MATHEMATICAL MODELING OF THE EFFICIENCY OF CAR PARKING SPACES ALONG THE ROADSIDE

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Abstract

Worldwide, the number of vehicles on the road has been rapidly increasing during the past decade, despite the continued development of public transport. Therefore, the problem of lack of parking spaces is occurring in many countries. Focusing on parallel car parking spaces along the roadside, there are two types of parking spaces: marked parking lots and parking zones (or undivided parking lots). This paper is dedicated to the comparison between the efficiency of these two types of parking spaces. Agent-based modeling is used to simulate the scenario of parking with particular input parameters such as driver behavior and car length. The experimental results show that various conditions of the society, namely driver behavior, car length, and marked parking lot length, are key parameters to determine whether a marked parking lot or a parking zone is more efficient.

Keywords: **Agent-Based Modeling; Marked Parking Lot; Parking; Parking Zone**

Introduction

The automobile has been one of the most important elements in human life since the early 20th century. Despite the fact that mass transit has been improved in many nations, the global trend of the automotive market saw an expansion from approximately 60 million vehicles produced in 2000 to almost 100 million automobiles manufactured in 2018. (Wagner, 2021). However, parking areas have not kept pace with this dramatic increase in the number of automobiles (Harris, 2021). The problem of inadequate parking space can frequently be found in developing countries where public transport is not quite good, yet many people can afford to purchase and operate an automobile. This problem cannot be solved by simply constructing or adding new parking spaces, because land is finite and construction costs are high. Therefore, the most efficient way of organizing the parking areas, which constitute a limited resource, should be achieved.

There are in general two types of parallel parking along the roadside: marked parking lots and parking zones or undivided parking lots (show Figure.1 and Figure. 2). Firstly, marked parking lots are the parking areas that are divided into individual parking spaces, and drivers must respect the separating lines and park only one car in a given

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space. Secondly, undivided parking lots or parking zones are parking spaces that have no markings between the cars, but may have a common marking line separating them from the road parallel to the road's kerb. Marked parking lots have been used for a long time as they can organize the cars to be parked properly. However, they have to be constructed with a width that exceeds the average length of a car in a particular country or society. This causes unwanted spaces between cars and leads to sparser parking than is possible. Therefore, this type of parking seems to not be the most efficient way, and parking zones might be an alternative that can solve this problem.

For parking zones, on the other hand, it may appear at first look that cars can be parked as close to another as possible because of the lack of markings, resulting in a greater number of parked automobiles without unused space between cars, as happens in parking lots. Unfortunately, in reality, nothing can guarantee that drivers will park as tightly as possible. Since automobiles are noticeably different in length, and the drivers freely choose parking spots in the parking zones, the gaps formed between the cars can be quite big, but still insufficient to allow for parking new arriving cars in them. Thus, parking zones can lead to even bigger undesired spaces between cars as compared to parking lots. Therefore, the primary goal of this paper is to compare the effectiveness of both types of parking and to decide which one is better in terms of the number of automobiles parked over a specified length of roadside parking area.

This paper is divided into four sections. The first section states a description of the research problem and important terms that are used in this study. The second section is the model and the simulation that is developed to determine the efficiency of parking zones. The third section presents the experimental results that come from the simulation described in the second section. Lastly, the fourth section contains the conclusion for the comparison between these two types of parallel parking.

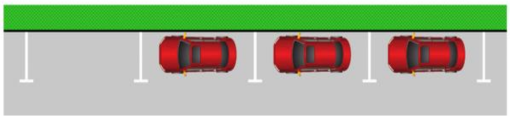

Figure 1. Marked parking lot

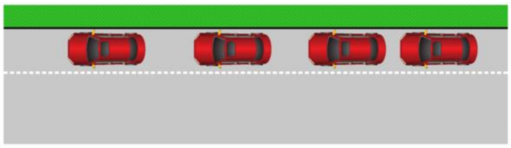

Figure 2. Parking zone

Description of the Research Problem

In this study, we make the following assumptions:

(1) The automobiles are parked exactly parallel to the kerb, every car is equally far from the kerb, and the kerb is straight.
(2) The road is wide enough to prevent swinging issues when parking in any position.
(3) The front-parked and the back-parked car are the widest cars possible. Since people cannot estimate the width of cars with the naked eye, they have to calculate the gap required from the widest car in a given data set.
(4) The first car and the last car can both exit the parking lot without being obstructed by anything.
(5) There are three types of people with regards to parking behaviour: responsible, careless, and selfish.
(6) All drivers leave at least a minimum gap required for the front car to exit the parking lot.

We use the nomenclature in Table 1 for this study.

Table 1. Indicates nomenclature used in this study and their meanings.

Variable	Meaning
r	Kerb-to-kerb turning circle radius (see Figure 3)
b	Wheel-base of a car (see Figure 4)
k	The gap between a car's front and its front wheel's center (see Figure 4)
w	Width of a front-parked car (see Figure 3)
g	Blank space between parked cars that drivers leave so they can exit the parking lot (see Figure 3)
n	Number of cars in the parking zone
a_i	Distance between the i^{th} car and the $(i-1)^{th}$ car
l_i	Minimal gap that the driver of the $(i-1)^{th}$ car needs to drive off the parking lot
R	Length of the road
m	Distance between two separate parking spaces in a marked parking lot
μ_a	Arriving rate of a car: a random variable with mean equal to 3 minutes per car
μ_l	Leaving rate of a car: a random variable with mean equal to 30 minutes per car
p_r	Percentage of responsible people in the society
p_c	Percentage of careless people in the society
p_s	Percentage of selfish people in the society

The Model

Required Minimum Gap Calculation

The geometry of a perfect parking scenario has been examined by Blackburn, (2009). Consider an ideal (in terms of minimizing the gap between the parked cars) parallel parking scenario (Show in Figure. 3).

When cars leave the parking lot, the red circle shows where corner A will be. The corner G is also on the perimeter of this red circle as it is the way that gives the smallest gap possible. The wheels are what rotate, but the size of the red circle is unknown. Thus, it is necessary to draw inner circles that show the direction of the wheels. Fortunately, it is possible to measure the radius of the kerb-to-kerb turning circle, which is the shortest circle that each car model's tires can turn. Other parameters are the cars' dimensions which surely affect the gap required for drivers and can also be measured. The Pythagorean Theorem can be used to efficiently calculate the minimal spacing between the parked cars by simplifying this scenario into Figure. 3 in accordance with Assumptions 1, 2, 3, and 6.

The final formula is introduced in equation 1.

$$g_{min} = \sqrt{(r^2 - b^2) + (b + k)^2 - (\sqrt{r^2 - b^2} - w)^2} - b - k. \quad (1)$$

From equation 1, it can be seen that the minimum gap increases as the width increases, which corresponds to the justification of the third assumption.

Simulation Development

A mathematical simulation for describing the process of parking in a parking zone and determining the average number of cars in a parking zone of a given length is developed. Important factors that are considered are the following: individual driver behaviour and the ratio of responsible, careless and selfish drivers, car length, and arriving and leaving rates of the cars. Figure 4.

Agent-based model (ABM) is utilized in this study. ABM is a computational model that simulates the behavior of an autonomous agent, in this case, a driver and a vehicle, in order to understand system behavior better (Bonabeau, 2002). The simulation works in accordance to the flowchart shown in Figure 5.

As shown in Figure 6, the parking area is initially configured to be empty. The first car is then simulated with a random length (according to the distribution of the given data in a particular society). Subsequently, equation 1 is used to determine the

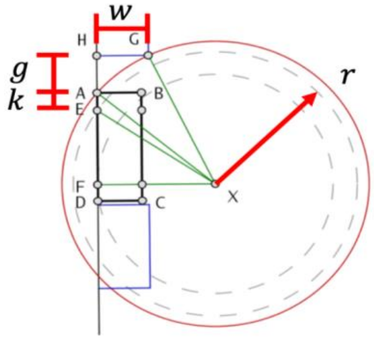

Figure 3. **Perfect parallel parking scenario**

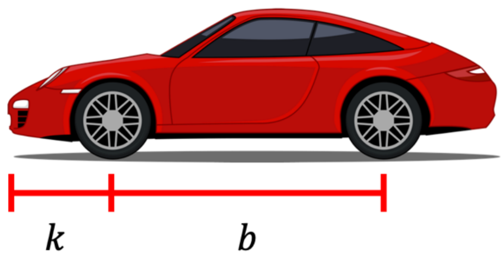

Figure 4. **Wheel-base of a car and distance from the center of the front wheel to the front of a car**

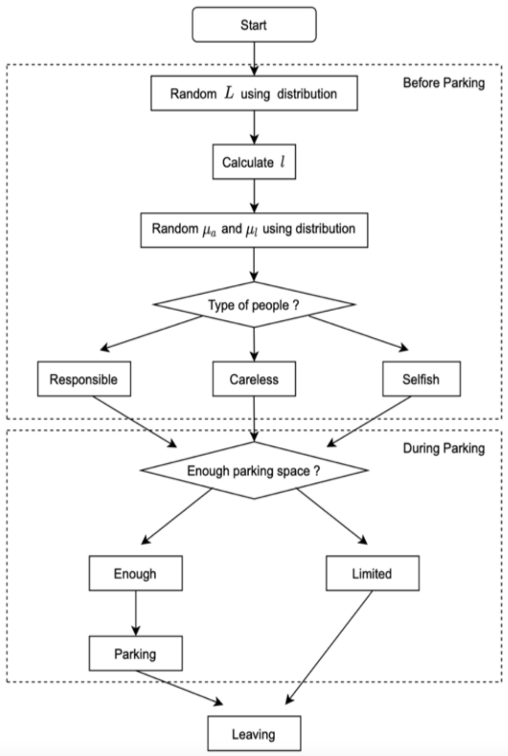

Figure 5. **Flow chart describing the process of simulating cars parking in a parking zone**

minimum gap required. The arrival and departure rates of that car are the following two variables that are simulated using an exponential distribution with means of μ_a and μ_l equal to 3 and 30 minutes, respectively. After that, the agent's behavior must then be simulated because the agent, which is the car, is autonomous. One of the three types of personal behaviors is assigned to the driver of that car according to the ratio of each kind in the community, which is adjustable.

Figure 6. **Empty parking scenario in the simulation (initial condition)**

Figure 7. **Parking probability distribution for a responsible person**

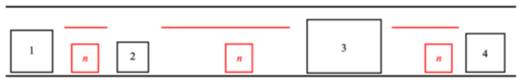

Figure 8. **Parking probability distribution for a careless person**

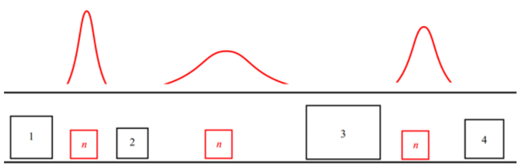

Figure 9. **Parking probability distribution for a careless person**

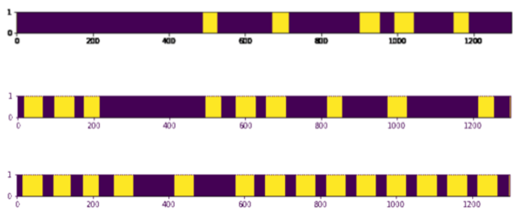

Figure 10. **Parking scenario in the simulation after some cars have entered the system**

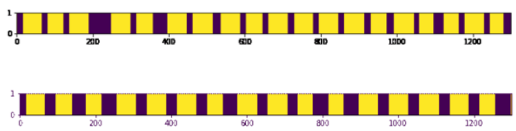

Figure 11. **Parking scenario in the simulation when capacity has been re ached**

The description of each type of people according to parking behaviour is as follows:

(1) A responsible person is one who is attempting to maximize the parking space by parking a car as closely as possible to any front-parked car or the available wall. (See the Figure 7.)
(2) A careless person tends to park a car any place that is accessible with the same likelihood. (See the Figure 8.)
(3) A selfish person is likely to park a car far away from other automobiles to avoid any accidents that could happen to the vehicle as a beta distribution with α = 5.00 and β = 5.00 with a higher likelihood when the space is wider (See the Figure 9.)

The beta distribution is a family of continuous probability distributions set on the interval [0,1]. The probability density function (PDF) of the beta distribution is a power function (see equation 2) where B (α, β) is defined in equation 3. Normally, the beta distribution is not symetric unless α = β, which is the case that we consider here to describe people's behaviors (for more details, see Ross (2014)).

$$f(x;\alpha;\beta) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1} \tag{2}$$

$$B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt \tag{3}$$

A Poisson process, which is a model for a series of discrete events where the average time between occurrences is known but the precise timing of events is random, is used to represent the process of arriving and departing (Koehrsen, 2019). A vehicle will remain in the parking space until the time determined by μ_a and μ_l has elapsed. The second car is mimicked in the same manner as the first one after it has been successfully parked. It will enter the parking space in accordance with its μ_a, and all other vehicles entering the system will be treated similarly.

This model is intended to calculate the average number of vehicles parked in a space of a specific length. This length can be adjusted according to the real situation that one wants to study. In this research, we assigned it to be 100 meters. The Monte Carlo method, which is employed in this experiment to estimate the potential results of an uncertain event, is used. The incoming and departing rates are viewed as a Poisson Process with μ_a and μ_l (See Ross (2014) for more details). The simulation operates as previously stated, and it is repeated 1,000 times for a total of 1,800 minutes (relative time utilized in the simulation, not real-time). The system's average car number is then determined. shown in Figures 10 and 11.

In the simulation, the purple color indicates the available space for cars to be parked, and the yellow blocks depict the cars that are completely parked in the simulated parking zone.

For a parking zone, we calculate the average number of cars in the parking zone at any moment in time. We consider nine different scenarios ('societies'), that differ by the ratio of each of the three behavioral driver types. The nine societies for the experiment are indicated in Table 2.

The calculations for the marked parking lot are also accomplished by varying the width of the marked spaces to be 6.5 meters, 8.0 meters, and 8.5 meters. It should be noted that driver behavior does not affect the simulation because each car can be parked in any available space at equal probability.

Apart from the average number of cars placed in the car parking spaces at any moment in time, rejection is also a significant factor that must be considered when comparing the efficiency of the parking spaces. In this study, rejection is defined as the number of cars that leave the system immediately without parking due to the following two reasons:

(1) There is no space available to accommodate a car of particular length and its required front gap.

(2) A car cannot be fitted into an available marked spaces because it is too large. This cause of rejection only applies to marked parking lots, as the parking zone allows cars of any size to park.

Table 2. **Percentage of each type of person by society**

Society	Responsible	Careless	Selfish
A	90	5	5
B	70	15	15
C	50	25	25
D	25	50	25
E	25	25	50
F	15	70	15
G	15	15	70
H	5	90	5
I	5	5	90

Table 3. **Average number of vehicles with SD and average number of rejected cars**

Society	Vehicles	SD	Rejected
A	8.64	1.12	0.39
B	8.46	1.21	0.41
C	8.35	1.15	0.50
D	8.30	1.18	0.65
E	8.37	1.09	0.67
F	8.40	1.11	0.81
G	8.39	1.11	0.83
H	8.60	1.03	1.18
I	8.58	0.99	1.24

Results and Discussion

Parking Zone (Behaviors Varied)

The average number of vehicles, the standard deviation of the average number of vehicles and the average number of rejected cars are presented in Table 3. Some graphic results are also shown in Figures 12-14.

According to these results, the largest average number of cars parked in the parking zone and the lowest number of rejected vehicles can be observed in society A. This is the society that contains the highest percentage of responsible people. Additionally, as the ratio of selfish people and careless people rises, the number of cars that are rejected goes up. Even though the average numbers of parked cars in all nine societies are close to another, it should be noted that this simulation is for a relatively short 100 meters of roadside, and the differences between the average number of cars will be greater for a longer roadside parking zone. Furthermore, the reason why the societies with a high percentage of selfish people and careless people, i.e., societies E-I, still show a close to maximum average number of parked cars, may be

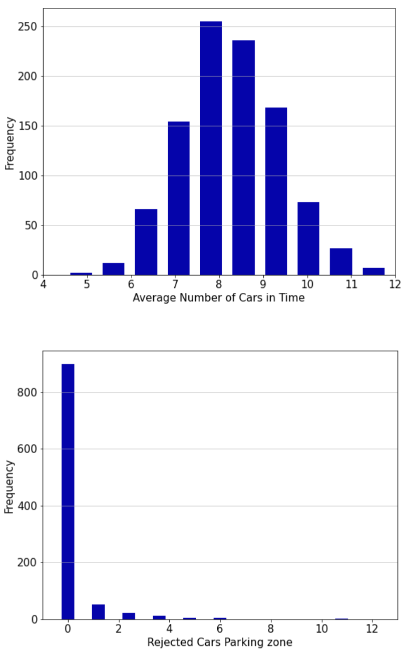

Figure 12. **Distribution of the average number of parked cars and of rejected cars in the parking zone in society A**

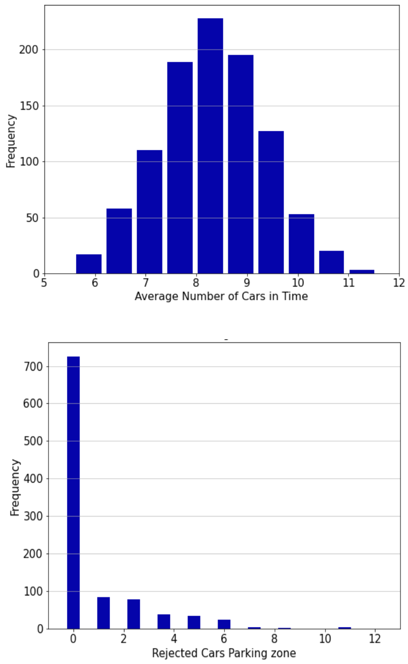

Figure 13. Distribution of the average number of parked cars and of rejected cars in the parking zone in society H

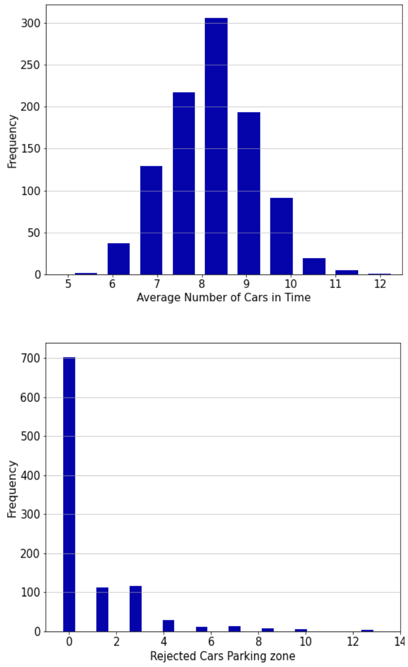

Figure 14. Distribution of the average number of parked cars and of rejected cars in the parking zone in society I

that when people park their cars far from another, they do not always block other cars from parking; the gap between those cars might be large enough for other cars to be parked in between, so this can sometimes be more effective.

Marked Parking Lot (Width Varied)

Space with 6.5 meters marks Figure 15 shows that the average number of cars is 2.68 cars with SD equal to 0.68 cars, and the average number of rejected cars is 56 cars.

Space with 8.0 meters marks Figure 16 shows that the average number of cars is 8.40 cars with SD equal to 1.13 cars, and the average number of rejected cars is 0.54 cars.

Space with 8.5 meters marks Figure 17 shows that the average number of cars is 8.34 cars with SD equal to 1.13 cars, and the average number of rejected cars is 0.97 cars.

From the experimental results, we can observe that the 8.0-meter marked parking lot shows the highest average number of parked cars at any given time and the least number of rejected cars as well. For the 6.5-meter parking lot, the significantly lower number of parked cars observed is due to the fact that most of the cars in the studied data set require spaces larger than 6.5 meters in order to accommodate car length plus the required minimum gap. A different reason accounts for the lower average number of parked cars in the 8.5-meter parking lot: one spot takes up more space than is required, resulting in the lower number of available parking spaces in the given roadside length. Increasing the width of the marked spaces does not result in an incline or decline in efficiency, as a wider length results in fewer rejected cars but also fewer individual parking spaces. Hence, for a marked parking lot, balancing between two types of rejections, one caused by marked spaces of too small a size and the other caused by a marked space of too large size, is crucial to maximize the parking area.

Lastly, when comparing the two types of parking spaces together, the parking zone tends to show a higher average number of parked cars at any given time than the marked parking lot. Nevertheless, there are societies that show a lower average of cars in the parking zone type. For example, the average number of parked cars in societies C-G are lower than those of the 8.0-meter marked parking lot. Moreover, for most societies the rejections in the 8.0-meter and 8.5-meter marked parking lots are lower than in the parking zone. Therefore, whether the parking zone is better, or the marked parking lot is better cannot be answered in a simple manner without any context; different conditions of the society, which are driver behavior, the length of each marked parking lot, or even the distribution of car length are key parameters to determine which method of parking is more efficient.

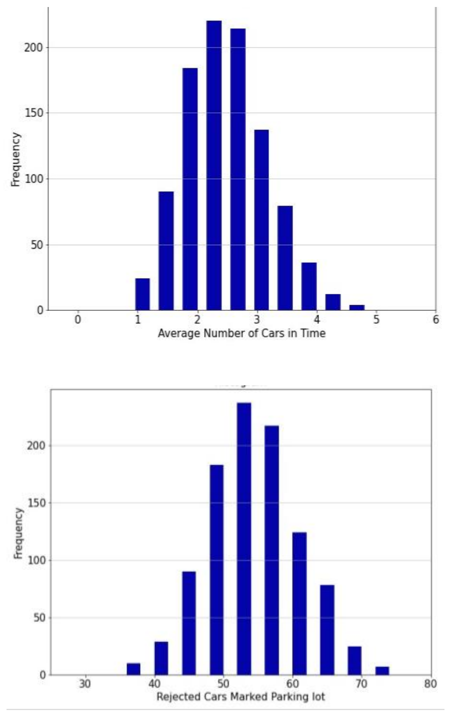

Figure 15. **Distribution of the average number of parked cars and of rejected cars in a 6.5 meters marked parking lot**

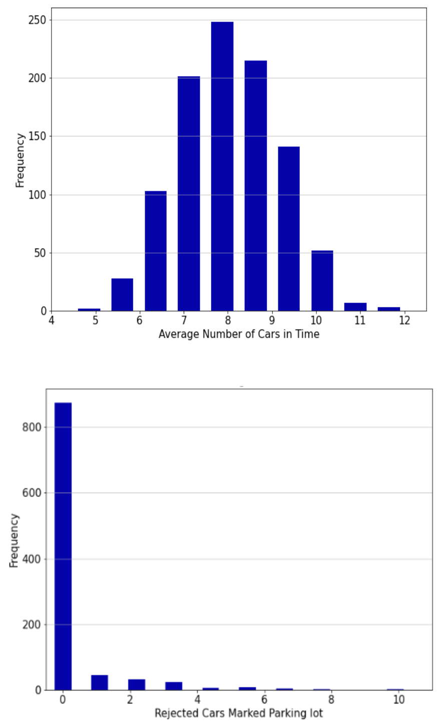

Figure 16. **Distribution of the average number of parked cars and of rejected cars in an 8.0 meters marked parking lot**

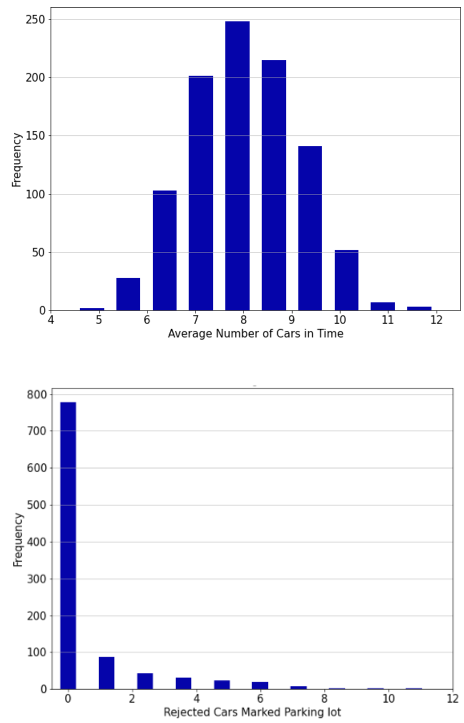

Figure 17. **Distribution of the average number of parked cars and of rejected cars in an 8.5 meters marked parking lot**

Conclusions

A mathematical model for calculating the efficiency of two types of parking spaces along the roadside, which are parking zone and marked parking lot, have been conducted by an agent-based approach. The presented model has been applied to various scenarios including parking zones in societies with different individual behaviors and marked parking lots with varied space widths. Interesting experimental results are observed. On average, the parking zone tends to show a higher number of parked cars for the nine societies studied as compared to the marked parking lots, and the highest number is observed in the society with the highest percentage of responsible people. Furthermore, the average number of rejected cars in the parking zone increases as the percentage of responsible people in the society decreases. However, when focusing on rejection, in some societies lower numbers of rejected cars are observed in both 8.0-meter and 8.5-meter parking lots as compared to the parking zone in some societies. Thus, according to all the results calculated in this study, no method of parking is recognized as the best, as the variation in driver behavior, the width of the parking space, the length of cars, and many other factors are essential to foretelling whether a parking zone or marked parking is more efficient to be applied in a particular situation. Therefore, our mathematical model presented in this study can effectively be applied as a useful tool to indicate the best parking method for any city or country with given essential data.

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