h_2}(z) = P_{h_2}(1) = P_{h_2}.$$

Following the determination of the limit values specified above, we have

$$P_B(1) = P_B = \frac{\gamma_1 P_{h_1} + \gamma_2 P_{h_2} + \zeta Q - \mu p_{1,B}}{\eta} \quad (12)$$

$$P_{h_1}(1) = P_{h_1} = \frac{\lambda}{\gamma_1 + \eta} p_{0,h_1} \quad (13)$$

$$P_{h_2}(1) = P_{h_2} = \frac{\lambda}{\gamma_2 + \eta} p_{0,h_2} \quad (14)$$

From Equations (3) and (5), which give

$$p_{0,h_1} = \frac{\mu}{\lambda + \eta + \gamma_1} p_{1,B} \quad (15)$$

$$p_{0,h_2} = \frac{\gamma_1}{\lambda + \eta} p_{0,h_1} \quad (16)$$

Let z_1 be the zero of the denominator $P_B(z)$. Therefore,

$$p_{1,B} = \frac{\zeta z_1 Q}{\mu - \left[\frac{\lambda \gamma_1 z_1}{\lambda + \gamma_1 + \eta - \lambda z_1} + \frac{\lambda \gamma_2 z_1}{\lambda - \lambda z_1 + \eta + \gamma_2} \frac{\gamma_1}{\lambda + \eta} \right] \frac{\mu}{\lambda + \eta + \gamma_1}}$$

Steady State Analysis: Model 2

In the following section, we discuss the steady-state balance equations of the model after the repair of the system server goes on a short vacation.

$$(\lambda + \mu + \eta) p_{1,B} = \mu p_{2,B} + \gamma_1 p_{1,h_1} + \gamma_2 p_{1,h_2} \quad (17)$$

$$(\lambda + \mu + \eta) p_{n,B} = \mu p_{n+1,B} + \gamma_1 p_{n,h_1} + \gamma_2 p_{n,h_2} + \lambda p_{n-1,B} n \geq 2 \quad (18)$$

$$(\lambda + \gamma_1 + \eta) p_{0,h_1} = \mu p_{1,B} + \zeta Q \quad (19)$$

$$(\lambda + \gamma_1 + \eta) p_{n,h_1} = \lambda p_{n-1,h_1} n \geq 1 \quad (20)$$

$$(\lambda + \eta) p_{0,h_2} = \gamma_1 p_{0,h_1} \quad (21)$$

$$(\lambda + \gamma_2 + \eta) p_{n,h_2} = \lambda p_{n-1,h_2} n \geq 1 \quad (22)$$

$$\zeta Q = \eta(1 - Q) \quad (23)$$

Define the partial probability-generating function

$$P_B(z) = \sum_{i=1}^{\infty} p_{i,B} z^i, P_{h_1}(z) = \sum_{i=1}^{\infty} p_{i,h_1} z^i \text{ and}$$

$$P_{h_2}(z) = \sum_{i=1}^{\infty} p_{i,h_2} z^i$$

Solving the Equations (17), (18), (19), (20), (21), (22) and (23), We obtain

$$P_B(z) = \frac{\mu z p_{1,B} - \gamma_1 z P_{h_1}(z) - \gamma_2 z P_{h_2}(z)}{\lambda z^2 - (\lambda + \mu + \eta) z + \mu} \quad (24)$$

$$P_{h_1}(z) = \frac{\lambda z}{\lambda + \gamma_1 - \lambda z + \eta} p_{0,h_1} \quad (25)$$

$$P_{h_2}(z) = \frac{\lambda z}{\lambda + \gamma_2 - \lambda z + \eta} p_{0,h_2} \quad (26)$$

$$p_{0,h_1} = \frac{\mu}{\lambda + \gamma_1 + \eta} p_{1,B} + \frac{\zeta Q}{\lambda + \gamma_1 + \eta} \quad (27)$$

$$p_{0,h_2} = \frac{\gamma_1}{\lambda + \eta} p_{0,h_1} \quad (28)$$

As $\lim_{z \rightarrow 1} P_B(z) = P_B$,
 $\lim_{z \rightarrow 1} P_{h_1}(z) = P_{h_1}(1)$ and $\lim_{z \rightarrow 1} P_{h_2}(z) = P_{h_2}(1)$.

After determining the above-mentioned limit values, we obtain

$$P_B(1) = \frac{\gamma_1 P_{h_1}(1) + \gamma_2 P_{h_2}(1) - \mu p_{1,B}}{\eta} \quad (29)$$

$$P_{h_1}(1) = \frac{\lambda}{\gamma_1 + \eta} p_{0,h_1} \quad (30)$$

$$P_{h_2}(1) = \frac{\lambda}{\gamma_2 + \eta} p_{0,h_2} \quad (31)$$

Following some calculations, we obtain

$$p_{1,B} = \frac{\frac{\zeta Q G}{\lambda + \gamma_1 + \eta}}{\mu z_1 - G \frac{\mu}{\lambda + \gamma_1 + \eta}}$$

where

$$G = \frac{\lambda \gamma_1 z_1^2}{\lambda + \gamma_1 - \lambda z_1 + \eta} + \frac{\lambda \gamma_2 z_1^2}{\lambda + \gamma_2 - \lambda z_1 + \eta} \frac{\gamma_1}{\lambda + \eta}$$

Special Cases

• $\gamma_1 \rightarrow \infty, \gamma_2 \rightarrow \infty$ and $\eta = 0$ then models will become $M/M/1$ queueing models with probability

$$p_{1,B} = \rho(1 - \rho)$$

This coincides with the result of Medhi (2003).

• If $\gamma_2 \rightarrow \infty$ then model 1 becomes single-vacation $M/M/1$ catastrophes, and after repair service starts immediately queueing model. Probability is as follows:

$$p_{1,B} = \frac{\zeta Q z_1}{\mu - \left(\frac{\lambda \gamma_1 z_1}{\lambda - \lambda z_1 + \gamma_1 + \eta} + \frac{\lambda z_1 \gamma_1}{\lambda + \eta} \right) \frac{\mu}{\lambda + \eta + \gamma_1}}$$

• If $\gamma_2 \rightarrow \infty$ then model 2 becomes single-vacation $M/M/1$ catastrophes, and service starts after a short vacation queueing model. Probability is as follows:

$$p_{1,B} = \frac{\frac{\zeta Q G}{\lambda + \gamma_1 + \eta}}{\mu z_1 - \frac{\mu G}{\lambda + \gamma_1 + \eta}}$$

where

$$G = \frac{\lambda \gamma_1 z_1^2}{\lambda + \gamma_1 - \lambda z_1 + \eta} + \frac{\lambda \gamma_1 z_1^2}{\lambda + \eta}$$

Results and Discussion

This section provides the critical system performance measurements, cost analysis, numerical illustration, and practical illustration for the given model.

Performance Measures

In this part, we define expressions for a few performance metrics that the system creator may find useful while creating the queueing system under consideration. These performance metrics are of broad importance for assessing how well our model performs.

The average number of customers per unit time on short- and long-term vacations for model 1 and for model 2:

$$E(L_1) = \frac{\lambda(\lambda + \gamma_1 + \eta)}{(\lambda_1 + \eta)^2} p_{0,h_1}$$

$$E(L_2) = \frac{\lambda(\lambda + \gamma_2 + \eta)}{(\lambda_2 + \eta)^2} p_{0,h_2}$$

The average number of customers served per unit time in a busy state for model 1:

$$E(L_B) = \frac{-\eta[\gamma_1 P'_{h_1}(1) - \gamma_2 P'_{h_2}(1)] - [\mu p_{1,B} - \gamma_1 P_{h_1} - \gamma_2 P_{h_2} - \zeta Q][\lambda - \mu]}{\eta^2}$$

The average number of customers served per unit time in a busy state for model 2 is:

$$E(L_B) = \frac{-\eta[\gamma_1 P'_{h_1}(1) - \gamma_2 P'_{h_2}(1)] - [\mu p_{1,B} - \gamma_1 P_{h_1}(1) - \gamma_2 P_{h_2}(1)][\lambda - \mu]}{\eta^2}$$

Cost Analysis

We define the total expense function, where the service rate is the key variable, and build an economic model in this part. In order to reduce the overall mean expenditure per quantity, our objective is to limit the service rate.

The following expenditure components are defined below (per unit of time):

C_1 = every customer observed in the system incurs a holding expense.

C_2 = waiting expense if one customer is to receive the service.

C_3 = expenditure for the time the server handled the service procedure.

C_i = expense when the server is on the i^{th} vacation ($i = h_1; h_2$).

C_μ = expense for service.

C_η = expense for disaster.

Cost function defined as follows

$$T = C_1 E(L) + C_2 E(W) + C_3 P_B + C_4 P_{h_1} + C_5 P_{h_2} + C_\mu \mu + C_\eta \eta$$

Numerical Illustrations

In this section, we provide some numerical examples that demonstrate the impact of various parameters, including the disaster rate η , vacation rate γ , arrival rate λ , service rate μ , and repair rate ζ on system performance measures. We randomly choose the values of various system parameters for the purpose of computational comfort. The mean system length $E[L]$ clearly reduces with an increase in μ , as seen in Figure 3. Moreover, the number of clients increases if the server takes a vacation after repairs. If clients are waiting in the system after repair, the server begins service immediately, resulting in fewer clients than the previous one. Figures 4 and 5 describe how the repair rate impacts the expected number of customers. That means if the disaster rate η increases, the number of clients will decrease in models 1 and 2. Simultaneously, if the repair rate ζ increases, the expected number of clients in the system will also increase. Figure 6 demonstrates how the average number of clients is declining while the rate of short vacations is increasing with different long vacation rates. In the same manner, as long-term vacation rates increase,

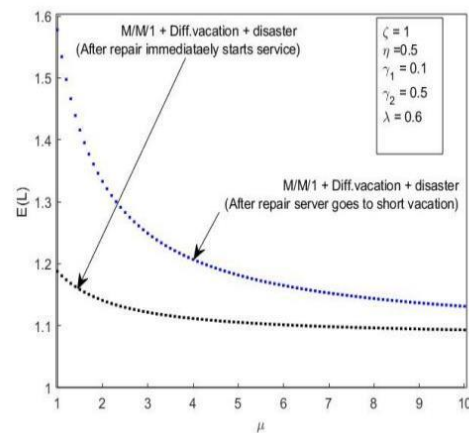


Figure 3. Expected number of customers in the system against μ

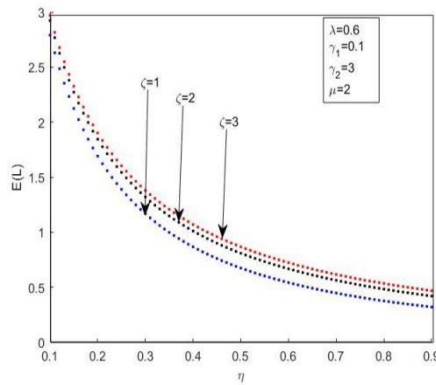


Figure 4. Expected number of customers in the system against η for model 1

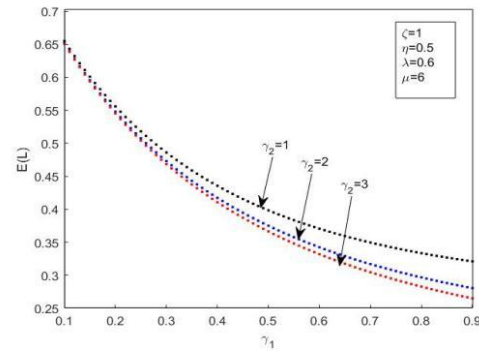


Figure 6. Expected number of customers in the system against γ_1 for model 1

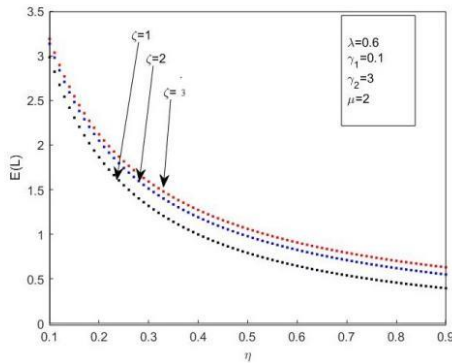


Figure 5. Expected number of customers in the system against η for model 2

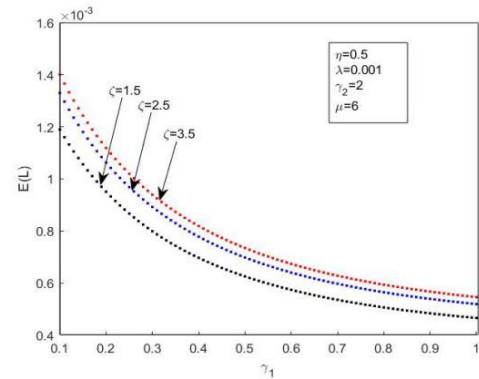


Figure 7. Expected number of customers in the system against γ_1 for model 2

the expected number of customers will decrease. Figure 7 represents the average number of clients against the short-term vacation rate. We see that the mean system length $E[L]$ drops as γ_1 grows, and that $E(L)$ also increases when repair rate increases. Figure 8 depicts the predicted waiting period in the event of a calamity. As the catastrophe rate rises, the expected waiting time decreases because, during a disaster, all clients are removed from the system, and at the same time, none of the clients wait in the system. Furthermore, if the server instantly returns to the system, the estimated waiting time decreases as the server goes on vacation following repair. The effect of service rate on the total cost for models 1 and 2 is shown in Figure 9. Total cost against μ is always convex, and the optimum service rate and total cost will exist. Moreover, the total cost of the first model is lower than the second model.

Practical Illustration

In our real-life situations, there are a number of practical situations that involve disaster and

vacation. One specific situation where this approach is used is the study of networking disasters, which is discussed below.

Networking services are offered so that customers can communicate. Networking services can occasionally impact a connecting facility's malfunction or crisis. That means natural disasters, such as earthquakes and floods, may have an impact on network issues. Moreover, some man-made problems may cause network disasters. Disaster management has become increasingly important in recent years. Disaster recovery provides guidelines for resuming connections for waiting customers in emergencies.

Following are some significant network disaster categories:

1. A device or internet activity that uses bandwidth excessively: When a single user uses a network connection excessively, it affects the system as a whole and reduces network performance for other users.

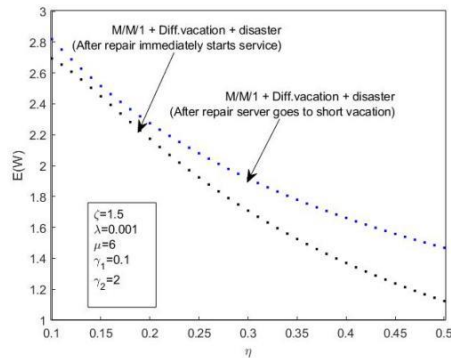


Figure 8. Expected waiting time in the system against η

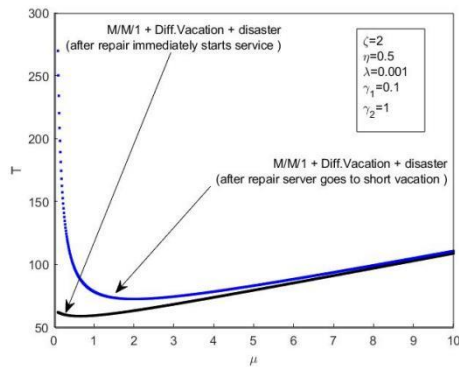


Figure 9. Comparison between total cost of above mentioned models

2. Configuration mistakes: Network frame errors happen if the manual changes frequently. These mistakes result in network catastrophes. So, the error connection will be shut down in order to resolve these errors. However, other users will not be able to access the network during the shutdown period.

3. Hardware failure: Hardware components are responsible for maintaining a network. Errors may occur after the sale ends or the component's life gets over. These mistakes also can be applied to network failures.

As a result of the aforementioned causes, networking services must be restored as soon as possible. Customers can then connect with others. Customers should be able to connect with others whenever they wish or need to do so. However, in the event of a network disaster, customers will be unable to connect to any location. As previously stated, networking service recovery has a substantial influence on the availability of services following disasters. Clients are flushed out following

a disaster, but the emergency repairs must also be finished before the next client comes. Whenever the first new customer arrives after the tragedy, the server should be ready to serve. A server is not ideal for a business's financial situation if it goes on vacation after repair. The most cost-effective option is to correct it right away and return to service mode. If customers are not waiting for repairs to be completed, the server can take a little vacation.

Conclusions

The $M/M/1$ vacation queueing mechanism is studied in this study with disastrous results. Two distinct types of resumptions are examined in model 1 and model 2. The probability-generating function approach is used to define the queueing model under consideration in order to obtain the steady-state probability and the corresponding performance metrics. Additionally, two distinct models' cost analyses have been addressed. Finally, it is determined that it is more practical to restart a server immediately following a repair rather than waiting for a little vacation. That means, model 1 is more useful than model 2 for customers. The following are the limitations of the model. The real-world scenario of the model is that the arrival process is not stationary, and there could not be enough waiting areas available. Added to that, customers are discouraged from joining the wait if there is a huge wait when they arrive. As a result, arrival rates vary per state. Queueing models for disasters can be used to evaluate the impact of a breakdown or a system reset in a service facility. Hence, disaster queueing models are useful in daily life. In future, the queue governed by the multi-server queueing system may be investigated in the presence of catastrophic failures and the financial status of different kinds of disasters that affect each server in the system. Furthermore, the aforementioned model is able to expand on one more research project on a traffic network disaster scenario.

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