

GENERALIZED REDEFINED ZAGREB INDEX ON SOME NANOSTRUCTURES

Shanmukha Mulleman Channappa^{1,*}, Shilpa Kalajji Channabasappa², Usha Arcot³, and Anil Kumar Kempasidpla Nagarajappa⁴

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Abstract

Chemical graph theory has become a significant part of the research. Topological indices are numerical descriptors used to determine physico-chemical properties of chemical compounds. In this work, the generalized redefined Zagreb index is defined. This index is applied on several topological indices viz., redefined Zagreb indices ($ReZG_1(G)$, $ReZG_2(G)$, $ReZG_3(G)$), sum-connectivity Zagreb index ($SCI(G)$), hyper Zagreb index ($HM(G)$), harmonic index ($H(G)$), arithmetic-geometric index ($AGI(G)$), SK index ($SK(G)$) and SK_1 index ($SK_1(G)$). The above said indices are calculated for aromatic compounds such as boron triangular sheet, borophene chain and melem chain nanostructures. The obtained results are useful in understanding the behaviour of compounds under the study.

Keywords: Topological indices, $ReZ_{(a, b)}$ -index, boron triangular sheet, borophene chain, melem chain

Introduction

Often mathematics and computer science are used in the applications to research development. One example of such applications is cheminformatics, a relatively recent area of research in mathematics. Among others, analyzing the structure of a molecule of a substance can be related to the tasks of cheminformatics.

There are many organic and inorganic chemical compounds, such that studying properties and structure is very important in industrial, pharmaceutical and laboratory chemistry (Farahani *et al.*, 2017). The study of these properties and structure seems promising to search and evaluate the use of a the structure (Mohajeri *et al.*, 2011). The

¹ Department of Mathematics, Bapuji Institute of Engineering and Technology, Davanagere-577004, Karnataka, India. E-mail: mcshanmukha@gmail.com

² Department of Computer Science and Engineering, Bapuji Institute of Engineering and Technology, Davanagere-577004, Karnataka, India. E-mail: shilpastjit21@gmail.com

³ Department of Mathematics, Alliance School of Applied Mathematics, Alliance University, Bangalore-562106, Karnataka, India. E-mail: usha.arcot@alliance.edu.in

⁴ Department of Mathematics, Bapuji Institute of Engineering and Technology, Davanagere-577004, Karnataka, India. E-mail: kn.anil5340@gmail.com

* Corresponding author

study of the structure of a molecule gives information about its chemical properties.

Graph theory has been a very useful branch of science, especially in the applications of chemistry. The topological index is a very powerful tool that provides information of chemical compounds. These topological indices are categorized based on the degree, distance and eccentricity. Most of these indices correlate with octane isomers properties, so that the compounds can be used for various purposes based on their correlation coefficient (Randic, 1975; Anil *et al.*, 2020; Shanmukha *et al.*, 2020a). Graph theory facilitates a mathematical model of a compound to draw insight into its physico-chemical properties. The properties of the compound are correlated with the properties of isomers. In modeling a compound into a graph, the hydrogen atoms are neglected without losing information of the molecule, as carbon atoms have four chemical bonds whereas hydrogen atom has one chemical bond. The recent trend in research has attracted study in information science. It provides a good correlation between the biological and structural properties of compounds. Scientists are working remarkably as the graph-theoretical techniques have led to the wide application in graph theory (Biggs *et al.*, 1986; Furtula *et al.*, 2010; Ghayasuddin *et al.*, 2019). Graph theory is not only extensively used in chemistry but also in various fields of science viz., pharmacy, aerospace, computer science etc. Graph theory is an ideal tool in chemists hands, involving representation, synthesis of compounds and numerous chemical activities. Also, chemists are always interested in breaking and making chemical bonds that result in different structures.

Graph theory is a mathematical representation of chemical structures with vertices and edges. Chemical graph theory is a study of chemical compounds using a simple graph, multiple edges, and finite graphs. Graph G consists of vertices (V) and edges (E) such that, vertices depicts atoms and the edges depicts links connecting the vertices. The chemical graph theory provides the study of characteristics of chemical compounds (Gutman and Polansky, 1986; Trinajstić, 1992; Zafar *et al.*, 2018; Ali *et al.*, 2019; Ali *et al.*, 2020; Huo *et al.*, 2020; Shanmukha *et al.*, 2020b; Siddiqui *et al.*, 2020; Shanmukha *et al.*, 2021).

The topological indices are the numerical descriptors that are used to determine the characteristics of chemical compounds. The mathematical chemistry provides several topological indices that are widely used in Quantitative Structure Activity Relationship / Quantitative Structure Property Relationship (QSAR/QSPR) studies (Shirdel *et al.*, 2013; Shanmukha *et al.*, 2020c).

Here finite, simple and connected graphs are considered. Let $G = (V, E)$ be a graph, where $V(G)$ and $E(G)$ are vertices and edges respectively. The degree of a vertex x is the total number of vertices adjacent to x and represented by ϑ_x .

(Akhter and Imran 2016; Aslam *et al.*, 2017) discussed some topological indices for chemical structures such as triangular boron nanotubes and benzenoid structures. (Ali *et al.*, 2017) recently obtained some topological invariants on boron triangular sheet, borophene chain and melem chain. Motivated by the earlier work on Zagreb indices, a generalized redefined Zagreb index (ReZ_(a, b)) is defined. First, second and third redefined Zagreb indices, sum connectivity index, hyper Zagreb index, harmonic index, arithmetic geometric index, SK index, and SK₁ index are computed using ReZ_(a, b) for nanostructures like boron triangular sheet, borophene chain and melem chain.

Materials and Methods

Definition 2.1.

The 1st, 2nd and 3rd redefined Zagreb indices proposed by (Ranjini *et al.*, 2013) are stated as follows:

$$\text{ReZG}_1(G) = \sum_{xy \in E(G)} \frac{\vartheta_x + \vartheta_y}{\vartheta_x \times \vartheta_y}$$

$$\text{ReZG}_2(G) = \sum_{xy \in E(G)} \frac{\vartheta_x \times \vartheta_y}{\vartheta_x + \vartheta_y}$$

$$\text{ReZG}_3(G) = \sum_{xy \in E(G)} (\vartheta_x + \vartheta_y) (\vartheta_x \times \vartheta_y)$$

Definition 2.2.

The sum connectivity index was introduced by (Zhou and Trinajstić, 2009) and is described as

$$\text{SCI}(G) = \sum_{xy \in E(G)} \frac{1}{\sqrt{\vartheta_x + \vartheta_y}}$$

Definition 2.3.

The hyper Zagreb index was proposed by (Shirdel *et al.*, 2013) and is described as

$$\text{HM}(G) = \sum_{xy \in E(G)} [\vartheta_x + \vartheta_y]^2$$

Definition 2.4.

Fajtlowicz (Fajtlowicz, 1988) proposed the harmonic index and is stated as

$$H(G) = \sum_{xy \in E(G)} \frac{2}{\vartheta_x + \vartheta_y}$$

Definition 2.5.

The arithmetic geometric index is introduced by (Shegehalli and Kanabur 2015) and is described as

$$AGI(G) = \frac{1}{2} \sum_{xy \in E(G)} \frac{\vartheta_x + \vartheta_y}{\sqrt{\vartheta_x \times \vartheta_y}}$$

Definition 2.6.

The SK index is introduced by (Shigehalli and Kanabur, 2016) and is described as

$$SK(G) = \frac{1}{2} \sum_{xy \in E(G)} \vartheta_x + \vartheta_y$$

Definition 2.7.

The SK₁ index is introduced by (Shigehalli and Kanabur, 2016) and is described as

$$SK_1(G) = \frac{1}{2} \sum_{xy \in E(G)} \vartheta_x \times \vartheta_y$$

Motivated by the work on Zagreb indices (Furtula *et al.*, 2014; Cangul *et al.*, 2017; Gutman *et al.*, 2020; Nilanjan, 2021) a degree based generalized redefined Zagreb index $ReZ_{(a,b)}$ is defined and it is stated as

$$ReZ_{(a,b)}(G) = \sum_{xy \in E(G)} [\vartheta_x + \vartheta_y]^a \times [\vartheta_x \times \vartheta_y]^b$$

where a and b are real numbers.

Below, Table 1 depicts the relationship of

$ReZ_{(a,b)}$ - index with other topological indices defined.

Table 1. Relationship between $ReZ_{(a,b)}$ -index and other topological indices

Topological index	Corresponding $ReZ_{(a,b)}$ - index
First redefined Zagreb index	$ReZ_{(1,-1)}(G)$
Second redefined Zagreb index	$ReZ_{(-1,1)}(G)$
Third redefined Zagreb index	$ReZ_{(1,1)}(G)$
Sum-connectivity index	$ReZ_{(-\frac{1}{2},0)}(G)$
Hyper Zagreb index	$ReZ_{(2,0)}(G)$
Harmonic index	$2ReZ_{(-1,0)}(G)$
Arithmetic - Geometric index	$\frac{1}{2}ReZ_{(1,-\frac{1}{2})}(G)$
SK index	$\frac{1}{2}ReZ_{(1,0)}(G)$
SK ₁ index	$\frac{1}{2}ReZ_{(0,1)}(G)$

The results consist of degree-based topological indices of three nanostructures. First, the generalized redefined Zagreb index is defined and then important degree based topological indices are computed by substituting some specific values to the parameters a and b . To calculate the results, methods such as combinatorial computing, vertex and edge partition, degree counting and graph-theoretical tools are used.

Results and Discussion

Boron triangular sheet (BTS (m, n)).

In a hexagonal carbon sheet, an atom is placed in each hexagon's center, the surrounding atoms being linked to the central atom. Similarly, this procedure is extended to obtain the boron triangular sheet. These are becoming increasingly important because of their structure stability, transport characteristics and electronic structure. These have good conducting capability than carbon hexagonal nanotubes. In this part, BTS (m, n) is studied. For a clear understanding, BTS (4, 4) is illustrated as in Figure 1.

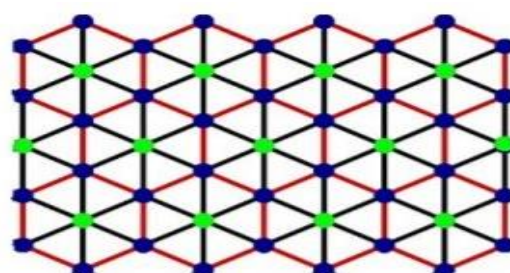


Figure 1. Boron triangular sheet (BTS (4,4))

To calculate the topological indices for the generalized redefined Zagreb index, the number of vertices, edges and edges types are found for the structure having m rows and n columns. The BTS structure consists of $m + n + 4$, $m + n - 2$, $m + n - 2$ and $2mn - m - n + 1$ vertices with degree equal to the values 3, 4, 5 and 6 respectively.

The BTS (m, n) edge set is classified into 8 partitions depending on the edges types. The detailed data about the types and the cardinality of edges is illustrated in Table 2.

Theorem 3.1.

The $ReZ_{(a,b)}$ -index of Boron triangular sheet BTS (m, n) for $m = n \geq 3$ is given by

$$\text{ReZ}_{(a,b)}(\text{BTS}(m,n)) = 4[(6)^a \times (9)^b] + 4[(7)^a \times (12)^b] + 2(m+n-2)[(8)^a \times (15)^b] + (m+n+4)[(9)^a \times (18)^b] + (m+n-4)[(8)^a \times (16)^b] + 2(m+n-2)[(10)^a \times (24)^b] + 3(m+n-2)[(11)^a \times (30)^b] + 6(mn - (m+n) + 1)[(12)^a \times (36)^b].$$

Proof. Table 2 gives the tabulated data of the number and types of edges for the structure BTS(m, n). Consider all the types of edges and their count in the definition of the generalized redefined Zagreb index, the result is proved.

$$\begin{aligned} \text{ReZ}_{(a,b)}(G) &= \sum_{xy \in E(\text{BTS}(m,n))} [\vartheta_x + \vartheta_y]^a \\ &\quad \times [\vartheta_x \times \vartheta_y]^b \\ &= \sum_{xy \in E_1(\text{BTS}(m,n))} [3+3]^a \times [3 \times 3]^b + \sum_{xy \in E_2(\text{BTS}(m,n))} [3+4]^a \times [3 \times 4]^b \\ &\quad + \sum_{xy \in E_3(\text{BTS}(m,n))} [3+5]^a \times [3 \times 5]^b + \sum_{xy \in E_4(\text{BTS}(m,n))} [3+6]^a \times [3 \times 6]^b \\ &\quad + \sum_{xy \in E_5(\text{BTS}(m,n))} [4+4]^a \times [4 \times 4]^b + \sum_{xy \in E_6(\text{BTS}(m,n))} [4+6]^a \times [4 \times 6]^b \\ &\quad + \sum_{xy \in E_7(\text{BTS}(m,n))} [5+6]^a \times [5 \times 6]^b + \sum_{xy \in E_8(\text{BTS}(m,n))} [6+6]^a \times [6 \times 6]^b \end{aligned}$$

$$\text{ReZ}_{(a,b)}(\text{BTS}(m,n)) = 4[(6)^a \times (9)^b] + 4[(7)^a \times (12)^b] + 2(m+n-2)[(8)^a \times (15)^b] + (m+n+4)[(9)^a \times (18)^b] + (m+n-4)[(8)^a \times (16)^b] + 2(m+n-2)[(10)^a \times (24)^b] + 3(m+n-2)[(11)^a \times (30)^b] + 6(mn - (m+n) + 1)[(12)^a \times (36)^b].$$

Using the generalized redefined Zagreb index, the other topological indices are calculated as shown below. For a and b of the generalized redefined Zagreb index, values are given as per Table 1. For each set of a and b values, different topological indices are calculated using Theorem 3.1.

Corollary 3.1.1. Let $G \cong \text{BTS}(m,n)$ be the triangular boron sheet, for $m = n \geq 3$, then

$$\begin{aligned} \text{a. } \text{ReZ}_1(G) &= \text{ReZ}_{(1,-1)}(\text{BTS}(m,n)) = 2mn + 2m + 2n + 1, \\ \text{b. } \text{ReZ}_2(G) &= \text{ReZ}_{(-1,1)}(\text{BTS}(m,n)) = 18mn + \frac{683}{250}m + \frac{683}{250}n - \frac{1303}{500}, \\ \text{c. } \text{ReZ}_3(G) &= \text{ReZ}_{(1,1)}(\text{BTS}(m,n)) = 2592mn - 592m - 592n - 140, \\ \text{d. } \text{SCI}(G) &= \text{ReZ}_{(-\frac{1}{2},0)}(\text{BTS}(m,n)) = \frac{433}{250}mn + \frac{599}{500}m + \frac{599}{500}n + \frac{77}{250}, \\ \text{e. } \text{HM}(G) &= \text{ReZ}_{(2,0)}(\text{BTS}(m,n)) = 864mn - 28m - 28n - 110, \end{aligned}$$

$$\begin{aligned} \text{f. } \text{H}(G) &= 2\text{ReZ}_{(-1,0)}(\text{BTS}(m,n)) = mn + \frac{459}{500}m + \frac{459}{500}n + \frac{237}{500}, \\ \text{g. } \text{AGI}(G) &= \frac{1}{2}\text{ReZ}_{(1,-\frac{1}{2})}(\text{BTS}(m,n)) = 6mn + \frac{159}{50}m + \frac{159}{50}n + \frac{9}{200}, \\ \text{h. } \text{SK}(G) &= \frac{1}{2}\text{ReZ}_{(1,0)}(\text{BTS}(m,n)) = 36mn + 7m + 7n - 5, \\ \text{i. } \text{SK}_1(G) &= \frac{1}{2}\text{ReZ}_{(0,1)}(\text{BTS}(m,n)) = 108mn - 7m - 7n - 14. \end{aligned}$$

Table 2. The edge partition of boron triangular sheet BTS(m, n)

$(\vartheta_x, \vartheta_y)$, where $xy \in E(G)$	No. of Edges
$E_1(3, 3)$	4
$E_2(3, 4)$	4
$E_3(3, 5)$	$2(m+n-2)$
$E_4(3, 6)$	$m+n+4$
$E_5(4, 4)$	$m+n-4$
$E_6(4, 6)$	$2(m+n-2)$
$E_7(5, 6)$	$3(m+n-2)$
$E_8(6, 6)$	$6(mn - (m+n) + 1)$

3.2. Borophene chain (B₃₆(n)).

Borophene is a linear arrangement of boron denoted by B₃₆(n). The arrangement of this chain has been illustrated for $n = 3$ in Figure 2 for better understanding. These are stronger than graphene and the configuration is flexible. Borophenes have a wide range of applications in batteries, storage of hydrogen and reduction of oxygen. To compute the generalized redefined Zagreb index, the linear arrangement of borophene is considered for $n \geq 2$. The B₃₆(n) consists of $2n + 4$, $8n + 4$, $8n - 2$ and $14n - 2$ vertices for degree 3, 4, 5 and 6 respectively. The complete information regarding the number of edges and types of edges and their cardinality is tabulated in Table 3. The edge set of B₃₆(n) is classified into eight partitions depending on the degree of end vertices.

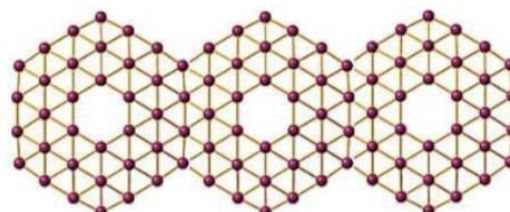


Figure 2. Borophene chain (B₃₆(n)(3))

Table 3. The edge partition of borophene chain $B_{36}(n)$

$(\vartheta_x, \vartheta_y)$, where $xy \in E(G)$	No. of Edges
$E_1(3, 4)$	$4n + 8$
$E_2(3, 6)$	$2n + 4$
$E_3(4, 4)$	$4n + 2$
$E_4(4, 5)$	$4n - 4$
$E_5(4, 6)$	$16n + 8$
$E_6(5, 5)$	$6n$
$E_7(5, 6)$	$24n - 6$
$E_8(6, 6)$	$21n - 9$

Theorem 3.2.

The $\text{ReZ}_{(a,b)}$ -index of borophene chain $B_{36}(n)$ for $n \geq 2$ is given by

$$\text{ReZ}_{(a,b)} B_{36}(n) = (4n + 8) [(7)^a \times (12)^b] + (2n + 4) [(9)^a \times (18)^b] + (4n + 2) [(8)^a \times (16)^b] + (4n - 4) [(9)^a \times (20)^b] + (16n + 8) [(10)^a \times (24)^b] + (6n) [(10)^a \times (25)^b] + (24n - 6) [(11)^a \times (30)^b] + (21n - 9) [(12)^a \times (36)^b].$$

Proof. Consider the number and types of edges for the borophene chain from Table 3 in the definition of the generalized redefined Zagreb index, the result of the theorem obtained is as follows:

$$\begin{aligned} \text{ReZ}_{(a,b)}(G) &= \sum_{xy \in E(BTS(m, n))} [\vartheta_x + \vartheta_y]^a \\ &\quad \times [\vartheta_x \times \vartheta_y]^b \\ &= \sum_{xy \in E_1(BTS(m, n))} [3 + 3]^a \times [3 \times 3]^b + \sum_{xy \in E_2(BTS(m, n))} [3 + 4]^a \times [3 \times 4]^b \\ &\quad + \sum_{xy \in E_3(BTS(m, n))} [3 + 5]^a \times [3 \times 5]^b + \sum_{xy \in E_4(BTS(m, n))} [3 + 6]^a \times [3 \times 6]^b \\ &\quad + \sum_{xy \in E_5(B_{36}(n))} [4 + 4]^a \times [4 \times 4]^b + \sum_{xy \in E_6(BTS(m, n))} [4 + 6]^a \times [4 \times 6]^b \\ &\quad + \sum_{xy \in E_7(BTS(m, n))} [5 + 6]^a \times [5 \times 6]^b + \sum_{xy \in E_8(BTS(m, n))} [6 + 6]^a \times [6 \times 6]^b \end{aligned}$$

$$\text{ReZ}_{(a,b)} B_{36}(n) = (4n + 8) [(7)^a \times (12)^b] + (2n + 4) [(9)^a \times (18)^b] + (4n + 2) [(8)^a \times (16)^b] + (4n - 4) [(9)^a \times (20)^b] + (16n + 8) [(10)^a \times (24)^b] + (6n) [(10)^a \times (25)^b] + (24n - 6) [(11)^a \times (30)^b] + (21n - 9) [(12)^a \times (36)^b].$$

For a and b of the generalized redefined Zagreb index, values are given in Table 1. For each set of a and b values, different topological indices are calculated using Theorem 3.2.

Corollary 3.2.1. Let $G \cong B_{36}(n)$ be the borophene chain for $n \geq 2$ then

- $\text{ReZG}_1(G) = \text{ReZ}_{(1, -1)}(B_{36}(n)) = 32n + 4,$
- $\text{ReZG}_2(G) = \text{ReZ}_{(-1, 1)}(B_{36}(n)) = \frac{1048}{5}n - \frac{3669}{500},$
- $\text{ReZG}_3(G) = \text{ReZ}_{(1, 1)}(B_{36}(n)) = 24224n - 3092,$
- $\text{SCI}(G) = \text{ReZ}_{(\frac{-1}{2}, 0)}(B_{36}(n)) = \frac{12591}{500}n + \frac{927}{500},$
- $\text{HM}(G) = \text{ReZ}_{(2, 0)}(B_{36}(n)) = 9066n - 702,$
- $H(G) = 2\text{ReZ}_{(-1, 0)}(B_{36}(n)) = \frac{787}{150}n + \frac{9}{5},$
- $\text{AGI}(G) = \frac{1}{2}\text{ReZ}_{(1, \frac{-1}{2})}(B_{36}(n)) = \frac{4081}{50}n + \frac{17}{5},$
- $\text{SK}(G) = \frac{1}{2}\text{ReZ}_{(1, 0)}(B_{36}(n)) = 425n - 11,$
- $\text{SK}_1(G) = \frac{1}{2}\text{ReZ}_{(0, 1)}(B_{36}(n)) = 1119n - 96.$

3.3. Melem chain $MC(n)$.

Melem is a white crystalline solid. It is prepared by the decomposition of carbon, nitrogen, and hydrogen atoms. It has wide applications in the preparation of resins and fireproof materials. Mathematically, these crystals are linearly arranged n times to get the Melem chain. The structure of $MC(4)$ for $n = 4$ is represented as shown in Figure 3.

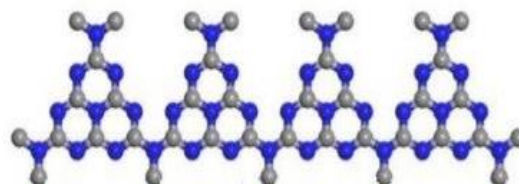


Figure 3. Melem chain (MC (4))

Consider the melem (2,5,8-triamino-tri-s-triazine), $C_6N_7(NH_2)_3$ nanotube chain. The vertices and edges of $MC(n)$ are $18n+4$ vertices and $21n+3$ edges. It has $3n + 3$, $6n$ and $9n + 1$ vertices with degrees 1, 2 and 3 respectively. Depending on the degree of end vertices, the $MC(n)$ edge set is classified into 3 partitions. The edge partition of the melem chain is shown in Table 4

Theorem 3.3.

The $\text{ReZ}_{(a,b)}$ -index of Melem chain $MC(n)$ for $n \in \mathbb{N}$ is given by

$$\text{ReZ}_{(a,b)} MC(n) = (3n + 3)[(4)^a \times (3)^b] + (12n)[(5)^a \times (6)^b] + (6n)[(6)^a \times (9)^b].$$

Table 4. The edge partition of Melem chain MC(n)

$(\vartheta_x, \vartheta_y)$, where $xy \in E(G)$	No. of Edges
$E_1(1, 3)$	$3n + 3$
$E_2(2, 3)$	$12n$
$E_3(3, 3)$	$6n$

Proof. Consider the number and types of edges for the melem chain from Table 4 in the definition of the generalized redefined Zagreb index, the result of the theorem is obtained.

$$\begin{aligned}
 \text{ReZ}_{(a,b)}(G) &= \sum_{xy \in E(\text{MC}(n))} [\vartheta_x + \vartheta_y]^a \\
 &\quad \times [\vartheta_x \times \vartheta_y]^b \\
 &= \sum_{xy \in E_1(\text{MC}(n))} (1+3)^a \times (1 \times 3)^b + \\
 &\quad \sum_{xy \in E_2(\text{MC}(n))} (2+3)^a \times (2 \times 3)^b + \\
 &\quad \sum_{xy \in E_3(\text{MC}(n))} (3+3)^a \times (3 \times 3)^b \\
 \text{ReZ}_{(a,b)}\text{MC}(n) &= (3n+3)[(4)^a \times (3)^b] + (12n)[(5)^a \times (6)^b] + (6n)[(6)^a \times (9)^b].
 \end{aligned}$$

For a and b of the generalized redefined Zagreb index, values are given in Table 1. For each set of a and b values, different topological indices are calculated using Theorem 3.3.

Corollary 3.3.1. Let $G \cong \text{MC}(n)$ be the Melem chain, for $n \in \mathbb{N}$, then

- $\text{ReZG}_1(G) = \text{ReZ}_{(1, -1)}(\text{MC}(n)) = 18n + 4,$
- $\text{ReZG}_2(G) = \text{ReZ}_{(-1, 1)}(\text{MC}(n)) = \frac{513}{20}n + \frac{45}{20},$
- $\text{ReZG}_3(G) = \text{ReZ}_{(1, 1)}(\text{MC}(n)) = 720n + 36,$
- $\text{SCI}(G) = \text{ReZ}_{(-\frac{1}{2}, 0)}(\text{MC}(n)) = \frac{233}{25}n + \frac{3}{2},$
- $\text{HM}(G) = \text{ReZ}_{(2, 0)}(\text{MC}(n)) = 564n + 48,$
- $\text{H}(G) = 2\text{ReZ}_{(-1, 0)}(\text{MC}(n)) = \frac{83}{10}n + \frac{3}{2},$
- $\text{AGI}(G) = \frac{1}{2}\text{ReZ}_{(1, -\frac{1}{2})}(\text{MC}(n)) = \frac{217}{10}n + \frac{35}{10},$
- $\text{SK}(G) = \frac{1}{2}\text{ReZ}_{(1, 0)}(\text{MC}(n)) = 54n + 6,$
- $\text{SK}_1(G) = \frac{1}{2}\text{ReZ}_{(0, 1)}(\text{MC}(n)) = \frac{135}{2}n + \frac{9}{2}.$

Conclusion

The work focuses on the nanostructures viz., boron triangular sheet, borophene chain and melem chain. The generalized redefined Zagreb index is defined and is calculated for the above-said structures. From the generalized redefined Zagreb index, some topological indices are derived for different values

of a and b of $\text{ReZ}_{(a, b)}$. As the above discussed chemical structures are application-based compounds, the topological indices help the chemists to do research on these compounds in the future.

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