

MATHEMATICAL MODEL AND SIMULATION FOR DESIGNING A COST-OPTIMIZED OFF-GRID HOUSE SOLAR ENERGY STORAGE SYSTEM

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Abstract

Solar power is a renewable energy source suitable for residential usage due to its affordability and the small area required for installation. However, it requires an effective energy storage system that can minimize power outages within a limited budget. This challenge was addressed by analyzing energy storage systems regarding climate and user behavior. In this study, a mathematical model has been developed to design a cost-effective energy storage system for an off-grid household. We utilized the Markov weather process and Monte Carlo simulation, considering various scenarios of available energy supply, energy consumption, and user behavior. In accordance with the outcome, the size and type of battery system from our model minimized power outages throughout the year. We applied our model to a 1600-square foot house in Rayong, Thailand, with the meteorological data and typical electricity loads from user data. For a budget threshold of 40,000 USD, our model suggests that the best battery is Tesla Powerwall+, which results in an average power outage of only around 3 hours per year, outperforming other batteries under identical economic conditions. Additionally, our model can be widely applied to real scenarios by considering a broader range of batteries, environmental impact and available space, and hybrid systems as well.

Keywords: Energy Storage System; Mathematical Modeling; Minimization Problem; Monte Carlo Simulation; Renewable Energy; Sustainable Development

Introduction

Solar power is considered one of the most critical, renewable, sustainable, clean, and environmentally friendly energy sources available. In line with technological advancements over the previous decade, the cost of solar energy has been rapidly decreasing, from \$359 per megawatt-hour in 2009

to \$36 per megawatt-hour in 2019 (Ray, 2019). One significant advantage of solar power over other energy sources is its availability, irrespective of location. Simple and convenient installation on a small scale is crucial for access in areas restricted from countries' national electric grids, and in these

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instances, an off-grid system is a necessity. One example is the study by Bugaje, which analyzed power supply options for remote areas of Nigeria. The analysis showed that solar or photovoltaic (PV) systems provided the most economic benefits, offering the least energy cost per annum and virtually maintenance-free operation (Bugaje, 1999). However, the limitation of an off-grid PV power system is its necessity to be optimally designed for maximum reliability and minimum cost (Khatib, 2016; Al-Falahi, 2017; Goel, 2017). Therefore, devising a method to design and develop an efficient off-grid PV system is necessary to meet the future demands for solar energy.

To address the problem, the PV system must be developed on the basis of technical, economic, and environmental criteria so that the number of PV arrays, the number of storage batteries, the capacity of the converter and inverter, and the PV tilt angle can be carefully chosen. Previously, several studies have developed mathematical models to determine the optimal PV module and its sizing for off-grid houses (Ghafoor and Munir, 2015; Werulkar and Kulkarni, 2015; Kamali, 2016; Okoye and Oranekwu-Okoye, 2018; Akinsipe, 2021). However, no studies have focused on the development of mathematical models for evaluating the optimal requirements of energy storage systems.

In typical standalone off-grid PV systems, the storage battery plays a significant role not only in increasing the reliability of the PV system but also in reducing the total capital cost during a specific period of time. Therefore, the storage battery should be carefully selected to operate with a high level of reliability and at the lowest cost. Moreover, the most efficient or optimum battery storage system for an off-grid residence should be based on the householder's needs. The main goal of this research is to develop a mathematical model to select the optimum battery storage option for every off-grid home from all accessible options on the market. Accordingly, the model must be capable of ranking all the options to determine the most viable choice for any off-grid home.

Mathematical Models

The overall model is separated into five components, the main model based on three sub-models and two add-on models. The main model is responsible for the compilation and decision to find the most suitable type of battery for the objective household and uses the information from its three compartments: the solar energy supply yield, the energy load demand, and the input data, such as the specifications of available batteries.

The main model consists of three models. The first of these is the PV-performance model. In this model, the results from the second and third (solar radiation, climate) models will be processed into the total energy that the solar cell can provide to the battery each hour. The second model, the solar radiation model, estimates the solar radiation in the targeted region, and the third model, the climate forecasting model, estimates the cloud cover in the targeted region. Both solar radiation and climate are the main factors that affect the energy yield from solar cells.

Besides the main model and its compartments, two add-on models are needed in special circumstances where the values of some parameters are unknown. The first is the load demand estimation model, and the second is the continuous and instantaneous power ratings estimation model.

Main Model

The task of the main model is to find the most suitable type of battery for the objective household from a given list of options. The input of this model is the energy production from the PV system array and the user's energy consumption, both on an hourly scale.

There are five parts to the calculation. The first is the state of charge, which shows the percentage of energy inside the battery. The second and the third are the energy charging rate and discharging rate, respectively. The fourth part uses the equation from the earlier steps to calculate the amount of energy inside the battery as a function of time. The last part involves the process of selecting the most suitable option.

The Stage of Charge

The percentage of energy stored in a battery is represented by the stage of charge (SOC), with a value of 100% indicating a completely charged state (Barua, 2009). To retain battery quality, the SOC cannot fall below the recommended SOC value (SOC_{min}). Thus,

$$SOC_{min} \leq SOC(t) \leq SOC_{max} = 1 \quad (1)$$

At the reference point, the battery SOC is reset to 100%, which means that it is fully charged before use. So,

$$SOC(0) = SOC_{max}. \quad (2)$$

When calculating the charged and discharged energy, there is energy loss due to the inefficiency of the processes. Round-trip efficiency (η_{RT}) is defined as the ratio of the energy output acquired

from the battery supplying the system to the energy input, which is the energy from the PV array. This value can be expressed as follows (Hubert and Stefan, 2017)

$$\eta_{batt,in} = \eta_{batt,out} = \sqrt{\eta_{RT}} \quad (3)$$

where

$\eta_{batt,in}$ is the charging efficiency,
 $\eta_{batt,out}$ is the discharging efficiency.

The Total Energy Charged

Due to process inefficiencies, energy is lost while charging from the PV array into the batteries. The possible energy charged into the batteries ($PEC_{pv}(t)$) can be expressed as

$$PEC_{pv}(t) = E_{pv}(t) * \eta_{batt,in} \quad (4)$$

Where

$E_{pv}(t)$ is the energy from the PV array mentioned in the PV performance model.
 $\eta_{batt,in}$ is the efficiency of energy charged into the battery.

The Total Energy Discharged

The energy consumption from load demand is used to calculate the total energy discharged from the battery. This can be given by the following equation:

$$DC_{total}(t) = \frac{U_{total}(t)}{\eta_{wire} * \eta_{inv} * \eta_{batt,out}} \quad (5)$$

where

$DC_{total}(t)$ is the total energy discharged from the battery,
 $U_{total}(t)$ is the total load demand,
 η_{wire} is the wire's efficiency,
 η_{inv} is the inverter efficiency.

The Battery Energy As A Function Of Time

The power input, power output, and battery capacity and charge rate are used to formulate the amount of energy stored in a battery at each instance, as follows:

$$E_{bat}(t) = E_{bat}(t-1) + PEC_{pv}(t) - DC_{total}(t) \quad (6)$$

where

$E_{bat}(t)$ is the energy in the battery at hour t ,
 $PEC_{pv}(t)$ is the total energy charged into the battery.
 $DC_{total}(t)$ is the total energy discharged from the battery,

The reliability of the battery system can be calculated using the loss of load probability (LLP), which refers to the ratio of the total unmet energy demand to the total energy demand in a year. The LLP values are between 0 and 1. Therefore, high reliability is indicated when the LLP value approaches 0, meaning that there is only a small chance of energy shortage. In contrast, if the LLP value approaches 1, blackouts happen frequently throughout the year. The LLP can be computed using the following equation (Ridha, 2020):

$$LLP = \frac{\sum_t^T LL(t)}{\sum_t^T U_{total}(t)} \quad (7)$$

where LL is loss of load defined as

$$LL(t) = \begin{cases} DC_{total}(t) - PEC_{pv}(t) - E_{bat}(t-1); & LL(t) > 0 \\ 0; & otherwise \end{cases} \quad (8)$$

$DC_{total}(t)$ is the total energy discharged from the battery,
 $PEC_{pv}(t)$ is the possible energy charged
 $E_{bat}(t-1)$ is the energy in the battery at hour $t-1$,
 $U_{total}(t)$ is the total load demand.

Battery Selection

Having obtained the LLP indicator from the previous calculations, a list of input batteries and a procedure for selecting the best battery system can be found as discussed below.

The first value to be considered is the instantaneous power rating. The battery must be able to provide enough power for the appliance that consumes the highest instantaneous power rating. If not, appliance damage can occur, and the battery will be considered as not practical, and the LLP value will be shown equal to 1.

Our procedure starts with spending the total budget given to purchase batteries. Then, the energy demand, energy supply, and battery specification (including capacity, rate of charge, and rate of discharge) are taken together for the simulation to find the LLP value of each battery. The best battery will give the lowest LLP. If several battery types give an LLP of 0, to find the best one, all these types of batteries will be analyzed for the lowest price that still achieves a 0 LLP. The method is reducing the number of batteries assembled in each type until the LLP becomes nonzero and comparing the prices at that moment for all types to find the cheapest option. The overall process of the main model is shown in the flow chart in Figure 1.

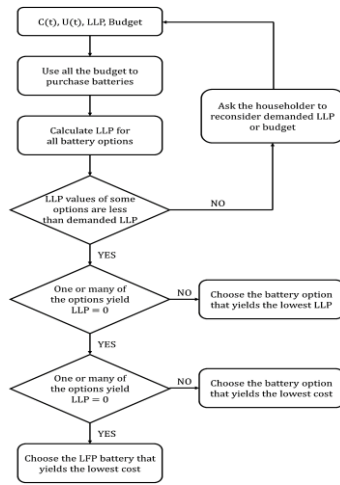


Figure 1. Flowchart of the main model

Compartments of the Model

The PV system is composed of solar panels, inverters, and other electrical and mechanical hardware. Due to the dependence on both environmental factors and the efficiency of the hardware, the performance of the PV system varies in different cases. Therefore, the main model was subdivided into three compartments.

PV Performance Model

The hourly PV output strongly depends on the amount of solar radiation absorbed by the solar panels. Other variables affecting performance include local climate conditions, ambient temperature, air mass, and cell temperature. The PV array current is enhanced if the solar radiation increases. In contrast, increasing the ambient temperature decreases the output voltage of the PV array. Thus, the power output of a PV system for a corresponding period ($P_{pv}(t)$) when t represents the time duration (equal to one hour) can be empirically illustrated using the following formula (Sarhan, 2018):

$$P_{pv}(t) = \left[P_{peak} \left(\frac{G(t)}{G_{standard}} - \gamma [T_c(t) - T_{standard}(t)] \right) \right] * \eta_{wire} * \eta_{inv} * \eta_{climate} \quad (9)$$

under the condition that $P_{pv}(t) > 0$, $G(t) > 0$ and otherwise equal to 0,

where

$G(t)$ is the solar irradiation for the corresponding period (W/m^2),

$G_{standard}$ is the solar radiation on the standard test conditions (STC) (1000 W/m^2),
 $T_{standard}$ is the ambient temperature on the standard test conditions (STC) (25°C),
 γ is the temperature coefficient of the PV modules,
 P_{peak} is the peak power at STC PV module,
 η_{wire} is the wire's efficiency,
 η_{inv} is the inverter's efficiency.

The cell temperature (T_c), which is determined under a solar radiation value of 800 W/m^2 , at an ambient temperature of 20°C and a 1 m/s wind speed (Sarhan, 2018), can be calculated as:

$$T_c(t) = T_{ambient}(t) + \frac{NOCT-20}{800} G(t) \quad (10)$$

where

$T_{ambient}(t)$ is the ambient temperature,
 $NOCT$ is the nominal operating cell temperature,
 $G(t)$ is the solar irradiation for the corresponding period (W/m^2).

Note that the parameter of ambient temperature is cited from the database of a reliable organization, such as Visual Crossing Corp (Corporation, 2022).

Since the hourly solar radiation data are input, the energy from the PV array is then equivalent to the total output power of a PV system, and the energy generated by the PV module can be empirically expressed as follows:

$$E_{pv}(t) = N_{pv} * P_{pv}(t) \quad (11)$$

where

$E_{pv}(t)$ is the energy from PV array
 N_{pv} is the number of PV modules,
 $P_{pv}(t)$ is the output power of a PV modules.

Solar Radiation Model

Solar radiation is the intensity of sunlight for a given ground area, and this depends on the angle between the sun and that location on Earth. Hence, in different regions, solar radiation is not equal, which leads to another factor that is needed to be included in the PV performance model. Two factors affect solar radiation: the number of daytime hours and maximum solar radiation in one day, both of which can be described by sinusoidal functions,

$$D(m) = a_1 + b_1 \sin(c_1 m + d_1) \quad (12)$$

where

m is the number of the month,
 $D(m)$ is daytime of each day in month m ,
 a_1 , b_1 , c_1 , and d_1 are parameters that
 rely on the location.

$$G_{max}(m) = a_2 + b_2 \sin(c_2 m + d_2) \quad (13)$$

where

m is the number of the month,
 $G_{max}(m)$ is the maximum solar radiation in
 month m , a_2 , b_2 , c_2 , and d_2 are
 parameters that rely on the location.

Regarding the data on sunlight intensity in many countries, the radiation in one day can be represented as a sinusoidal function and reaches its maximum point at noon. Assuming that noon is the midpoint of any particular day, the solar radiation model can be described by the following equation:

$$G(t) = \begin{cases} G_{max}(m) \cos\left(\frac{(t-12)\pi}{D(m)}\right); & 12 - \frac{D(m)}{2} < t < 12 + \frac{D(m)}{2} \\ 0; & \text{otherwise} \end{cases} \quad (14)$$

where

m is the labeled number of each month,
 $G(t)$ is sunlight intensity at hour t in the 24-hour system,
 $D(m)$ is daytime of each day in month m ,
 $G_{max}(m)$ is maximum solar intensity in month m .

Climate Model

Cloud cover is another factor that affects PV performance; for example, the sunlight intensity is reduced when the sky is overcast compared to when the sky is clear, which leads to a lower yield. Thus, cloud cover must be considered in an effective model. The Markov chain is one mathematical tool that can predict the upcoming cloud cover. The chain consists of three states describing the cloud cover in the upcoming hour, which are sunny (less than 30%), cloudy (between 30% to 80%), and overcast (more than 80%). On a sunny hour, the efficiency of the solar panels will be at their maximum, which is 100%. On a cloudy hour, it will be 70%, and in an overcast hour, it will drop to 10%. (Wolf, 2022). The transition probabilities were calculated from the historical climate data, which was obtained from a reliable weather database, such as Visual Crossing Corp. (Corporation, 2022), using the following equation:

$$Tran_{ij} = \frac{H_{ij}}{H_i} \quad (15)$$

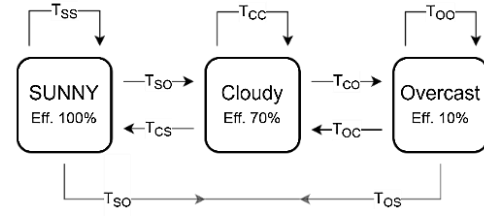


Figure 2. Transition probability of Markov

where

$Tran_{ij}$ is transition probability from State i
 to State j ,
 H_{ij} is number of hours that change from
 State i to State j ,
 H_i is number of hours of State i .

The example of the overall Markov model computed from the above equation is displayed in Figure 2.

However, the transition probability should differ season by season because the cloud cover is typically different in each season. Thus, in each season, the transition probability should be separately calculated.

Add-on models

These models will be applied when there is a lack of appropriate information, or the cited information is not informative enough because data such as the load demand needs to be given on an hourly scale.

Energy Load Demand Model

The load of electrical appliances is another parameter determined by the user. Since the model needs to be applicable in every region, the input format is designed to be flexible.

The energy consumption of each major electrical appliance can be estimated by the expression:

$$U_i(t) = W_i \times N_i(t) \quad (16)$$

where

$U_i(t)$ is the energy consumption of
 electrical appliance type i at time t ,
 W_i is the power rating of electrical
 appliance type i ,
 $N_i(t)$ is the number of active electrical
 consumers of type i .

After the energy consumption of each type has been estimated, the total energy consumption can be simply computed using the following equation:

$$U_{total}(t) = \sum_{i=1}^n U_i(t) \quad (17)$$

where

$U_{total}(t)$ is the total electrical consumption,
 n is the number of appliance types considered,
 $U_i(t)$ is the energy consumption of electrical appliances type i at time t .

Estimation of Continuous Power Rating and Instantaneous Power Rating

Apart from calculating $U(t)_{total}$ to afford an estimation of LLP, the other two criteria that the battery needs to meet are the continuous power rating and the instantaneous power rating. For continuous power rating, the upper bound can be approximated by considering the case when all the high-energy-consuming appliances are used simultaneously. The threshold value is calculated to be the sum of the power by all appliances using the following formula:

$$CPR = \sum_{i=1}^n WA_i * NA_i \quad (18)$$

where

CPR is the continuous power rating,
 WA_i is the wattage of appliance of type i ,
 NA_i is the number of appliances of type i .

The instantaneous power rating of a battery storage system is required for a few seconds when each electrical appliance is switched on. Not all appliances require a high instantaneous power rating. To sufficiently supply the appliance when starting, our model overestimates this from the typical value by 30%. Since the required period of instantaneous power is very short, the required instantaneous power can be estimated by assuming the case that all appliances except the highest instantaneous power-

consuming appliance (HIPC) are on. Then, the power required when that appliance is switched on is added using the following formula:

$$IPR = CPR - CP_{HIPC} + IP_{HIPC} \quad (19)$$

where

IPR is the instantaneous power rating,
 CPR is the continuous power rating,
 IP_{HIPC} is the instantaneous power when HIPC is on,
 CP_{HIPC} is the continuous power when HIPC is on.

Experimental Design

To illustrate the performance of the model, we applied our model to a 1600-square-foot off-grid house located in Rayong, Thailand. The electrical consumption behavior of the residents in the house was synthesized to provide data on the typical load role of all users on an hourly scale. Weather forecasting and solar radiation inputs were raw data collected in Rayong, Thailand. Besides load demand and environmental conditions, money and the list of available batteries are also fundamental inputs. We considered available funds of 40,000 US dollars. The directory of the available batteries is shown in Table 1.

Input Power from Solar Panel

In the first part of our model, the input power from the solar panel charged into the battery is calculated. All electrical appliances in the home are powered only by the solar energy system; the whole energy is yielded by the Panasonic HIT® Black solar panel, whose specifications are shown in Table 2.

Table 1. Battery Specifications

Battery No.	Battery Type	Cost per unit (USD)	Continuous Power Rating (kW)	Instantaneous Power Rating (kW)	Round-Trip Efficiency	Charging Wattage (W)	Charging Wattage per module(W)	Usable Capacity (kWh)
1	Sonnen Eco 5 Gen 3.1	15,000	3.00	6.00	85.0%	-	3,015	5.00
2	SonnenCore	9,500	4.80	9.60	85.0%	9,000	-	10.00
3	Discover AES 7.4 kWh	6,478	6.65	14.40	95.0%	-	7,839	7.40
4	Electriq PowerPod 2	13,000	7.60	9.00	96.6%	11,400	-	10.00
5	Tesla Powerwall+	8,500	7.00	10.00	90.0%	5,000	-	13.50

Table 2. Solar panel specifications

Details	Value
Peak power (P_{peak})	330 W
Power temperature coefficients of P_{max}	-0.258 %/°C
NOCT	44 °C
Module Area	30.09 m ²

Climate parameters are the next factor that affects the output power. The data of cloud cover stated in the weather database from Visual Crossing Corp. was applied to create the Markov model. This information was collected at Rayong, Thailand, from 1 January 2020 until 31 December 2020. The data also included the ambient temperatures which were made use of in the PV performance calculation.

To approximate the number of daytime hours and the maximum sunlight intensity, solar radiation and PV radiation data were modeled through graph regression using data from the European Commission on PVGIS Photovoltaic Geographical Information System database (Huld, 2012). Following this, the power gained from the solar panel can be estimated by simulating the constants of the Panasonic HIT® Black solar panel with a wire efficiency of 1 and inverter efficiency of 0.95, and the parameters from the previous sub-model:

$$P_{pv}(m, t) = \begin{cases} \left[330 \left(\frac{G(t)}{1000} + 0.258[T_c(t) - 25] \right) \right]^{0.95} * \eta_{climate} & ; P_{pv}(t) > 0, G(t) > 0 \\ 0 & ; otherwise \end{cases} \quad (20)$$

Output Power from Load Demand

The actual load demand can be challenging to measure or calculate. It is possible to simplify the calculation and consider only the loads of electrical appliances that significantly consume the stored energy and impact the possible power shortages, which include space temperature control, water heating, lighting, cooking, and laundry. Not all these appliances have a high wattage per hour. An LED light bulb, for example, consumes only 10 W. Nevertheless, the numbers and the operating times accumulate to a significant proportion of the total. Also, some appliances operate for only a few hours each week, so in total, they do not contribute a large amount to the total energy consumption. However, at the time of operation, their high power draws a large portion of the total electricity used, which can result in load loss. The power rating of each type of appliance is shown in Table 3.

Table 3. Wattage power consumption of selected appliances

Electrical Appliances	Typical Continuous Power Consumption (W)
LED Light Bulb	10
Washing Machine	500
Air Conditioner	1000
Clothes Dryer	3000
Water heater	4000
Heat Pump	4700

Besides individual power consumption, the model also needs to consider the number and activation time interval of each type of appliance on an hourly scale to calculate the power consumption, according to this equation:

$$U_i(t) = W_i \times N_i(t) \quad (21)$$

where

$U_i(t)$ is the energy consumption of electrical appliance type i at time t ,
 W_i is the power rating of electrical appliance type i ,
 $N_i(t)$ is the number of active electrical consumers of type i .

Thus, the user's energy consumption schedule is created to be such a sample for the input format of this model. Users must enter the schedule of energy consumption in each season specifically. In each of the five appliance types in Table 3, the input information must contain an approximate number of people using that appliance in every hour of one day. This difference in behavior was a crucial factor in the model used to indicate which battery is the most suitable to maintain the lifestyle of each household.

Battery Evaluation

The first priority is to measure the capability of each type of battery to bear with the instantaneous power rating. If the instantaneous rate is beyond the limits of a battery type, then the LLP of that type of battery will be automatically set to 1.

Subsequently, the models that surpass the first criterion will be put into a simulation of the weather and electricity consumption conditions that have been prepared, calculating the energy inside the battery as a function of time. When the energy demand exceeds the battery's capacity, the LLP value, the energy deficit, and the estimated blackout time will be determined.

Results and Discussion

Input Power from Solar Panel

Figure 3 represents the results of the Markov chain model for the weather simulation for each season: summer (15 February-16 May), rainy (16 May-15 October), and winter (16 October-15 February) at Rayong, Thailand.

From the simulation of the climate by the Markov chain on a day-to-day basis over one calendar year the climate pattern shown in Figure 4 could be generated. Darkest bars represent overcast days. Dark gray bars represent cloudy days. Light gray bars represent summer days. A pattern can be

seen during different times of the year: overcast conditions are more common towards the beginning and end of the year, which is the winter season. Even though sunshine appears throughout the year, most of it occurs during the summer season.

Once the information about climate and solar radiation has been collated, the last stage in obtaining the input power from the solar panel is through calculating the PV performance. The hourly output power of a PV system can be calculated and plotted for each day. Applying the above equation with weather simulation, PV performance from the simulation as a function of time can be analyzed, as shown by the plots in Figure 5, which show weather performance for four separate days during the year.

The examples represented in the graph span different times of the year: January, April, August, and December, and show that there is a significant wattage input variance in different months. The 1st of April has the highest PV output energy gained from the sun, followed by the 1st of August, the 1st of January, and the 1st of December, respectively.

Output Power from Load Demand

Four examples of daily electrical consumption obtained from simulation are displayed in Figure 6. The variation in loads across different days is due to the seasonal use of household appliances such as air conditioners, water heaters, and those used for cooking.

The time each appliance is active may be dictated by the season and other special occasions.

This function is specific to the month and to whether the day is a weekday or a weekend. This variation in behavior was a key aspect in the model's determination of which battery is best for a given household.

Battery Selection Under a 40,000 USD Budget

Considering the charging of energy to the battery each hour from the PV performance model and the discharge of energy from the electricity consumption by the user, it was found that the Sonnen eco 5 Gen 3.1 battery cannot deliver the instantaneous power required by high-energy-consuming appliances. As a comparison, the LLP result for all batteries tested in this study is displayed in Table 4.

Based on these results, the Sonnen Eco 5 Gen 3.1 gives a 1.00 LLP value, indicating that this battery cannot meet the instantaneous power rating and cannot be further considered. Our model shows that the best battery for a 40,000-dollar budget is the Tesla Powerwall+, which has an average power outage of only 3 hours per year, surpassing other batteries under identical economic, locational, and lifestyle conditions.

The option to change variables allows the model described herein to be applied to households with differing power usage requirements, and different climatic regions. Changing the input conditions will produce different sets of results, each depending on the scenario. For example, in a situation for two houses located under the same

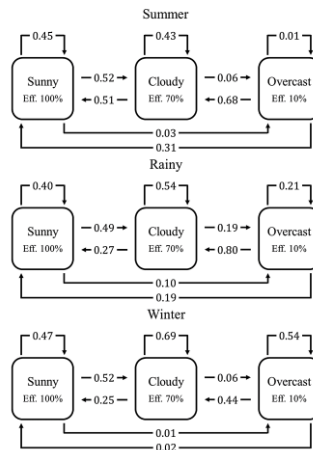


Figure 3. Transition probability of Markov



Figure 4. Heatmap representing climate pattern simulated by the Markov model

Table 4. Results of battery selection under 40,000 USD budget

No.	Battery Type	LLP Value	Total Deficit (kW)	Estimated Blackout (Hours)
1	Sonnen Eco 5 Gen 3.1	1.000000	-	-
2	SonnenCore	0.044456	878.045	389.43
3	Discover AES 7.4 kWh	0.001952	38.548	17.09
4	Electriq PowerPod 2	0.007265	143.483	63.64

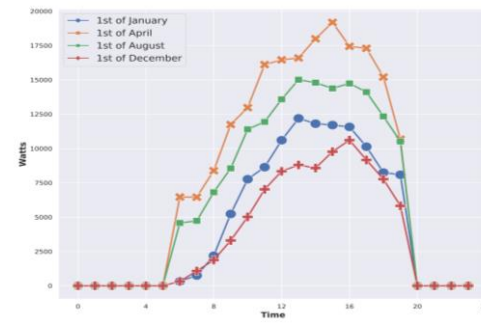
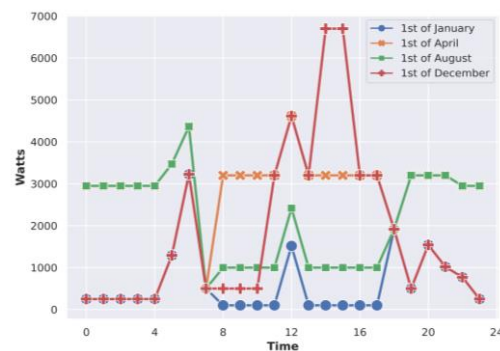
climatic conditions and having the same battery option, the model can give distinct preferences for each depending on their daily activities. Moreover, this model is very adaptable in terms of data intake. Additional data, such as more accurate location-based climates, more precise energy consumption in certain activities, or the availability of new battery options can be factored in, allowing for enhanced resolution and more certainty in regard to electricity utilization and battery selection.

Conclusions

In this paper, a mathematical model for the optimization of battery storage systems for off-grid households has been developed to facilitate consumers who desire the most suitable and cost-effective battery system for their own households. This is a newly devised model which can aid users in the selection of a battery system, which no previous studies have tried to develop. Thus, the proposed model is the basis for improvements to become a viable and reliable tool. Users can enter information about their electrical consumption behavior, the location, and the specification of the solar panels and set the budget limitations in the model to estimate the most suitable battery system for their household.

Proposals for a new climate-dependent PV performance model are developed using Markov models. Even though Markov chains are data-intensive, the model is stochastic. Although relying on a large quantity of data, the weather-appropriate stochastic model is suitable for a simulation based on real scenarios since the model can approximate randomness while keeping realistic features.

Add-on models and sub-models were developed as vital tools to help homeowners feed the data demand of the model. These additional extensions allow the primary model to be applied not only in an ideal situation but also in an actual real scenario. To test and evaluate, we applied our model to a synthetic 1,600-square-foot house in

**Figure 5. Output power of a PV system examples on different dates of the year****Figure 6. Daily electricity consumption examples on different dates of the year**

Rayong, Thailand. Using the computed LLP value for each battery, the best battery was effectively identified under various conditions.

Our approach takes financial constraints into consideration. The concept relates to determining the most suitable battery in accordance with global standards and limitations. Regardless, prioritizing quality above cost brought an additional concern. A slight adjustment to a limited budget may result in an entirely new outcome, which can be resolved by testing on multiple budget.

Lastly, this convenient and accurate mathematical model of battery system performance can help people have a more straightforward battery design process. Deciding the most suitable battery system for different conditions while also maintaining an economic advantage can encourage people to use solar energy, a green technology supporting a sustainable world.

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