

DETERMINATION OF OPTIMUM PARAMETERS FOR THE DESIGN OF COMPOSITE SHAFT

Pinank Patel* and Divya Adalja

Received: November 06, 2023; Revised: February 28, 2024; Accepted: April 01, 2024

Abstract

In this research work, emphasis is given to investigate the dynamic properties of a revolving composite shaft. The experimental analysis is used to calculate the Eigen pairs for different modes of vibration of revolving shafts. A Finite Element Method (FEM) prototype is used to articulate the stiffness matrices. The stiffness matrices consider the property of distortion about an axis, the shear distortion on the Eigen trend of the revolving shaft. The perilous speed of the revolving shaft is calculated based on the Campbell diagram. It also helps to identify the volatility areas to attain accuracy and regulate the dynamic performance of the physical system in a resonance state. The effect of laminate constraints like piling orders, fiber direction, boundary conditions, and percentage of fiber volume on the natural rate of recurrence and perilous speed of the shaft is investigated. The results obtained by mathematical analysis and by performing experiments were compared. It is observed that the results obtained by the FEM method and experimental results for the composites shaft are in good agreement with each other.

Keywords: Composite material; Critical speed; Damping; Dynamic properties

Introduction

The main advantage of Composite materials is that they provide higher strength than metallic components with similar weight. The composite materials also provides better material characteristics as well as certain material properties can be controlled using proper combination of constitutes of composite material. The investigations are focused on replacing the metallic shafts with compound shafts in many equipment and vehicle transmission shafts. The composite materials also have the advantage of providing pre-fixed performances, like dynamic behavior and

perilous speeds. That can be achieved by altering the array of the various combined layers, direction of fibers and number of plies. The composite materials have high-damping characteristics as compared to metallic materials. The internal damping in a rotor made of compound material is significantly higher than that of a rotor made from metal. A helicopter rotor shaft made from epoxy resins and boron composite was prepared and analyzed. The vital speeds at different vibration modes were calculated with the help of Equivalent Modulus Beam Theory (EMBT), considering the shaft having a thin walled

Marwadi University, Rajkot-Morbi Highway, Rajkot, Gujarat, India. 360003. E-mail: pinank.patel@marwadieducation.edu.in

* Corresponding author

DOI: <https://doi.org/10.55766/sujst-2024-02-e02719>

Suranaree J. Sci. Technol. 31(2):010297(1-10)

round tube and the support condition at ends is simply supported beam (SSB) (Zinberg and Symonds, 1970).

Numerous studies have investigated the mechanical behavior of various composite materials, including CFRP, glass fiber composites, and hybrid laminates. For instance, research by Dipen kumar *et al.* (2019) explored the mechanical properties of Carbon fiber reinforced polymers (CFRP) composites, highlighting their high strength-to-weight ratio and stiffness characteristics. Similarly, Venkateshan *et al.* (2020) conducted comparative analyses of different composite materials, emphasizing the importance of material selection in achieving desired performance objectives.

Understanding the dynamic behavior of composite shafts is essential for ensuring operational reliability and mitigating potential vibration-induced failures. Previous research, such as that by Sino *et al.* (2008) has investigated the dynamic characteristics of composite shafts under various loading conditions. These studies have elucidated the influence of factors such as material damping, geometric parameters, and boundary conditions on critical speeds and vibrational modes.

The significant speed of the shaft was calculated based on the unbalance results and the reaction graphs acquired in the non-critical space. Dos *et al.* (1987) presented systematic examinations on cylindrical pipes made from layer composite. Further research by Reis, indicates that the extended beam element helps analyze the problem of a rotor. Chun and Charles (1993) analyzed the bending and twisting effects on the critical speed of the shaft.

The fatigue behavior and long-term durability of composite shafts are critical considerations in engineering design. Research by Alberto and Luigi (2019) has explored fatigue mechanisms in composite materials, including delamination, fiber breakage, and matrix cracking. Moreover, studies have proposed methodologies for predicting fatigue life and assessing durability performance in composite shafts, contributing to enhanced reliability and service life.

The gyroscopic effects on shafts were also considered. Chun and Charles (1993) analyzed the composite shafts to understand the unsteadiness under dynamic conditions for a shaft acted upon by a varying torque and/or changing rotational speeds.

Optimization techniques play a pivotal role in determining the optimum parameters for composite

shaft design. Research by Meng *et al.* (2018); Chen *et al.* (2018) has utilized computational methods, such as finite element analysis (FEA) and genetic algorithms, to optimize composite shaft configurations for specific performance criteria. These studies have demonstrated the efficacy of optimization approaches in achieving lightweight designs while meeting structural requirements and performance targets.

With the help of an in-depth analysis of previous research, including studies on material properties, structural behavior, and dynamic response, key factors influencing the design process are determined for the analysis.

Manufacturing Method and Details of Material Used for Composite Shaft:

The composite shaft is manufactured using Standard Electrical grade glass and vinyl ester resin matrix for the composite material and Glass fiber is used as reinforcement. The mixture rule is used for calculating the mechanical properties of the composite (You, 2017). The hand lay-up method and curing under pressure are used to produce five groups of similar laminates. Each laminate consists of a total of five layers.

Group 1: [0/0/0/0/0].

Group 2: [0/30/0/30/0].

Group 3: [0/45/0/45/0].

Group 4: [0/60/0/60/0].

Group 5: [0/90/0/90/0].

The angle of fiber inclination in degrees is shown in the above groups. At the end of the curing process, the dimensions of the laminated composite shaft are as follows: Diameter 60 mm, length 800mm. A representative sample prepared from Fiber reinforced Polymers (FRP) using three layers, each with a thickness of 1mm, is shown in Figure 1 and Figure 2. For each group of composites, three levels of composites were fixed. The three levels are:

1. Fiber volume fraction $V_f = 25\%$.
2. Fiber volume fraction $V_f = 45\%$.
3. Fiber volume fraction $V_f = 70\%$.

For each volume fraction, five groups of specimens were manufactured to experimentally determine the effect of various orientations of lamina, and stacking sequence of layers on the mechanical properties of the composite shaft. Jeppesen *et al.* (2021)

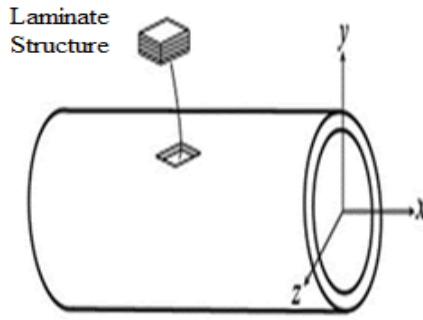


Figure 1. Three-dimensional diagram of composite shaft

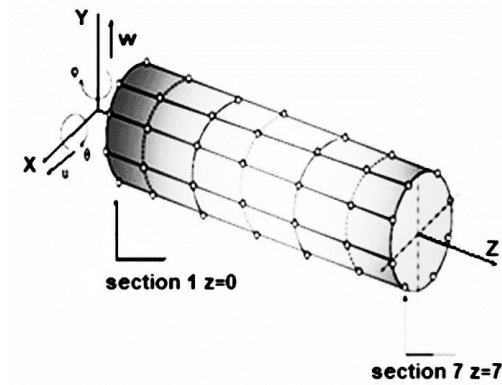


Figure 2. Finite Element model of composite shaft with eight-node elements

Design of Composite shaft based on various parameters

Analytical Approach:

The analytical study helped to determine the mechanical properties of composite shafts. The mixture rule is used to determine different properties. Classical lamination theory is used to find the stiffness matrices related to bending and coupling, for different lamina orientations (Charles W. Bert 2012). The composite shaft is constructed with the help of a number of laminas, which are joined together without any discontinuity. The composite shaft is considered to have no relative deformation between bonded layers.

The displacement field can be within the element and can be represented by global coordinates Mehdi and Mohammad *et al.* (2017):

$$[x \ y]^T = [N^k] \{s^k\} \quad (1)$$

where $[N^k]$ is shape function matrix,
 $\{s^k\} = [x \ y \ \theta \ \phi]^T$

Where,

x = Linear displacement along X-axis.

y = Linear Displacement along Y-axis.

θ = Angular Displacement about X-axis.

Φ = Angular Displacement about Y- axis.

The strain matrix is given by:

$$\{\epsilon\} = [B^k] \{S^k\} \quad (2)$$

Where, $\{\epsilon\}$ is Strain Matrix

The value of the natural coordinate fluctuates between -1 to 1. It is calculated at the midpoint of each layer, and the stress-strain and stiffness are computed at the mid-section of each layer. The volume integral can be written in two parts, integral over area of surface and thickness.

The stiffness can be given by the following equation,

$$[K_e] = \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 B^T [D] [B] |J| d\xi d\eta \quad (3)$$

$[K_e]$ = Stiffness Matrix

$[D]$ = Material Transformation Matrix.

Different elements of material transformed stiffness matrix are given by

$$[D] = \begin{bmatrix} D_1 & D_{12} & 0 & 0 & 0 \\ D_{12} & D_2 & 0 & 0 & 0 \\ 0 & 0 & G_{12} & 0 & 0 \\ 0 & 0 & 0 & KG_{12} & 0 \\ 0 & 0 & 0 & 0 & KG_{12} \end{bmatrix} \quad (4)$$

Where, $D_1 = \frac{E_1}{\Delta}$, $D_2 = \frac{E_2}{\Delta}$,

$D_{12} = \frac{E_2 v_{12}}{\Delta}$, $\Delta = 1 - v_{12}v_{21}$

E_1 = Modulus of elasticity of Matrix material of the composite

E_2 = Modulus of Elasticity of fibers

v_{21} and v_{12} are axial poison's ratio

The elastic modulus in x direction is 29 N/mm² Elastic modulus in y direction is 21 N/mm² and poison's ratio is 0.35.

The mass matrix at layer j is given Müzel *et al.* (2020) by:

$$[M] = \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 [N_i] \rho [N_j] d\xi d\eta \quad (5)$$

Where, $[M]$ = Mass matrix;
 $[N_i]$ and $[N_j]$ = Shape functions;
 ρ = Density in kg/m^3 ; and
 ξ = Damping factor.

The free vibration of an un-damped system is equivalent to an Eigen-value problem (Gupta and Singh, 1996).

$$[K] - \omega^2[M]\{X\} = 0 \quad (6)$$

Where $[K]$ = Overall Stiffness Matrix,
 ω = Natural frequency
 $[M]$ = Mass Matrix
 $\{X\}$ = Displacement Matrix. (Boukhalifa *et al.*, 2008).

The Matlab program calculates the Eigen-values and Eigenvectors for five different lamina orientations for four different end conditions. The four end conditions are: Clamped - clamped, clamped- free, clamped - simply supported and simply supported - simply supported. The results for the fiber volume fraction of 25% are shown in Table 1.

Table 1. First harmonic Frequency in Hz for various lamina orientations and different limiting conditions

Orientation of Lamina	Clamped-Clamped	Clamped-Simply	Simply-Simply
[0/0/0/0/0]	245.2	152.1	91.2
[0/30/0/30/0]	201.4	122.4	72.1
[0/45/0/45/0]	153.8	90.5	50.4
[0/60/0/60/0]	122.3	79.3	43.2
[0/90/0/90/0]	100.5	65.3	27.3

Dunkerley's theorem suggests that the total number of critical speeds will same as the number of f_n diagonal vibration.

The theoretical value of critical speed can be calculated from the equation of natural frequency of transverse vibration Harry. Harry *et al* (2021).

$$f_{tv} = \left[\frac{E C I g}{W L^4} \right] \quad (7)$$

Where,

f_{tv} = frequency of transverse vibrations in Hz.

E = Modulus of Elasticity in N/mm^2 .

C = Damping coefficient in Kg/sec .

I = Moment of Inertia in Kg.m^2 .

g = Gravitational Force in m/sec^2 .

W = Weight in Kg .

L = Length in meter.

Finite Element Analysis

The Finite element analysis is performed by preparing the geometry first and the discretization is done by using Shell elements since layers prepare the composite. The shell element will be most suitable for layered structure. The element has four degrees of freedom at each node: translation along x and y-axis and rotation about x and y-axis. Eight nodes, layer thickness, direction angle, and material properties define the element. The load applied includes the externally applied forces, gravity force, and imposed non zero displacements. The boundary condition considered here is that one end of the shaft is fixed and at the other end torque is applied.

In analysis of effect of lamina orientation on frequency, and other parameters, different orientations are labelled as 1,2,3,4 and 5.

The orientation of [0/0/0/0/0] is labeled as 1, similarly [0/30/0/30/0] as 2, [0/45/0/45/0] as 3, [0/60/0/60/0] as 4, and the orientation of [0/90/0/90/0] is labelled as 5, in following Figures 3,4 and 5.

The tests result for a clamped-clamped end condition with different stacking sequences and different fiber volume fractions are presented in Figure 3.

Figure 3 clearly indicates that there is a significant change in frequency with reference to changes in stacking sequences.

Such behavior is observed because the stiffness of the shaft is largely affected by outer staking layer, the inner layer has comparatively less impact on stiffness. The fiber volume fraction has less effect on Eigen parameters, which is obvious from the results. This is represented in the results with the specimen having an orientation of [0/0/0/0/0] and a volume fraction of 70%.

Considering the placements of lamina, the variation of the perilous speed relative to V_f is comparatively high, as shown in the above Figure.

The results in Figure 3 and Figure 4 reveal that, the frequency of vibration and the perilous speed of the shaft are less affected by the percentage of fiber in the composite shaft. This is because they are largely dependent on the angular positioning of layers of lamina in the composite material.

The Figure 5 indicates the consequence of piling arrangements and V_f on damping factor ξ . The results indicates that the angular positioning of layers of lamina affects the damping factor more significantly. Patel *et al.* (2020).

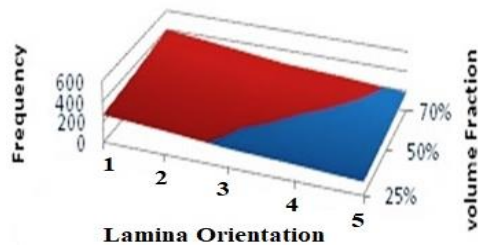


Figure 3. Effect of Lamina orientation and fiber Volume fraction on frequency

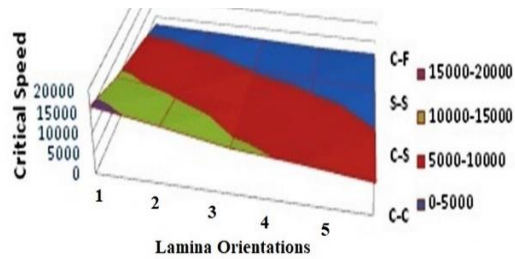


Figure 4. Effect of Stacking sequence and lamina orientation on Critical speed

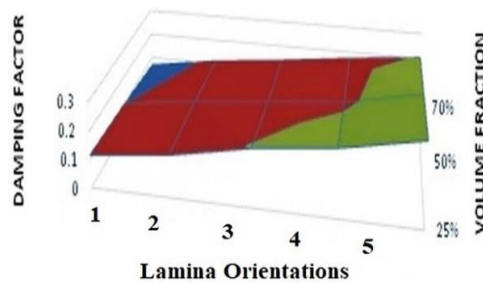


Figure 5. The effect of Fiber volume fraction and Lamina orientation on the damping factor

From Figures 3,4, and 5, it is apparent that the sample with positioning [0/90/0/90/0] has the lowest perilous speed in contrast with the other samples; at the same time, the sample with [0/0/0/0/0] positioning has the highest value of perilous speed. The reason for this is that the flexural elasticity is minimum with orientation [0/90/0/90/0] and maximum with [0/0/0/0/0] orientation.

From the analysis, it is observed that the stacking sequence significantly affects the natural frequency. The stiffness of shaft is mainly dependent on outer staking layer. The natural frequency is also dependent on the percentage of fiber in the composite shaft. The variation in critical speed with change in lamina orientation is recorded in Table 2.

Table 2. Values of Critical speed of the shaft for different lamina orientations

Lamina orientation	1 st Mode	2 nd Mode
[0/0/0/0/0]	2750	4200
[0/30/0/30/0]	2200	3750
[0/45/0/45/0]	1900	3500
[0/60/0/60/0]	1650	3200
[0/90/0/90/0]	1350	3000

It is observed that when the r.p.m. of shaft increases, the f_n and the ξ both are divided in two parts one with higher value and the other with lower value and as the speed continues to increase the two values diverse further apart.

The Campbell diagrams represent the increase of natural frequency w.r.t. the speed of rotation. It also indicates that the stacking sequence has large effect on frequencies.

In Figures 6 to 10 the Campbell charts associated with the four structures are presented. These outcomes display the advantage of using piling orders as an optimization factor for natural frequencies and for the instability regions. The difference in frequency for the first forward whirl is about 45 percent between the two configurations, and for the instability, the difference is about 30 percent. For the stacking sequence of [0/90/0/90/0], rotor speed can go beyond the third critical speed devoid of producing any instability. However, for the stacking sequence of [0/30/0/30/0], instability is caused at speeds greater than the second perilous speed. This kind of response is because of the fact that if the no. of fibers oriented close to the longitudinal direction is higher, then the rigidity of the rotating shaft will be more, and with that, the frequency will be higher in such cases. At the same time, if the orientation angle is less, the internal damping will be low, which will result in instability.

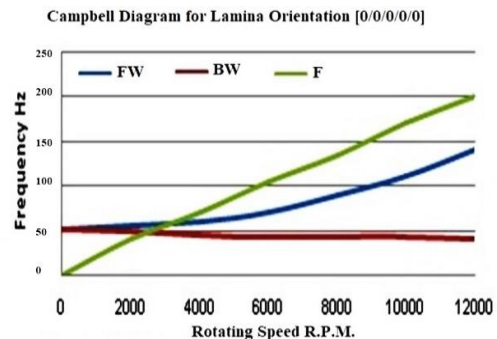


Figure 6. Campbell diagram [0/0/0/0/0]

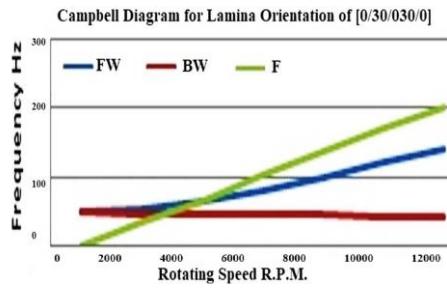


Figure 7. Campbell diagram [0/30/0/30/0]

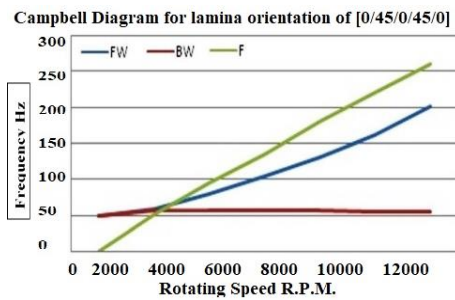


Figure 8. Campbell Diagram [0/45/0/45/0]

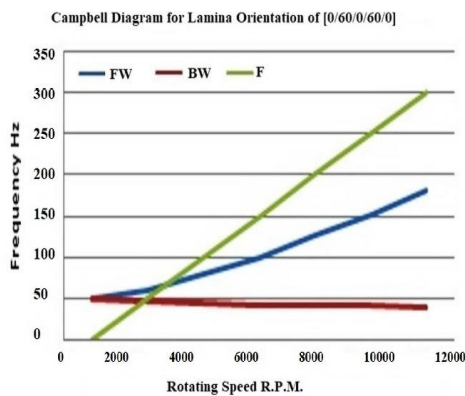


Figure 9. Campbell Diagram [0/60/0/60/0]

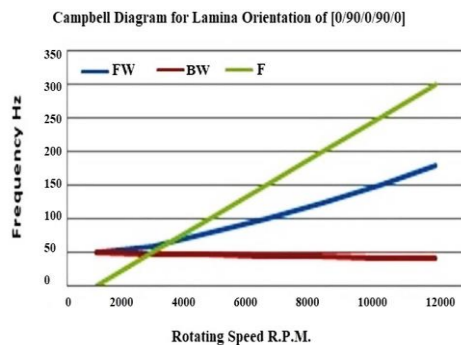


Figure 10. Campbell Diagram [0/90/0/90/0]

Experimental Investigation

For the Experimental investigations, the composite shaft of required numbers is prepared with the help of winding and layup technique. Experiments are to be conducted for 5 different orientations of lamina, there will be four different end conditions for lamina and the experiments will be performed for three shafts made from 3 different volume fractions.

The experimental analysis is performed to determine the perilous speeds, Natural frequencies of vibration and damping characteristics of composite shaft with respect to different parameters. The different specimens are prepared using winding and layup method for composites. The mathematical equation to find natural frequency of vibration for a shaft is given by:

$$f_n = \frac{60}{2\pi} \sqrt{\frac{g}{\delta}} = \text{critical speed of shaft}$$

Where,

f_n = Natural frequency of vibrations in Hz;

g = Gravitational Force in m/sec^2 ;

δ = Deflection of the shaft in m.

δ can be calculated by using the equation

$$\delta = \frac{WL^4}{EI}$$

The critical speed is calculated experimentally for the composite shafts made from different fiber volume fraction. Different specimen prepared for the composite material are shown in Figure 11.

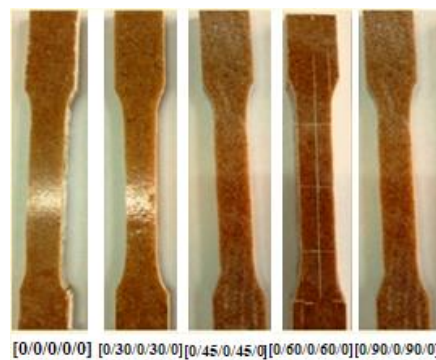


Figure 11. Composite material specimen with different orientations

The results of perilous speed for shafts with volume fraction of 70% are tabulated in Table 3.

Table 3. Critical speed for different lamina orientations and boundary conditions

Orientation of Lamina	Clamped - Clamped		Clamped- Simply supported	
	MI	MII	MI	MII
[0/0/0/0/0]	16227	37732	10834	34112
[0/30/0/30/0]	13697	31653	9123	28607
[0/45/0/45/0]	10892	24821	7277	22672
[0/60/0/60/0]	9466	25321	6365	19645
[0/90/0/90/0]	7645	17432	5143	15745

Where, MI denotes Mode: I and MII denotes Mode II of vibration.

It is observed from above results that for the lamina orientation of [0/90/0/90/0] the critical speed during mode I and mode II are very less and when the lamina orientation is [0/0/0/0/0], the critical speed during both modes is very high.

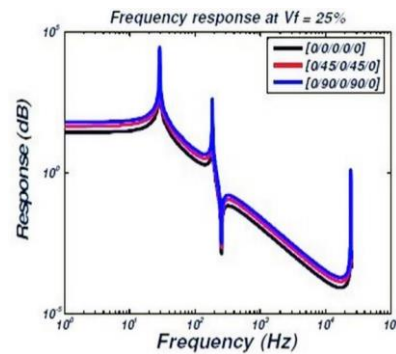
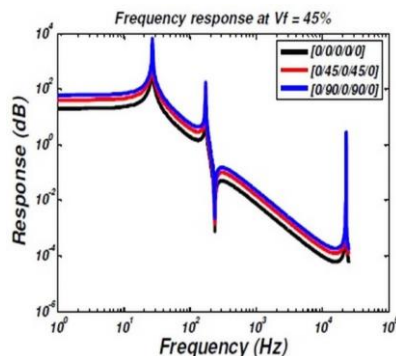
The Frequency effects on composite shaft dynamic behavior are experimentally determined using a fast Fourier transformation analyzer and computer interfacing. The experiments were conducted for frequencies up to 700Hz to determine the frequency response. The damping ratio ξ was calculated with the help of quality factor Q from natural frequency of vibration at mode I (ω_1) and natural frequency of vibration at mode II (ω_2).

The dynamic investigation of composite revolving shaft with different angular arrangements of fiber and different boundary conditions were carried out. The angular arrangements of fiber, perilous speed of shaft and end conditions of shaft have an impact on the dynamic characteristics of composite shaft. The Eigen parameters can be subtracted using drunkenly method. The specimens of different angular arrangements of fibers [0/0/0/0/0] and [0/90/0/90/0] have uppermost and lowermost frequencies respectively. Chang *et al.* (2004).

In the analysis three different set of results are presented. The results of first set provide the validation of the mathematical model and FEM model. The second set of results provides outcomes of the effect of material properties and geometric properties on the damping ratio and natural frequency of vibration. The effect of rotating speed on the damping ratio and natural frequency of vibration are determined with the help of results obtained from third set of specimens. If we apply lighter and very stiff material for manufacturing the composite shaft i.e. fiber or glass of high modulus and polyester

then we may get higher flexural frequency and higher damping ratio. The use of special materials will lead to an increase in cost and the increase in material properties is very less so it is not practical to use highly stiff material for improving material properties.

The experiments were performed on the shaft made from composite material, different magnitudes of the disturbing forces were applied on the shaft to check its dynamic behavior. The parameters like frequency, mode shapes and perilous speed were measured by experiments. The results were obtained for three different values of V_f and with different angular arrangements of the lamina in the composite shaft. The analytical results were also calculated using programming in Matlab software. The results obtained with experimental analysis and mathematical calculation were in agreement with each other. The Figures 12 to 14 shows the results of frequency response for different volume fractions.

**Figure 12. Frequency response for volume fraction 25%****Figure 13. Frequency response for volume fraction 45%**

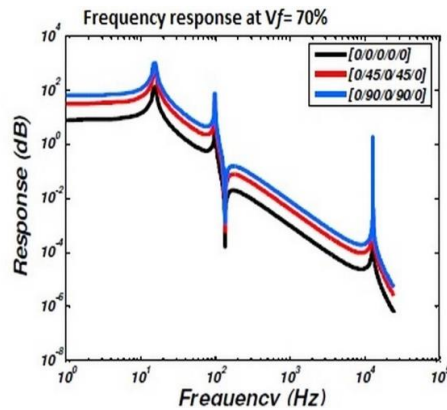


Figure 14. Frequency Response for volume fraction 70%

Results and Discussion

The design parameters of composite shafts were investigated using analytical, finite element analysis (FEA), and experimental approaches.

- The analytical results align with the trends observed in FEA and experimental results, indicating consistency in predictions.
- Discrepancies between analytical, FEA, and experimental results are due to simplifications or assumptions made in analytical models and the complexities involved in real-world behavior captured by FEA and experiments.
- There is a noticeable trend of decreasing frequency with increasing lamina angle across all methods.
- Different lamina orientations significantly affect the natural frequencies and critical speeds of composite shafts.
- Changes in boundary conditions (e.g., from clamped-clamped to clamped-simply supported) lead to variations in critical speeds, highlighting the importance of considering different operating conditions during design.
- The consistency between analytical, FEA, and experimental results validates the accuracy of the analytical and numerical models used.
- These results can be utilized for further optimization of composite shaft designs, aiming to enhance structural integrity, minimize vibration, and improve overall performance in various applications.

The combined analysis of analytical, FEA, and experimental results provides valuable insights into the behavior and performance of composite shafts under different conditions. The results demonstrate a substantial difference in natural frequencies between different piling configurations. Specifically, the first forward whirl frequency exhibits a remarkable 45 percent difference between two distinct configurations. This underscores the significant impact that piling orders can have on the dynamic behavior of rotating shaft systems. The results of Campbell diagram for different orientations depicts that piling sequence greatly affects the instability region in shaft system. By identifying potential instability points in advance, it is possible to implement necessary design modifications or operational controls to ensure safe and reliable operation of rotating machinery.

Conclusions

The results of analysis and experiments performed on composite shaft provide following conclusions:

- The study reveals a notable variation in frequency in response to changes in stacking sequences. Different configurations significantly affect the dynamic behavior of the shaft, highlighting the importance of careful selection during design optimization.
- The outer staking layer predominantly influences the stiffness of the shaft, while the inner layer exerts comparatively less impact. This highlights the critical role of outer layers in determining the structural integrity and mechanical properties of the shaft.
- The results of dynamic analysis reveals that the fiber volume fraction has minimal effect on Eigen parameters. The other factors, such as stacking sequence and orientation angles, play a more significant role in determining vibration characteristics of the composite shaft.
- The frequency of vibration and the critical speed of the shaft exhibit less sensitivity to changes in the percentage of fiber in the composite material. Instead, their behavior is largely dependent on the angular positioning of lamina layers within the shaft.
- The flexural elasticity of the shaft varies across different orientations, being minimum

with the [0/90/0/90/0] orientation and maximum with [0/0/0/0/0] orientation. This highlights the significance of orientation in determining mechanical properties.

- As the rotational speed of the shaft increases, both the frequency (f_n) and damping ratio (ξ) exhibit a split into two distinct parts, with one part having higher values and the other lower values. This divergence becomes more pronounced as the speed continues to increase.
- The rigidity of the rotating shaft is directly influenced by the concentration of fibers oriented close to the longitudinal direction. Higher concentrations result in increased rigidity and higher frequencies, while lower orientation angles lead to reduced internal damping, potentially resulting in instability.

The findings from this study highlight the complex relationship among stacking sequences, fiber orientation, and mechanical properties in determining the dynamic behavior and stability of composite shafts. These insights are crucial for design decisions aimed at optimizing performance and ensuring the reliability of rotating machinery in various industrial applications.

References

- Alberto D'A. and Luigi G. (2019). Principal Features of Fatigue and Residual Strength of Composite Materials Subjected to Constant Amplitude (CA) Loading. *Material*, 12(16): 2,586. <https://doi.org/10.3390/ma12162586>
- Boukhalfa, A., Hadjoui, A. and Hamza Cherif, S.M. (2008). Free Vibration Analysis of a Rotating Composite Shaft Using the p -Version of the Finite Element Method. *International Journal of Rotating Machinery*, 1-10. <https://doi.org/10.1155/2008/752062>
- Charles W.B. (2012). *Manual on Experimental Methods for Mechanical Testing of Composites*. In Springer, Dordrecht, p.1-4
- Chen, F., Liu, L., Lan, X., Li, Q., Leng, J., and Liu, Y. (2018). The study on the morphing composite propeller for marine vehicle. Part I: Design and numerical analysis. *Composite Structures*, 168:746-757. <https://doi.org/10.1016/j.compstruct.2017.02.072>
- Chun-Do, K. and Charles. W.B. (1993). Critical speed analysis of laminated composite, hollow drive shafts. *Composites Engineering*, 3(7-8):633-643. [https://doi.org/10.1016/0961-9526\(93\)90087-Z](https://doi.org/10.1016/0961-9526(93)90087-Z)
- Dipen, K.R., Durgesh, D.P., Emanoil, L.M. and Emanoil, L. (2019). Fiber-Reinforced Polymer Composites: Manufacturing, Properties, and Applications. *Polymers Journal*, 11(10):1,667. <https://doi.org/10.3390/polym11101667>
- Dos Reis HLM, Goldman RB, Verstrate PH. (1987) Thin-walled laminated composite cylindrical tubes: part III - critical speed analysis. *Journal of Composites Technology and Research*; 9(2): 58-62 <https://doi.org/10.1520/CTR10430J>
- Harry V., Bermingham, M., Dunn, M. and Veidt M. (2021). Layup optimization of laminated composite tubular structures under thermomechanical loading conditions using PSO Composite Structures, 276(2021):114483. <https://doi.org/10.1016/j.compstruct.2021.114483>
- Gupta, K. and Singh, S.E. (1996) Dynamics of composite rotors. *International Journal of Rotating Machinery*, 2(3):179-186. <https://doi.org/10.1155/2008/752062>
- Venkateshan, K., Ramanathan, K., Vijayanandh, R., Selvarajand, S., Kumar, G.R., and Kumar, M.S. (2020). Comparative structural analysis of advanced multi-layer composite materials. *Materials Today Proceedings*, 27(3):2,673-2,687 <https://doi.org/10.1016/j.matpr.2019.11.247>
- Hajianmaleki, M. and Mohammad S.Q. (2013) Vibrations of straight and curved composite beams: A review. *Composite Structures*, 100:218-232, <https://doi.org/10.1016/j.compstruct.2013.01.001>
- Meng, Y.S. Yan, L., Huang, W., Zhang, T.T., Du, Z.B. (2018). Structural design and analysis of a composite wing with high aspect ratio. *Journal of Zhejiang University-SCIENCE A*, 20:781-793. <https://doi.org/10.1631/jzus.A1900271>
- Chang M.-Y., Chen, J.-K. and Chang, Y.C. (2004). A simple spinning laminated composite shaft model. *International Journal of Solids and Structures*, 41(3-4):637-662. <https://doi.org/10.1016/j.ijsolstr.2003.09.043>
- Jeppesen, N., Mikkelsen, L.P., Dahl, V.A., Christensen, A.N. and Dahl, V.A. (2021). Quantifying effects of manufacturing methods on fiber orientation in unidirectional composites using structure tensor analysis. *Composites Part A: Applied Science and Manufacturing*, 149:106541. <https://doi.org/10.1016/j.compositesa.2021.106541>

- Patel, A., kaneria, A.J., Sharma, D.S. and Trivedi, R. (2020). Dynamic analysis of micro-cantilever with electrostatic Force. Australian Journal of Mechanical Engineering, 20(3):1448-4846.
<https://doi.org/10.1080/14484846.2020.1761006>
- Müzel, S.D. Bonhin, E.P., Guimarães, N.M. and Guidi, E.S. (2020). Application of the Finite Element Method in the Analysis of Composite Materials: A Review. Polymers Journal, 12(4):818.
<https://doi.org/10.3390/polym12040818>
- Sino, R., Baranger, T.N., Chatelet, E. and Jacquet, G. (2008). Dynamic analysis of a rotating composite shaft. Composites Science and Technology, 68(2):337-345.
<https://doi.org/10.1016/j.compscitech.2007.06.019>
- You, Y.-J.; Jay Kim, J.-H., Park, K.-T. Seo, D.-W. and Lee, T.H. (2017). Modification of Rule of Mixtures for Tensile Strength. International Journal of Polymers, MDPI publication. 9(12):682-694.
<https://doi.org/10.3390/polym9120682>
- Zinberg H., Symonds M., and Corp, W. (1970). The Development of an Advanced Composite Tail Rotor Driveshaft. Engineering material science, 15.