

EARTH SIZE MEASUREMENT IN SUMMER SOLSTICE USING “ÉRATOSTHÈNE” METHOD

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Abstract

The knowledge of solar deposit is very necessary in sizing and proper operation of any system powered by solar energy. Solar radiation data are used in the design development, construction, as well as in the solar energy systems performance evaluation, but also for the construction of buildings with a view to better thermal insulation adapted to the geographical location. According to the summer solstice 2022 occasion, in this work the measure of the earth radius is proposed using “Eratosthenes method”. For this, three fundamental conditions have been taken for the correct application in our calculations. Which cities to choose? When to make the measurements? And what precautions to take? This document may be beneficial for students and the public

Keywords: Summer solstice 2022, Method of Eratosthenes, Radius of the earth, Movement of the earth, solar radiation

Introduction

Assuming that the earth is round and that the sun was so far from the earth, around 200 years BC, Eratosthenes had a brilliant idea that said when the sun illuminates, the objects shadow depends directly on its height. Precisely at noon of June 21th, it was observed that in Aswan (Syene at the time) the sun was lighting up the bottom of a well; on the same date, in Alexander an obelisk of the city cast a shadow with an angle of 7.2°, which allowed him to measure the radius of the earth with an astonishing precision.

In this work according to the occasion of summer solstice 2022, Based on the information in two different books (Bion and Stone, 1758; Butterfield; 1906), despite the later work’s seemingly learned study of geodesy cited in the works of WYLD, Samuel et MANTHEY, David. (Wyld and Manthey, 2001), we tried again the experiment of Eratosthenes where we seek to determine the precision of our measurement and reflect on what should have been done to correct the errors and ambiguities scattered throughout our

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text. We have chosen two cities in Algeria located almost at the same longitude; Adrar and Naama.

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Materials and Methods

Test protocol

Hypothesis1:

Three conditions must be satisfied for the correct calculation used in the method:

The two chosen cities are on the same meridian.

When the earth is farthest from the sun: the day of the summer solstice June 21.

The objects used to measure the shadows will be placed perpendicular to the surface of the place considered.

Hypothesis2:

precise knowledge of the distance between the two cities. In our method we used Google maps.

Biography and geographical data

In the following Figures 1 and 2 we can see that the two cities Adrar and Naama are located almost on the same meridian.

Measurement of the air distance between Adrar and Naama:

Distance as the crow flies Naama-Adrar (IGI, 2015) is: 599.6 km as shown in Figure 3 bellow (372.6 miles) direction: 179°

GPS coordinates (Yaiche *et al.* 2014)

Naama: 33.26667,-0.31667

Adrar: 27.87429,-0.29388

We made the measurements with two sticks of the same length 1197mm each placed in a perforated brick so that the stick remains perpendicular on the horizontal plane as shown in Figure 4 bellow.



Figure 1. Geographical map of Algeria



(a) Adrar	(b) Naama
Country: Algeria	Country: Algeria
Wilaya Adrar (Chief town)	Wilaya Naama (Chief town)
Daira Adrar (Chief town)	Daira Naama (Chief town)
Postal code: 01000	Postal code: 45000
Population: 62657 residents (2013)	Population: 6 251 residents (2008)
Area: 427 368 Km ²	Area: 2 482 Km ²
Latitude: 27.88° Nord	Latitude: 33.27° Nord
Longitude: 0.28° West	Longitude: 0.3° West
Altitude: 263 m	Altitude: 1166 m

Figure 2. Biography and geographical data of both cities Adrar and Naama

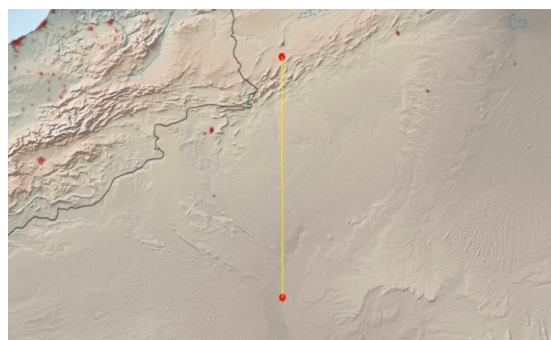


Figure 3. Air distance Adrar -Naama (as the crow flies)

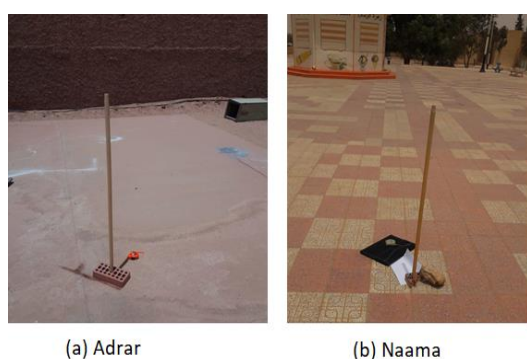


Figure 4. Shadow measurement in Adrar and Naama

The data below for both cities have been taken from (Capderou, 1985; Capderou, 1986; Yaiche et al., 2014)

For Adrar:

Latitude = 27.88° N ; longitude = 0.28 West;
Alt = 263m
Adrar stick length: $j_k = 1197$ mm

The equation of time as a function of the day is given by the following Equation (1)

$$T_{SV} = T_L + ET - 61.1 \text{ (in minutes)} \quad (1)$$

For Naama:

Latitude = 33.27°N; longitude = 0.3° West;
Alt = 1166m

Naama Stick Length: $AN = 1217$ mm (first test) and 1197mm (last test)

The equation of time as a function of the day is given by the following Equation (2)

$$T_{SV} = T_L + ET - 61.3 \text{ (in minutes)} \quad (2)$$

It is seen that solar noon changes every day in order to be ready for measurements a few minutes before, the second method is recommended which gives the solar noon respectively for Adrar and Naama on 21/06/2022 from the time of sunrise and sunset, by using the LunaSol software (Omraam, 2022) in Equation (3) below and compare it with that of Michel Capderou given in Equations (1, 2) above.

$$\text{Solar noon} = \text{Sunrise time} + (\text{Duration of day}) / 2 \quad (3)$$

Results and measures

We summarize the different values measured in the following Table 1

As can be seen in the diagram below, the angle (b-a) which is the latitude difference between both cities, it is equal to the difference in angle between both angles shadows, so:

Equations (4) and (5) below are obtained by applying Pythagoras theorem respectively to triangles jkm and NAB :

$$\tan(a) = \frac{j_m}{j_k} \quad (4)$$

$$\tan(b) = BN/AN \quad (5)$$

So

$$a = \text{Arctan} \left(\frac{j_m}{j_k} \right) \quad (6)$$

$$b = \text{Arctan} (BN/AN) \quad (7)$$

The circumference of the earth is then given by the Equation (8) (see Figure 5)

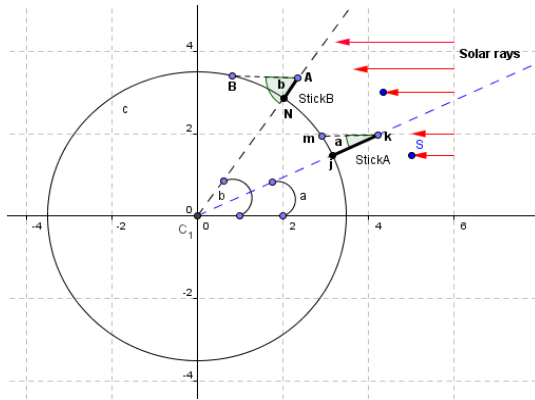
Table 1. Measurement reading in Adrar and Naama

Date		Adrar	Naama
Test 1	21/05/2022 ($E_T = +03.5$)	$T_{SV} = T_L + E_T - 61.1$	$T_{SV} = T_L + E_T - 61.3$
		$T_L = 12h57'36''$	$T_L = 12h57'48''$
		Stick length=1197 mm Shadow $j_m = 191$ mm	Stick length=1217 mm Shadow $BN_1 = 299$ mm
Preliminary test	12/06/2022	$T_L = 13h00'54''$	$T_L = 13h01'6''$
		Stick length=1197 mm	Stick length=1197 mm
		Shadow $j_m = 142$ mm	Shadow $j_m = 256$ mm
Test 2	21/06/2022	Method 1 $E_T = -01.7$ min	Method 1 $E_T = -01.7$
		Method 2 $E_T = -1.61$ min	Method 2 $E_T = -1.61$ min
		$T_{SV} = T_L + E_T - 61.1$	$T_{SV} = T_L + E_T - 61.3$
		$T_L = 13h02'48''$	$T_L = 13h03'$
		Shadow $j_m = 136$	Shadow $BN_{21} = 252$
		Stick length=1197 mm	Stick length=1197 mm

$$P = 2\pi R = \frac{jN \cdot 360}{(b-a)} \quad (8)$$

Then;

$$R = \frac{jN \cdot 360}{2\pi \cdot (b-a)} \quad (9)$$



Figures 5. Schematic arrangement of the sticks on both considered regions

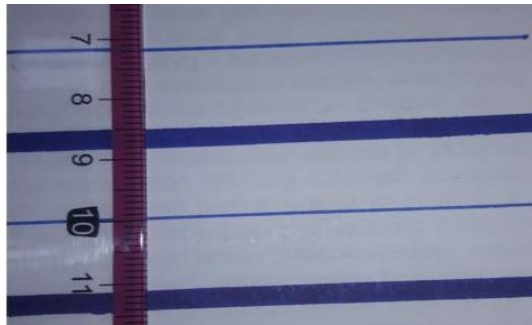


Figure 6. Type of marker used to locate shadows

Improved measurement quality

In order to improve accuracy in shadow measurements, we will use the most accurate type of marker as shown in the Figure 6 below

Discussion

By exploring table 1 and equations 6, 7 and 9 we obtain:

The measurements result in 21/05/2022:
 $R_{mes} = 7252 \text{ Km}$

The measurements result in 12/06/2022:
 $R_{mes} = 6475 \text{ Km}$

The measurement result in 21/06/2022:
 $R_{mes} = 6354 \text{ Km}$

Knowing that the reference value for the radius of the earth at the equator is 6378 Km; the relative uncertainty of R is given by the following formula:

$$\text{As of 21/05/2022} ; \frac{\Delta R}{R} = \frac{|R_{mes} - R_{ref}|}{R_{ref}} = \frac{|7252 - 6378|}{6378} = 13.7\%$$

$$\text{As of 12/06/2022} ; \frac{\Delta R}{R} = \frac{|R_{mes} - R_{ref}|}{R_{ref}} = \frac{|6475 - 6378|}{6378} = 1.5\%$$

$$\text{As of 21/06/2022} ; \frac{\Delta R}{R} = \frac{|R_{mes} - R_{ref}|}{R_{ref}} = \frac{|6354 - 6378|}{6378} = 0.37\%$$

Using the second method we find an accuracy of $\frac{\Delta R}{R} = \frac{|6378 - 6139.4|}{6378} = 3.7\%$; this method gives us a result 10 times less good compared to that of Michel Capderou. This can be explained by the fact that the rays of the sun are not parallel to each other or begin to deviate from parallelism in these times taken as solar noon.

Conclusions

According to the measurement results achieved in 21/06/2022, it is found that the earth radius is equal to 6354 Km with an accuracy of 0.37 %, while the real value of the earth radius calculated with the most evolved scientists instruments is 6378 Km at the equator.

The preliminary tests allowed us to detect possible errors and to avoid them on the day of the true summer solstice (June 21).

The arrangements have been made to further improve the accuracy of the calculation. Indeed the lengths of the sticks have been taken equal and the choice of markers for identifying the shadows of the sticks was made. The watches in both localities have been synchronized.

The difference in angles between Adrar-Naama ($b-a=5.4^\circ$) have almost the same longitude also proves to us the roundness of the earth. That is to say that the displacement between both places on the surface of the earth is done according to, an arc of a circle; this was predicted by Pythagoras 2300 years ago.

We can see that this calculated angle difference of 5.4° is obtained by subtracting the latitudes of the two cities considered, i.e. ($33.27^\circ - 27.88^\circ = 5.39^\circ$).

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Declarations

The authors have no competing interests to declare that are relevant to the content of this article.

Nomenclature

T_{sv} : True solar time

T_L : Local time

E_T : Equation of time, min

T_{srt} : Sunrise time

D_{day} : Duration of day

jm_1 : Measurement n°01 of the shade in Adrar, on 21/05/2022, mm

jm_2 : Measurement n°02 of the shade in Adrar, on 21/06/2022, mm

BN_1 : Measurement n°01 of the shade in Naama, on 21/05/2022, mm

BN_2 : Measurement n°02 of the shade in Naama, le 21/06/2022, mm

Jm_{xy} , BN_{xy} : Measurement of the shade, mm

X: test number

Y: Method number

Jk: Adrar Stick length, mm

AN: Naama Stick length, mm

h, ' , '' : Hour, Minute, Second

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