

COSECANT-SQUARED BEAM PATTERN SYNTHESIS OF CONCENTRIC CIRCULAR ANTENNA ARRAY IN A RANGE OF AZIMUTH PLANE USING EVOLUTIONARY ALGORITHM

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Abstract

This paper presents, a pattern synthesis method based on Differential Evolutionary Algorithm. A cosec² beam pattern has been generated from different concentric circular ring arrays of isotropic elements by finding the optimum set of elements amplitude and phase using the Differential Evolution algorithm. The shaped beam pattern (cosec²) is generated in three predefined azimuth planes ($\varphi=0^\circ$, 5° and 10°) from a Concentric Circular Antenna Array (CCAA) having two rings, three rings, and four rings. The total number of isotropic elements are equal for array geometries. The optimum excitation for array geometries is also verified for a range of arbitrarily chosen azimuth planes. The obtained main beam is similar to the desired pattern with some minor variations in peak Side Lobe Level (peak SLL) and ripple. Dynamic range ratio (DRR) is improved by using 4-bit discrete Amplitudes and 5-bit discrete Phases. These discrete excitations also simplify the design complexity of the feed networks.

Keywords: Concentric Circular Array Antenna (CCAA), Differential Evolution Algorithm (DE), Dynamic range ratio (DRR), Discrete Amplitudes and Phases, Isotropic Element

Introduction

This shaped beam pattern is very beneficial for radar and satellite-related communication systems. Concentric circular Antenna array (CCAA) geometry is valuable for its azimuth symmetry. Array synthesis is the process of finding out radiation patterns of various array geometries and makes the obtained patterns closer to their desired patterns either by changing its elements amplitude and phase or by varying the array geometries

(Balanis 1997, Elliott 2002) Shaped beams are often required in different applications, but in most cases, the shaped beams are generated with high sidelobe levels and ripple (Azevedo 2008, Lei *et al.* 2010, and Mandal *et al.* 2015). An efficient evolutionary algorithm has been used for finding out 4-bit discrete amplitudes and 5-bit discrete phases of the array elements to get the desired shaped beams with minimum sidelobe level and ripple. Various

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approaches have been reported in the literature for generating shaped beams are as follows;

Shaped Beam Patterns (cosec² and Flat-top) have been generated using Fast Fourier Transform (FFT) method from linear array antenna by J.A.R. Azevedo (2008). Lei *et al.* (2010) generate a cosecant-square-shaped beam with a low sidelobe level using the modified least square method. Mandal *et al.* (2015) generates shaped beams from CCAA having 30 isotropic elements using Evolutionary Algorithms in a single azimuth plane. The large valued DRR is minimized by making the weakly excited elements off. The cosecant-squared pattern with low PSLL and ripple has been generated using GA from a planar array by Sallam and Attiya (2020). To obtain an array To obtain an array pattern with shaped beam and pencil beam from the linear antenna array, a modified version of IWO is used, namely MIWO. Here the author Kenane *et al.* (2015) also discussed the fundamental difference in the reproduction process of both IWO and MIWO. A cosecant-square shaped beam pattern is synthesized from a uniformly spaced but non-uniformly excited linear antenna array by Behera *et al.* (2013) using Differential Evolutionary Algorithm. Mandal *et al.* (2012) generates a Cosec² beam pattern of a circular array of 30 isotropic elements using the Genetic Algorithm (GA).

In this paper, a cosec² beam pattern of Concentric Circular Antenna Array (CCAA) of isotropic elements in three pre-defined azimuth planes has been reported. Three different cases have been considered. In the first case, a cosecant-squared beam is generated from 2 rings circular array by finding out the optimum set of elements amplitudes and phases. And in the second and third case, the cosec² pattern with the same design parameters are generated from the three-ring and four-ring concentric circular array by finding out an optimum set of amplitudes and phases of the array elements, where the optimum normalized Amplitudes and Phases are discrete in nature, and the number of isotropic elements is identical with different orientation for each case.

For all the circumstances, the optimum excitations are computed using Differential Evolution (DE) Algorithm (Storn and Price 1997; Yang *et al.* 2002; Price *et al.* 2005 Das *et al.* 2009; Guo and Li 2009; Xiaomei *et al.* 2013; Massa *et al.* 2006).

Dynamic Range Ratio (DRR) is improved by using only 16 distinct levels of the discrete excitation.

Results clearly show the effectiveness of DE for all the design cases presented in this problem.

Problem Formulation

In this literature, the peak SLL and ripple of the cosec² beam pattern in three different planes of ($\theta=0^\circ$, $\varphi=0^\circ$), ($\theta=0^\circ$, $\varphi=5^\circ$) and ($\theta=0^\circ$, $\varphi=15^\circ$) are reduced by finding out the optimum set of elements amplitude and phase using Differential Evolution (DE) algorithm. The shaped beam patterns are also generated from three different array geometries, and all these obtained patterns are not restricted to a single azimuth plane $\varphi=0^\circ$ rather a range of azimuth planes using the same excitations.

The patterns retain their shape with minor variations within the pre-specified range of azimuth planes. For achieving the better Dynamic Range Ratio (DRR) discrete 4-bit normalized Amplitude and 5-bit phases have been used as excitation for all three cases. Three Concentric Circular Antenna Array of isotropic element is taken.

The far-field pattern of these arrays are shown in Figure 1, Figure 2 and, Figure 3 can be defined as (Balanis, 1997; Elliott 2002).

$$AF(\theta, \varphi) =$$

$$\sum_{m=1}^M \sum_{n=1}^{N_m} I_{mn} e^{j[k r_m \sin \theta \cos(\varphi - \varphi_{mn}) + \alpha_{mn}]}$$

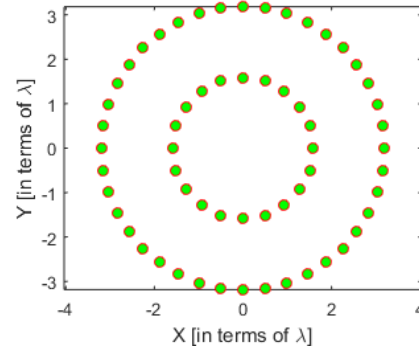


Figure 1. Geometry of 3 ring CCAA

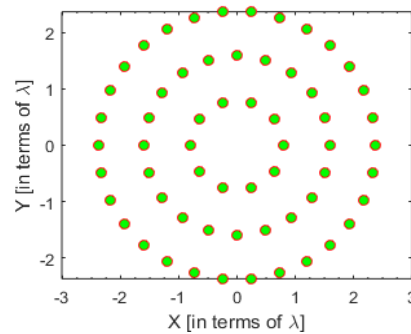


Figure 2. Geometry of 3 ring CCAA

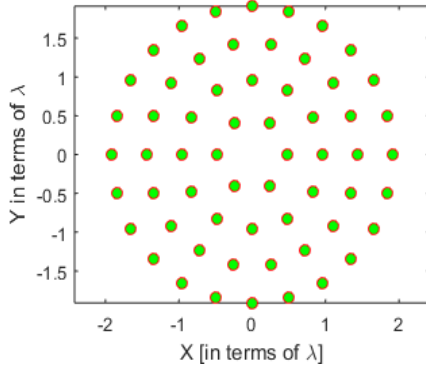


Figure 3. Geometry of 4 ring CCAA

Where, M = number of concentric rings; I_{mn} excitation amplitude of mn -th element; N_m = number of isotropic elements in m -th ring; $r_m = N_m d_m / 2\pi$ radius of the m -th ring; $k = 2\pi/\lambda$ represents wave number; λ is the wavelength; (θ, φ) Elevation and azimuth angle; d_m inter-element arc spacing of m^{th} circle; $\varphi_{mn} = 2n\pi N_m / N_m$; N_m shows the angular location of the mn^{th} element with, $1 \leq n \leq N_m$ and α_{mn} is the phase excitation of mn^{th} element. The fitness function of the cosecant-squared beam pattern for this approach is defined as:

$$F(\rho) = k_1 \left\{ \text{peak SLL}^d - \max_{\theta \in A} (AF_{dB}^\rho(\theta, \varphi)) \right\}^2 + k_2 \times \Delta \quad (2)$$

where, Δ is defined for cosec² beam pattern as:

$$\Delta = \sum_{\theta_{\text{ripple}} \in \{0^\circ \text{ to } 30^\circ\}} |AF_{dB}^\rho(\theta_{\text{ripple}}, \varphi) - D(\theta_{\text{ripple}}, \varphi)|_{dB} \quad (3)$$

In equation, (2) and (3) $\varphi \in (0^\circ \text{ to } 10^\circ)$ at $\theta = 0^\circ$ plane. p is the unknown parameter set responsible for the desired beam patterns, defined as follows:

$$\rho = \{I_{mn}, \alpha_{mn}\}, \quad 1 \leq m \leq M \text{ and } 1 \leq n \leq N_m \quad (4)$$

peak SLL^d is the desired value of peak SLL for cosec² beam pattern. A is the sidelobe region for the cosec² pattern. $D_{dB}(\theta, \varphi)$ is the desired pattern shown in Figure 4 at $\varphi = (0^\circ, 5^\circ \text{ and } 10^\circ)$ plane. The range of θ_{ripple} for cosec² beam pattern is 0° to 30° and 10° for each case. k_1 and k_2 are the weighing factors chosen as one. The fitness function F_p has to be minimized separately for each array geometry by

computing the optimum sets of discrete 4-bit I_m and optimum sets of discrete 5-bit α_{mn} among the elements using the Differential Evolution (DE) Algorithm. For solving such an optimization problem, the population size is considered as 50, the scale factor (F) is taken 0.8, and crossover rate (CR) of DE is chosen as 0.2 (Storn and Price 1997; Yang *et al.* 2002; Price *et al.* 2005; Das *et al.* 2009; Guo and Li 2009; Xiaomei *et al.* 2013; Massa *et al.* 2006) whereas the scheme is used in this nonlinear problem is 'DE/best/1/bin' with maximum iteration number of 3500.

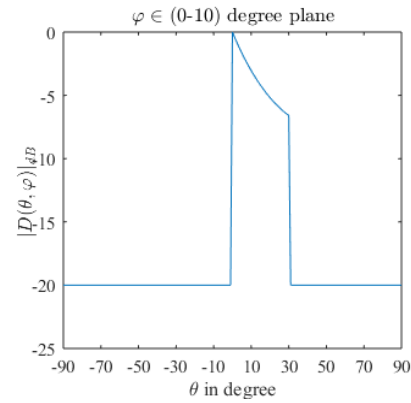


Figure 4. Desired Cosec² beam pattern

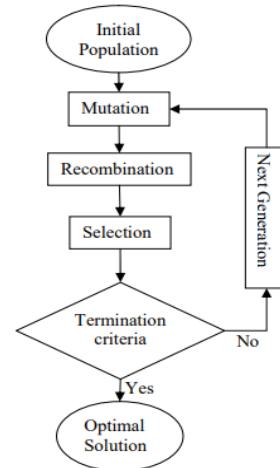


Figure 5. Flow chart of Differential Evolution Algorithm (DE)

Simulation Result

Three concentric circular ring arrays of 60 isotropic elements with 0.5λ inter-element arc spacing has been considered. In case I, the CCAA consist of two rings, where the 1st ring contains 20 isotropic elements and the 2nd ring contains 40

isotropic elements, shown in Figure 1. In case II, the CCAA having three rings where the 1st ring contains 10 isotropic elements, and 2nd ring contains 20 isotropic elements, and the last round contains 30 isotropic elements (Figure 2). In case III, the array has shown in Figure 3 has four rings, and each circle from 1st to 4th contains the isotropic elements 6, 12, 18, and 24, respectively. The number of elements present in each circle with their respective ring radius is given in Table 1. For each case, the inter-element arc spacing has been considered as 0.5λ . The number of elements present on a circle having 0.5λ arc spacing, the radius $r_m = N_m d_m / 2\pi$ of each circle has been calculated and expressed in Table 1. Figure 4 shows the desired pattern in a predefined range of the azimuth plane. ($\varphi \in 0^\circ - 10^\circ$). Figure 6 shows the cosecant-squared pattern generated from two rings CCAA (Figure 1) by applying optimum discrete amplitudes and phases (Figure 9) using.

The generated pattern in three predefined azimuth planes is nearly similar to the desired one with some variations. Similarly, Figure 7 and Figure 8 depict the obtained cosec² beam pattern in all redefined azimuth planes, generated from 3 rings (Figure 2) and 4 rings CCAA (Figure 3) using the excitation given in both Figure 10 and Figure 11, respectively. The design specification of the Cosecant squared beam pattern with its corresponding desired and obtained outcomes are mentioned in Table 2. Table 2 clearly shows that the peak SLL for case I is -15.7224 dB with 45.36 dB ripple for $\varphi = 0^\circ$ plane. The values of peak SLL are -15.9839 dB and -16.4169 dB respectively for $\varphi = 5^\circ$ and 10° plane. Similarly, the obtained values of peak SLL for case II (3 ring CCAA) is -17.2103 dB, -20.7001 dB, and -20.9339 dB corresponding to its desired value -20 dB are given in Table 2. For case

III (4 ring CCAA) the desired and obtained values of peak SLL and ripple for all three predefined azimuth planes are also shown in Table 2. Optimum values of excitation for the array elements obtained using the DE algorithm are presented in Table 3 for case I, Table 4 for case II, and Table 5 for case III. The element-wise 4-bit discrete amplitudes (I_m) and 5-bit discrete phases α_{mn} are shown in Figure 9, Figure 10, and Figure 11. The Normalized amplitude has 2^4 different levels in between (0 to 1), whereas phases have 2^5 different levels between (-180° to $+108^\circ$). The obtained values of normalized amplitudes having a minimum value of 0.0625 and a maximum of 1 provide the DRR value 16, and this solved the complexity of designing the feed networks. Table 3, Table 4, and Table 5 also show that every element contribute producing.

Table 1. Number of Rings, Ring Radius and Number of Elements in each ring for all Cases

Case No.	Total ring (M)		Ring Number			
			1	2	3	4
I	2	N_m	20	40	--	--
		$r_m(\lambda)$	1.5915	3.1831	--	--
II	3	N_m	10	20	30	--
		$r_m(\lambda)$	0.7958	1.5915	2.3873	--
III	4	N_m	6	12	18	24
		$r_m(\lambda)$	0.4775	0.9549	1.4324	1.9099

Table 2. Desired and obtained result for all cases in predefined φ planes

φ in Degree	Parameters (dB)	Desired Values (dB)	Obtained Values (dB)		
			2 Ring	3 Ring	4 Ring
0°	peakSLL	-20.00	-15.7224	-17.2103	-19.0538
	(Δ)	0.00	45.36	40.38	48.95
	peakSLL	-20.00	-15.9839	-20.7001	-19.8025
5°	(Δ)	0.00	43.19	45.66	49.24
	peakSLL	-20.00	-16.4169	-20.9339	-20.3728
	(Δ)	0.00	42.95	42.99	49.40

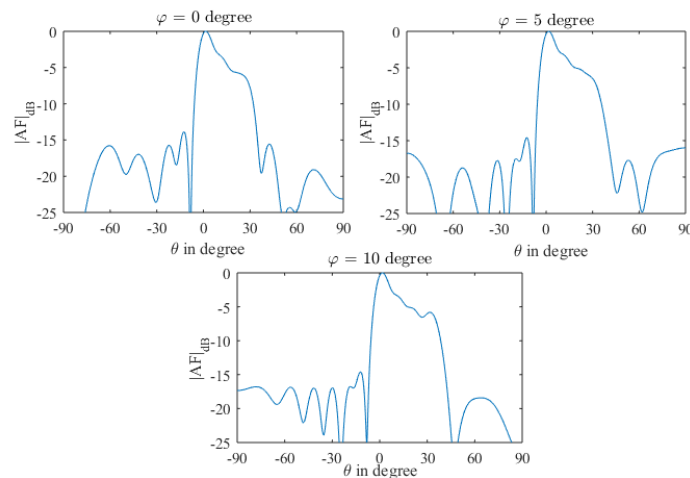


Figure 6. Shows cosec² pattern of 2 Ring CCAA (Case I)

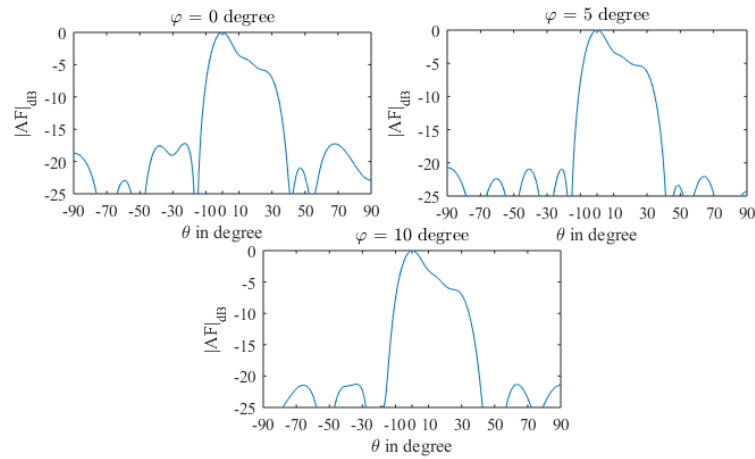


Figure 7. Shows cosec² pattern of 3 Ring CCAA (Case II)

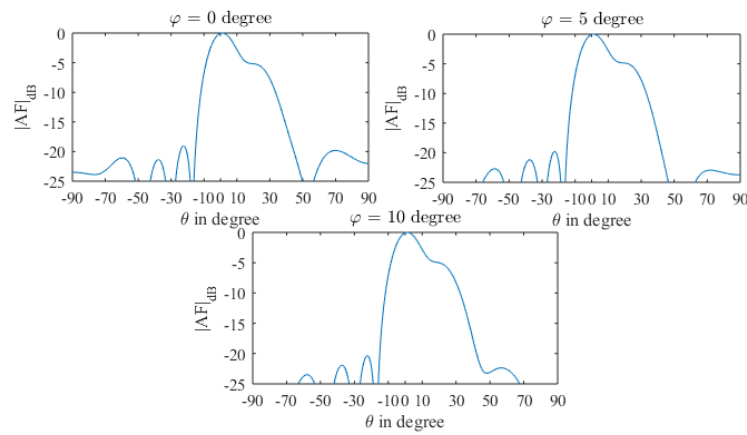


Figure 8. Shows cosec² pattern of 4 Ring CCAA (Case III)

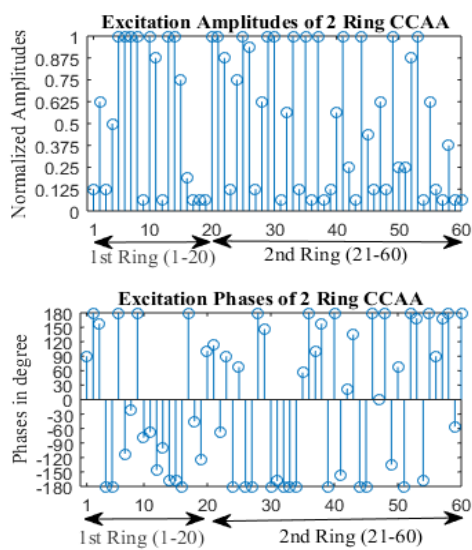


Figure 9. Optimum excitations for 2 Ring CCAA (Case I)

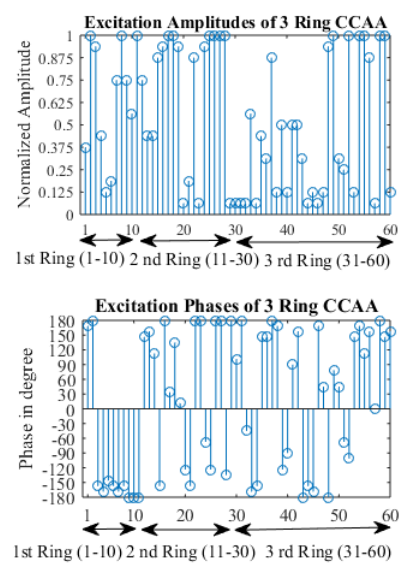


Figure 10. Optimum excitations for 3 Ring CCAA (Case II)

The desired pattern. No such elements having amplitude 0, i.e., every element is in the 'ON' state as the Dynamic Range Ratio (DRR) is less, so fewer attenuators and phase shifters are required. All the

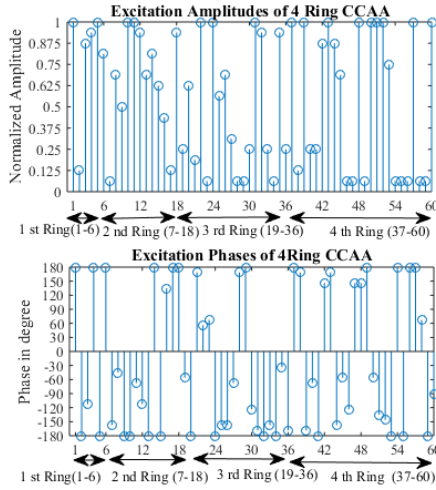


Figure 11. Optimum excitations for 4 Ring CCAA (Case III)

sets of excitations for each case are also verified by considering three arbitrarily chosen azimuth planes ($\varphi=2.5^\circ$, 7.5° , and 12.5°) presented in Figure 12, Figure 13, and Figure 14. The cosec² beam pattern has been generated with some minor variations concerning its desired parameters illustrated in Table 6. From Table 6, it can be observed that for an arbitrary angle of 2.5° which is in-between the range of predefined plane 0° to 5° has a value of peak SLL -16.3221 dB with ripple (Δ) 42.49 dB for case I, peak SLL -18.6652 dB and ripple (Δ) 44.59 dB for case II and peak SLL -19.4612 dB with ripple 49.05 dB for case III. Whereas for the arbitrary angle of 7.5° the value of peak SLL and ripple (Δ) are -16.3177 dB and 42.54 dB respectively for case I. For case II peak SLL -20.7578 dB with 42.00 dB ripple and for case III peak SLL -20.0990 dB with 49.26 dB ripple. This is adequate to its desired value, but for an arbitrary angle of 12.5° which is beyond the predefined range of the azimuth plane ($\varphi=10^\circ$) the obtained peak SLL and ripple far deviated from its desired value shown in Table 6. The convergence curves of the DE are shown in Figure 15.

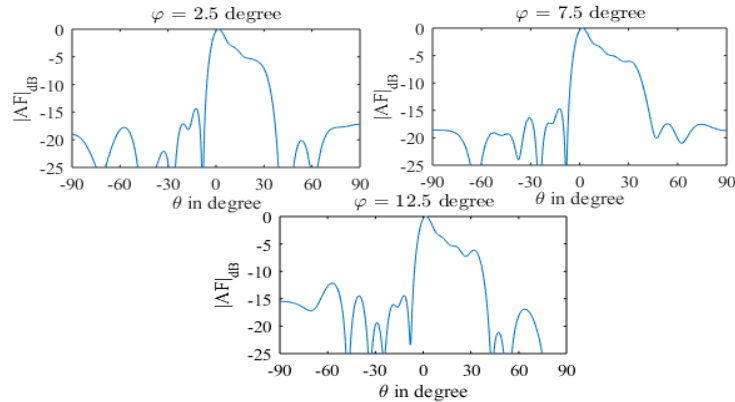


Figure 12. Cosec² patterns from 2 Ring CCAA in arbitrary planes (Case I)

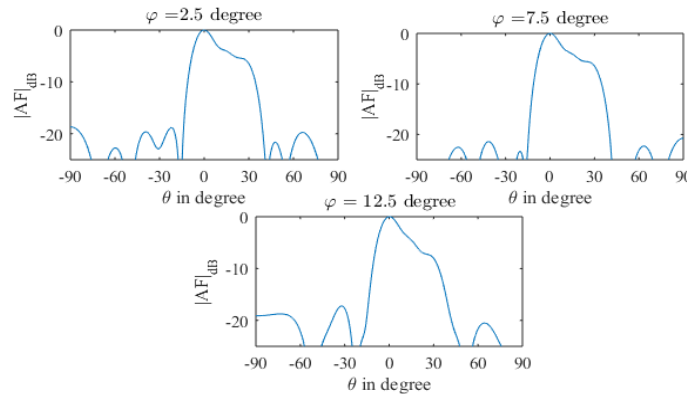


Figure 13. Cosec² patterns from 3 Ring CCAA in arbitrary planes (Case II)

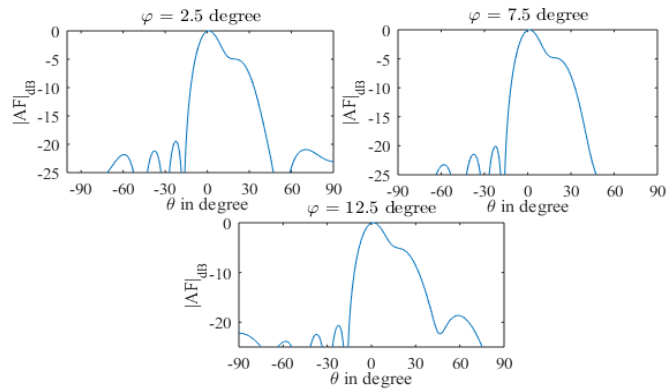
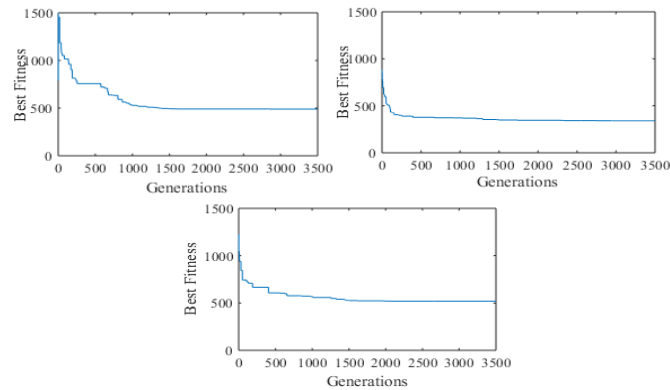
Figure 14. Cosec² patterns from 4 Ring CCAA in arbitrary planes (Case III)

Figure 15. Convergence curve of Differential Evolution Algorithm (DE) for all three cases

Table 3. Amplitudes and Phases of the Array Elements for Case I

Ring No	Normalized Amplitudes	Phases in Degrees
1	0.1250, 0.6250, 0.1250, 0.5000, 1.0000, 1.0000, 1.0000,	90.00, 180.00, 157.50, -180.00, 67.50,
	1.0000, 0.0625, 1.0000, 0.8750, 0.0625, 1.0000, 1.0000,	-180.00, -180.0, 180.0, 146.25, -180.00, -168.75, -180.0,
	0.7500, 0.1875, 0.0625, 0.0625, 0.0625, 1.0000	-180.00, -180.00, 56.25, -180.00, 180.00, -45.00, -123.75, 101.25
2	1.0000, 0.8750, 0.1250, 0.7500, 1.0000, 0.9375, 0.1250,	112.50, -67.50, 90.00, -180.00, 67.50,
	0.6250, 1.0000, 1.0000, 0.0625, 0.5625, 1.0000, 0.1250,	-180.0, -180.0, 180.00, 146.25, -180.00, -168.75, -180.0, -180.00,
	1.0000, 0.0625, 1.0000, 0.0625, 0.1250, 0.5625, 1.0000,	-180.00, 56.25, 180.00, 101.25, 157.50, -180.00, 180.00,
	-0.2500, 0.0625, 1.000, 0.4375, 0.1250, 0.6250, 0.1250,	-157.50, 22.50, 135.00, -180.0, -180.00, 180.00, 0.00, 180.00, -
	1.0000, 0.2500, 0.250, 0.8750, 1.0000, 0.0625, 0.6250,	135.00, 67.50, -180.00, 180.00, 168.75, -168.75, 180.00, 90.00,
	0.1250, 0.0625, 0.3750, 0.0625, 0.0625	168.75, 180.00, -56.25, 180.00

Table 4. Amplitudes and Phases of the Array Elements for Case II

Ring No	Normalized Amplitudes	Phases in Degrees
1	0.375, 1.00, 0.9375, 0.4375, 0.1250, 0.1875, 0.7500, 1.00,	168.75, 180.0, -157.5, -168.75, 146.25, -157.5, -168.75, -157.5, -
	0.7500, 0.5625	180.0, -180.0
2	1.000, 0.7500, 0.4375, 0.4375, 0.875, 0.9375, 1.000,	-180.0, 146.25, 157.50, 112.5, -157.5, 180.00, 33.75, 135.00,
	1.000, 0.9375, 0.0625, 0.1875, 0.875, -0.0625, 0.9375,	11.25 -123.75,
	1.00, 1.000, 1.000, 1.000, 0.0625, 0.0625	-157.5, 180.0, 180.0, -67.50, -123.75, 180.0, 180.00, -135.00,
3		180.00, 101.25
	0.0625, 0.0625, 0.5625, 0.0625, 0.4375, 0.3125, 0.8750,	180.00, -45.00, -168.75, -157.50, 146.25, 146.25, 180.00,
	0.1250, 0.5000, 0.1250, 0.5000, 0.5000, 0.3125, 0.0625,	168.75, -123.75, -90.00, 90.00, 157.50, -180.00, -157.50, -
	0.1250, 0.0625, 0.1250, 0.9375, 1.0000, 0.3125, 0.2500,	168.75, 168.75, 45.00, -180.00, 78.75, 45.00,
	1.0000, 0.1250, 1.0000, 1.0000, 0.8750, 0.0625, 1.0000,	-67.50, -101.25, 146.25, 168.75, 112.50, 157.50, 0, 180.00,
	1.0000, 0.1250	146.25, 157.50

Table 5. Amplitudes and Phases of the Array Elements for Case III

Ring No	Normalized Amplitudes	Phases in Degrees
1	1.00, 0.125, 0.875, 0.9375, 1.00, 0.8125	180, -180, -112.50, 180.00, -180.00, 180.00
2	0.0625, 0.6875, 0.5, 1.00, 1.00, 0.9375, 0.6875, 0.8125, 0.625, 0.4375, 0.125, 0.9375	-157.50, -45.00, -180, -180, -67.50, -112.5, -180.0, 180.0, -180.0, 135.0, 180.0, 180.0
3	0.250, 0.6250, 0.1875, 1.00, 0.0625, 1.00, 0.5625, 0.6875, 0.3125, 0.0625, 0.0625, 0.25, 1.00, 0.9375, 0.250, 0.0625, 0.9375, 0.250,	-56.25, -180.0, 168.75, 56.25, 67.50, -180, -157.5, -157.5, -67.5, 168.75, 180, -123.75, -168.75, -180, -157.5, -180, -33.75, -168.75, 180, 168.75, -168.75, -67.5, -180, 146.25, 168.75, -
4	1.00, 0.125, 1.00, 0.2500, 0.2500, 0.8750, 1.00, 0.875, 0.6875, 0.0625, 0.0625, 1.00, 0.0625, 1.00, 1.00, 1.00, 0.7500, 0.0625, 0.0625, 0.0625, 1.00, 0.0625, 0.0625, 1.00	157.5, -56.25, -123.75, 146.25, 46.25, 180, -56.25, -135, 146.25, -180.00, 180.00, -180, 180, 180.00, 67.50, -180.00, -90.00

Conclusions

Synthesis of a cosec² (Cosecant-Squared) beam pattern of three different concentric ring array antennas has been proposed and assessed using the Differential Evolution algorithm (DE). The pattern has been generated in three predefined azimuth planes instead of any particular ϕ plane for all three cases. The optimum discrete excitations used for generating the shaped beam patterns for three distinct array geometries are also verified for three arbitrarily chosen azimuth angles within and beyond the range of predefined azimuth planes. Those excitations are helpful not only in some predefined ϕ angles rather a range of azimuth planes. Sidelobe and ripple are minimized by minimizing perfectly formulated fitness function using an Evolutionary Algorithm to synthesize the Cosecant-Squared beam patterns. Dynamic Range Ratio (DRR) has been improved by using digital excitations among the array elements. The presented method can also be applied to synthesize other array geometries using different Optimization Algorithms.

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