

A NEW SPHERICALLY SYMMETRIC NEUTRON STAR MODEL EMBEDDING CLASS I IN DURGAPAL METRIC

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Received: July 19, 2023; Revised: March 23, 2024; Accepted: April 09, 2024

Abstract

In this paper, we develop a novel spherically symmetric neutron star model by combining the two useful mathematical aspects of astrophysics, Karmakar's condition and Durgapal metric for anisotropic stellar objects. We use Karmakar's condition to embed a class I reducibility criteria in the model. The solutions are simplified by considering $e^v = B(1+x)^6$, where $x = Cr^2$, beyond the fifth model given by Durgapal. The model does not have the central singularity. The model well behaves for the realistic physical properties of the neutron stars. The energy density, the pressure in the radial direction, and the pressure in the tangential direction are positively finite and decrease from the center to the surface of the neutron stars. The redshift and the compression moduli are also positive finite. In the offered model, we take the values of the constants as $1.5760 \leq A \leq 2.295$, $0.0602 \leq B \leq 0.5577$, and $7.492 \times 10^{-10} \leq C \leq 3.397 \times 10^{-9}$ for the neutron stars EXO 1785-248, SMC X-1, Her X-1, 4U 1538-52, LMC X-4, and Cen X-3. The Buchdahl condition is well satisfied by the model.

Keywords: Anisotropy; Class I Reducibility; Durgapal Metric; Neutron Star; Spherical Symmetry

Introduction

The study of general relativity started with Einstein's hypothesis of curved spacetime. Einstein proposed that massive bodies curve the spacetime around them, which causes the formation of a gravitational field due to those bodies. In theoretical physics, these fields are studied through a mathematical model known as field equations. The field equations are the geometrical representation of the curve spacetime due to the mass of a body. The massive bodies curve the spacetime around them significantly. Neutron stars, white dwarf stars, quark stars, and strange stars are some massive stars in the universe having mass as

a real number multiple of the mass of the Sun. In order to study such astrophysical objects, the researchers try to obtain the exact solutions of Einstein's field equations by considering the different conditions and parameters. Schwarzschild gave the first solution in 1916 (Schwarzschild, 1916) just after the field equations produced by Einstein (Einstein, 1915). The solution given by Schwarzschild was an early attempt to discover the exact solutions of the field equations and was appreciated by Einstein. But, the solution given by Schwarzschild was having limitations of singularity. Nevertheless, it was a fruitful attempt and

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DOI: <https://doi.org/10.55766/sujst-2024-02-e01814>

Suranaree J. Sci. Technol. 31(2):030180(1-14)

acceptable for all at the time. Tolman (1939) took the solutions of field equations to another level and proposed eight solutions of the field equations. All these eight solutions are not suitable for studying the inner assembly of the super dense astral matters. But, the fourth and the seventh solutions are well known for relating the inner assembly and physical properties of colossal compressed astral matters.

Researchers have been continuously trying to develop useful exact solutions to the field equations given by Einstein to study general relativity in the different perspectives of stellar objects in a Riemannian metric of four dimensions. In contemporary research of theoretical physics in general relativity and extended general relativity, neutron stars are widely studied (Mak and Harko, 2003; Astashenok *et al.*, 2013; Pili *et al.*, 2017; Saha and Debnath, 2019; Thirukkanesh *et al.*, 2019; Estevez-Delgado *et al.*, 2020a; Estevez-Delgado *et al.*, 2021).

Karmakar suggested the study of complex curved spacetime of lower dimension can be simplified by embedding it in the flat spacetime of higher dimension (Karmakar, 1948). Karmakar's condition gained popularity among researchers due to its simplified curved spacetime nature. In the very recent past Saklany (Saklany *et al.*, 2023) studied the dense stellar with united dark energy in Karmakar-connected relativistic spacetime. Khan (Khan *et al.*, 2022) used Karmakar's condition, generalized polytropes, and complexity factor to study compressed objects. Parida *et al.* (2022) attained a novel class of results for a relativistic anisotropic dense star by exploiting the Karmakar's reducibility criterion. Pant *et al.* (2022) discovered novel results for Einstein-Maxwell field equations in four dimensional spacetime taking charged form of anisotropic stellar matters inserting Karmakar's method and the gravitational decoupling through minimal geometric decoupling. Mathias *et al.* (2021) generated exact solutions to Einstein-Maxwell field equations by analyzing the embedding condition. Abbas *et al.* (2021) investigated anisotropic stellar dense spheres in $f(R)$ gravity by using Karmakar's method. Maurya and Nag (2021) presented an anisotropic results of Einstein's field equations through the minimally distorted class I generator. Prasad *et al.* (2021) also investigated relativistic model for anisotropic spheres in $f(R, T)$ gravity by using Karmakar's method. Mustafa *et al.* (2021) obtained wormhole solutions from the Karmakar conditions in $f(Q)$ formalism. Ramos *et al.* (2021) studied class I interior solution supported by anisotropic polytropes. Das *et al.* (2020) used Vaidya and Tikekar's spheroidal model to develop a dense astral model inserting class I Karmakar's condition.

Hansraj and Moodly (2020) proposed stellar modelling of isotropic Einstein-Maxwell perfect fluid spheres by inserting class I. Gomez-Leyton *et al.* (2020) developed a well-recognized systematic results of the Einstein-Maxwell field equations using the class I embedding conditions. Bhar (2019) obtained an anisotropic compressed star model via inserting class I reducibility. Prasad *et al.* (2019) obtained a relativistic model for anisotropic compressed stars using Karmakar condition. Singh *et al.* (2016) proposed newfangled results by inserting class I to represent an anisotropic fluid sphere in general relativity, Bhar and Govender (2017) presented a model of a compressed relativistic star in the presence of an electric charge embedding Karmakar's class I reducibility criterion. Maurya and Maharaj (2017) discussed a generalized model of a compressed star as an alternative of Karmakar's class I spacetime. Bhar *et al.* (2016) created a new relativistic anisotropic dense stellar model with a spherically symmetric embedding class I metric. Bhar and Govender (2017) generated a model of anisotropic dense stars satisfying Karmakar's reducibility criterion. Singh *et al.* (2016) obtained a new static and spherically symmetric model by applying Karmakar's condition.

Durgapal-type solution is a popular method to simplify the solutions of Einstein's field equations. Landeta (2022) constructed astral models on the complexity factor for static and spherically symmetric self-gravitating systems as a supplementary state for approaching to the system of equations consequent from gravitational decoupling by using Durgapal IV and other solutions as the core solutions. Santana *et al.* (2022) projected a new method to join the Lane-Emden equations for relativistic anisotropic polytropes by investigating the solutions originating from Durgapal IV and others. Contreras *et al.* (2022) used the vanishing complexity factor to formulate uncharged and charged symmetric to Durgapal models. Estevez-Delgado *et al.* (2020b) presented a dense star model produced by quintessence and ordinary matter using Durgapal metric. Zubair *et al.* (2020) used Durgapal metric potential to investigate the charged static spherically symmetric models taking $f(R)$ theory of gravity. Tello-Ortiz *et al.* (2020) reported the local anisotropic effects on Durgapal's fourth model in the framework of gravitational decoupling thru the minimal geometric deformation method. Islam *et al.* (2019) presented a new model for strange stars constructed thru Durgapal IV metric. Morales and Tello-Ortiz (2018) extended Durgapal's fifth isotropic solution into an anisotropic scenario. Nasim and Azam (2018) proposed physical models for anisotropic charged

matters using Durgapal metric in the generalized polytropic equation of state. Mehta *et al.* (2013) presented a well-behaved class of charged analogous of Durgapal solution. Fuloria and Tewari (2012) obtained a charged analogous of Durgapal solution. Mehta *et al.* (2013) presented a charged analogous of Durgapal solutions. Murad and Fatema (2013) presented a novel class of interior solutions of Einstein-Maxwell field equations analogous to Durgapal's exact solution in general relativity for a perfect fluid. Maurya and Gupta (2011) attained a class of models for charged superdense star models in spherically symmetric curvature coordinates by taking Durgapal type metric.

In this paper, we have applied Karmakar embedding condition in Durgapal metric to attain a novel model for the anisotropic neutron stars. We took $n = 6$ in Durgapal Model to achieve more accurate physical characteristics of the neutron stars (Durgapal, 1982) and embedded class I reducibility criterion to obtain a simplified form of the model. The entire work of this paper is distributed into six sections. In the second section, we have taken Durgapal metric to formulate the field equations. In the third section, we are embedding Karmakar condition to obtain a mathematical model. In the fourth section, we discuss the physical acceptability conditions of the model. In the fifth section, we have discussed the physical properties of the model. In the sixth section, we have discussed the matching condition at the boundary.

Field Equations and Durgapal Metric

Let us take a line element with the spherically symmetric metric in curvature coordinate as follows:

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2, \quad (1)$$

where, ν and λ are the functions of r only.

The Einstein field equation can be given by

$$R_j^i - \frac{1}{2} R \delta_j^i = -\frac{8\pi G}{c^4} T_j^i, \quad (2)$$

where R_j^i is Ricci tensor, R is Ricci scalar and T_j^i is the energy-momentum tensor.

Generally, the matter inside a neutron star is of anisotropic nature. Therefore, the energy momentum tensor for such a distribution of matter can be given by

$$T_j^i = [(P_r + \rho c^2)v^i v_j - P_r \delta_j^i + (P_r - P_\tau)\chi_j \chi^i], \quad (3)$$

where P_r is radial pressure, P_τ is tangential pressure, ρ is the energy density, v^i is the velocity vector in radial direction, v_j is the velocity vector in tangential direction, χ^i is the unit space vector in radial direction, and χ_j is the unit space vector in tangential direction.

If we take the metric of the line element given by equation (1) and put it in equation (2) we obtain the following field equations:

$$\frac{8\pi G}{c^4} P_r = \frac{\nu'}{r} e^{-\lambda} + \frac{1}{r^2} e^{-\lambda} - \frac{1}{r^2}, \quad (4)$$

$$\frac{8\pi G}{c^4} P_\tau = e^{-\lambda} \left(\frac{\nu''}{2} - \frac{\lambda' \nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu'}{2r} - \frac{\lambda'}{2r} \right), \quad (5)$$

$$\frac{8\pi G}{c^2} \rho = \frac{\lambda'}{r} e^{-\lambda} - \frac{1}{r^2} e^{-\lambda} + \frac{1}{r^2}, \quad (6)$$

where the prime on each quantity represents the derivative of the quantity with respect to r that is with respect to the radius.

We proceed beyond the fifth model proposed by Durgapal (1982) and make the following assumptions:

$$e^\nu = B(1+x)^6, \quad x = Cr^2, \quad \text{and} \quad e^{-\lambda} = Y, \quad (7)$$

Where B and C are real constants.

The Conditions of Physical Appropriateness of the Solutions

The model obtained in Durgapal metric embedding Karmakar condition for the curvature coordinate can be considered well performing if the following conditions are satisfied by the solution:

1. The model is free from all the singularities i.e., geometrical and physical singularities. It must be free from infinite and negative values of the physical components at the center that is pressure density and the metric potentials, e^λ and e^ν .
2. The pressure and density obtained through the model should be decreasing as the radius of the object is increasing.
3. The red shift must be finite-positive and monotonically decreasing with respect to the radius of the object.
4. As the radial and tangential pressure is equal at the center of the object so the anisotropy factor must be zero at the center of the object and must be increasing as we move from inner core to the crust of the object.
5. Buchdahl condition is well satisfied by the model. (Buchdahl, 1959)

6. The velocity of sound in the radial (v_r) and the tangential (v_t) direction satisfies $0 \leq |v_t^2 - v_r^2| \leq 1$ for the stability.

Karmakar's Condition

Karmakar suggested that a purely gravitational field represented by Riemannian metric can be engrossed in a flat space of higher dimension. Therefore, a 4-dimensional spherically symmetric curved spacetime can be immersed into a 5-dimensional flat spacetime. The class I reducibility condition was proposed as:

$$R_{1414} = \frac{R_{1212}R_{3434} - R_{2142}R_{3143}}{R_{2323}}, \quad (8)$$

where $R_{2323} \neq 0$.

Now, we put the values of all the components in equation (8) from the line element taken in equation (1), we obtain

$$\frac{v''}{v'} + \frac{v'}{2} = \frac{\lambda' e^\lambda}{2(e^\lambda - 1)}. \quad (9)$$

Now, applying the assumptions for the metric potential taken in equation (7) into equation (9), we get:

$$\frac{dY}{dx} + \frac{(1+5x)}{x(1+x)} Y(1-Y) = 0. \quad (10)$$

This is simple differential equation which produces the following result

$$Y = e^{-\lambda} = \frac{1}{1+e^A x(1+x)^4}, \quad (11)$$

Where A is an arbitrary integral constant. With the help of equation (11) and using the equations (4), (5) and (6) we can calculate the pressures and the density as follows:

$$\frac{8\pi G}{Cc^4} P_r = \frac{1}{1+e^A x(1+x)^4} \left[\frac{12}{1+x} - e^A (1+x)^4 \right]. \quad (12)$$

$$\frac{8\pi G}{Cc^4} P_t = \frac{1}{(1+x)(1+e^A x(1+x)^4)} \left[\frac{12+36x}{1+x} - \frac{e^A (1+x)^3 (1+5x)(1+7x)}{1+e^A x(1+x)^4} \right]. \quad (13)$$

$$\frac{8\pi G}{Cc^2} \rho = \frac{e^A (1+x)^3}{1+e^A x(1+x)^4} \left[1 + \frac{2+11x}{1+e^A x(1+x)^4} \right]. \quad (14)$$

We calculate the radial and tangential pressure gradient as well as the density gradient by differentiating equations (12), (13) and (14) with respect to $x = Cr^2$.

$$\frac{8\pi G}{Cc^4} \frac{dP_r}{dx} = \frac{e^{2A}(1+x)^8}{(1+e^A x(1+x)^4)^2} - \frac{4e^A(1+x)^2(4+19x)}{(1+e^A x(1+x)^4)^2} - \frac{12}{(1+x)^2(1+e^A x(1+x)^4)^2}. \quad (15)$$

$$\frac{8\pi G}{Cc^4} \frac{dP_t}{dx} = \frac{12-36x}{(1+x)^3(1+e^A x(1+x)^4)^3} + \frac{2e^{2A}(1+x)^9(1+3x-3x^2)}{(1+x)^3(1+e^A x(1+x)^4)^3} - \frac{2e^A(1+x)^4(13+89x+196x^2)}{(1+x)^3(1+e^A x(1+x)^4)^3}. \quad (16)$$

$$\frac{8\pi G}{Cc^2} \frac{d\rho}{dx} = \frac{4e^A(1+x)^2(5+11x)}{(1+e^A x(1+x)^4)^3} - \frac{5e^A(1+x)^4}{(1+e^A x(1+x)^4)^3} - \frac{e^{2A}x(1+x)^8(1+2x)}{(1+e^A x(1+x)^4)^3} - \frac{6e^A x(1+x)^4(4+11x)}{(1+e^A x(1+x)^4)^3}. \quad (17)$$

The anisotropy factor can be given as:

$$\Delta = \frac{1}{1+e^A x(1+x)^4} \left[\frac{12+36x}{(1+x)^2} - \frac{12}{1+x} + e^A (1+x)^4 - \frac{e^A (1+x)^2 (1+5x)(1+7x)}{1+e^A x(1+x)^4} \right], \quad (18)$$

where $\Delta = \frac{8\pi G}{Cc^4} (P_t - P_r)$.

In Figure 1-6, we are showing the variation of metric potential for EXO 1785-248, SMC X-1, Her X-1, 4U 1538-52, LMC X-4, and Cen X-3. The variation shown in Figure 1-6 are analogous to the figure of metric potential shown by Tamta (Tamta and Fuloria, 2017) for a compact star candidate. Figure 7-12 are showing the variation in the energy density from the center to the surface. Figure 13-18 are showing the variation in the radial pressure and tangential pressure. The variation shown in Figure 13-18 are analogous to the figure shown by Maurya (Maurya *et al.*, 2018). It is clear from the figure that the radial pressure and tangential pressure are equal at the center and the largest difference is at the boundary. The radial pressure approaches to zero at the boundary. The change in anisotropy for EXO 1785-248, SMC X-1, Her X-1, 4U 1538-52, LMC X-4, and Cen X-3 is shown in Figure 19-24.

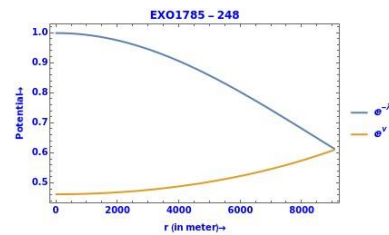


Figure 1. Plot of the variation of $e^{-\lambda}$ and e^v for EXO 1785-248, with respect to r , the radial coordinate

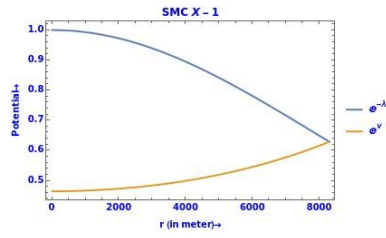


Figure 2. Plot of the variation of $e^{-\lambda}$ and e^{ν} for SMC X-1, with respect r , the radial coordinate

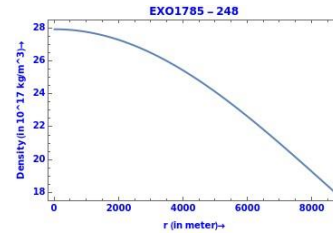


Figure 7. Plot of the variation of ρ for EXO 1785-248 with respect to r the radial coordinate

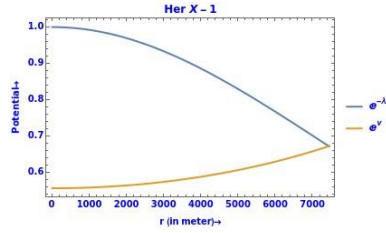


Figure 3. Plot of the variation of $e^{-\lambda}$ and e^{ν} for Her X-1, with respect r , the radial coordinate

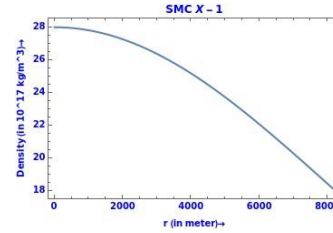


Figure 8. Plot of the variation of ρ for SMC X-1 with respect to r the radial coordinate

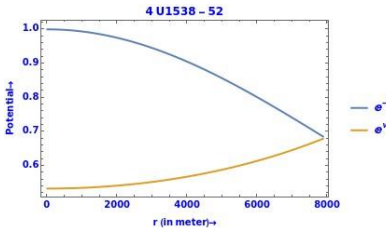


Figure 4. Plot of the variation of $e^{-\lambda}$ and e^{ν} for Her X-1, with respect r , the radial coordinate

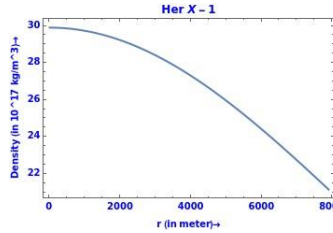


Figure 9. Plot of the variation of ρ for Her X-1 with respect to r the radial coordinate

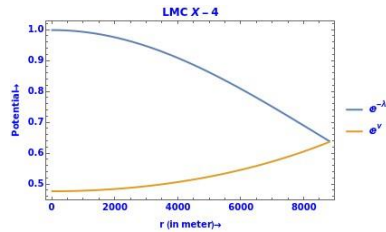


Figure 5. Plot of the variation of $e^{-\lambda}$ and e^{ν} for LMC X-4, with respect r , the radial coordinate

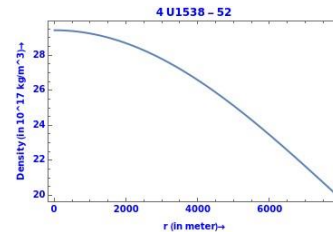


Figure 10. Plot of the variation of ρ for 4U 1538-52 with respect to r the radial coordinate

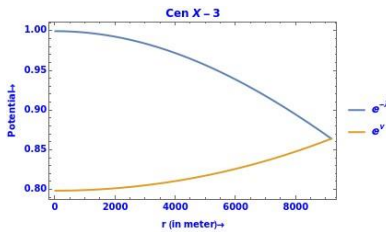


Figure 6. Plot of the variation of $e^{-\lambda}$ and e^{ν} for Cen X-3, with respect r , the radial coordinate.

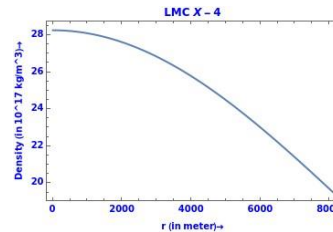


Figure 11. Plot of the variation of ρ for LMC X-4 with respect to r the radial coordinate

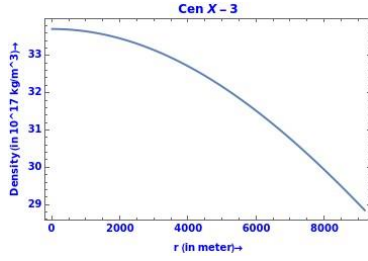


Figure 12. Plot of the variation of ρ for Cen X-3 with respect to r the radial coordinate

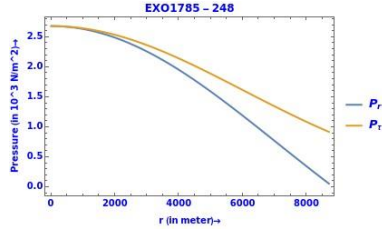


Figure 13. Plot of the variation of P_r and P_t for EXO 1785-248 with respect to r , the radial coordinate, taking $\frac{8\pi G}{Cc^4} = 1$

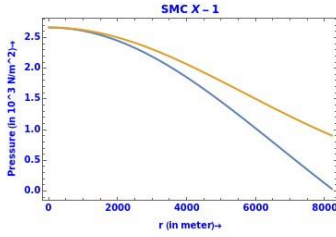


Figure 14. Plot of the variation of P_r and P_t for SMC X-1 with respect to r , the radial coordinate, taking $\frac{8\pi G}{Cc^4} = 1$

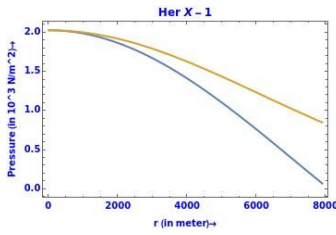


Figure 15. Plot of the variation of P_r and P_t for Her X-1 with respect to r , the radial coordinate, taking $\frac{8\pi G}{Cc^4} = 1$

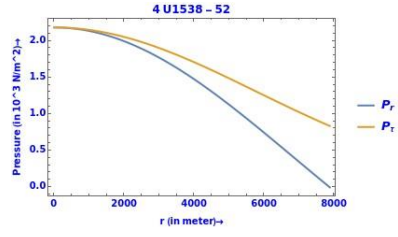


Figure 16. Plot of the variation of P_r and P_t for 4U 1538-52 with respect to r , the radial coordinate, taking $\frac{8\pi G}{Cc^4} = 1$

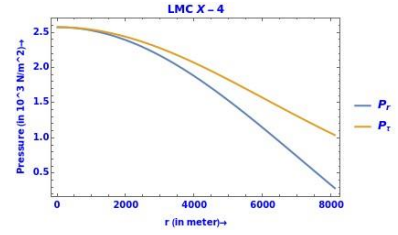


Figure 17. Plot of the variation of P_r and P_t for LMC X-4 with respect to r , the radial coordinate, taking $\frac{8\pi G}{Cc^4} = 1$

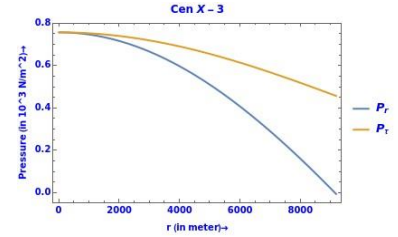


Figure 18. Plot of the variation of P_r and P_t for Cen X-3 with respect to r , the radial coordinate, taking $\frac{8\pi G}{Cc^4} = 1$

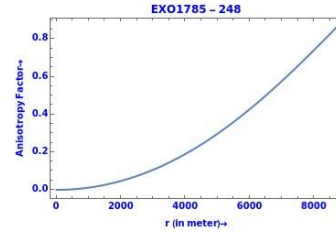


Figure 19. Plot of Anisotropy in EXO 1785-248 with respect to r , the radial coordinate, taking $\frac{8\pi G}{Cc^4} = 1$

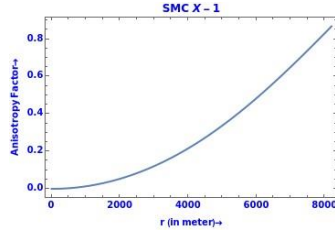


Figure 20. Plot of Anisotropy in SMC X-1 with respect to r , the radial coordinate, taking $\frac{8\pi G}{Cc^4} = 1$

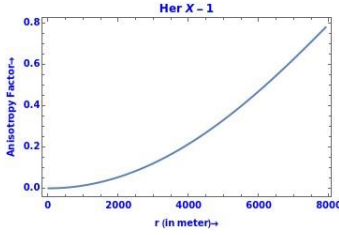


Figure 21. Plot of Anisotropy in Her X-1 with respect to r , the radial coordinate, taking $\frac{8\pi G}{Cc^4} = 1$

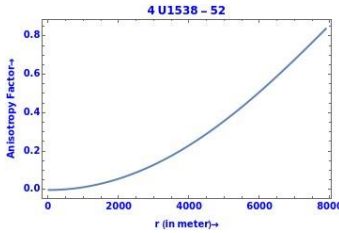


Figure 22. Plot of Anisotropy in 4U 1538-52 with respect to r , the radial coordinate, taking $\frac{8\pi G}{Cc^4} = 1$

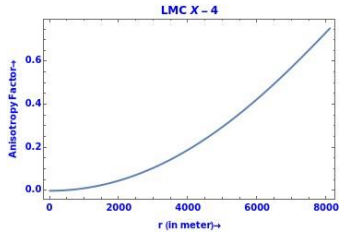


Figure 23. Plot of Anisotropy in LMC X-4 with respect to r , the radial coordinate, taking $\frac{8\pi G}{Cc^4} = 1$

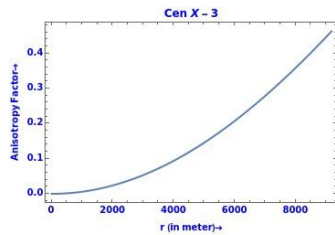


Figure 24. Plot of Anisotropy in Cen X-3 with respect to r , the radial coordinate, taking $\frac{8\pi G}{Cc^4} = 1$

Properties of the New Solution

The pressure and density at the center are non-zero and finite, which fits well into the acceptability condition of the solution. The radial and tangential pressure are equal at the center.

$$\frac{8\pi G}{Cc^4} P_r = \frac{8\pi G}{Cc^4} P_t = 12 - e^A. \quad (19)$$

$$\frac{8\pi G}{Cc^2} \rho = 3e^A. \quad (20)$$

Further, we obtain that P_r , P_t and ρ are decreasing from the center to the exterior of the neutron stars because

$$\left(\frac{8\pi G}{Cc^4} \frac{d^2 P_r}{dx^2} \right)_{r=0} < 0; \left(\frac{8\pi G}{Cc^4} \frac{d^2 P_t}{dx^2} \right)_{r=0} < 0; \quad (21)$$

$$\left(\frac{8\pi G}{Cc^4} \frac{d^2 \rho}{dx^2} \right)_{r=0} < 0.$$

The speed of sound in the radial and tangential direction can be calculated by

$$v_r = \sqrt{\frac{dP_r}{d\rho}}; v_t = \sqrt{\frac{dP_t}{d\rho}}, \quad (22)$$

$$\frac{1}{c^2} \frac{dP_r}{d\rho} = - \frac{e^{-A}(1 + e^A x(1+x)^4) \left(-12 + e^A(1+x)^4(e^A(1+x)^5 - 4(4+19x)) \right)}{(1+x)^4 \left(-4(5+11x) + e^A(1+x)^4(5 + e^A x(1+x)^4(1+2x) + 6x(4+11x)) \right)} \quad (23)$$

$$\frac{1}{c^2} \frac{dP_t}{d\rho} = \frac{2e^{-A}(-6 + 18x + e^A(1+x)^4) \left(13 + x(89 + 196x + e^A(1+x)^5(-1 + 3(-1+x)x)) \right)}{(1+x)^5 \left(-4(5+11x) + e^A(1+x)^4(5 + e^A x(1+x)^4(1+2x) + 6x(4+11x)) \right)} \quad (24)$$

From equation (23) and equation (24) $\frac{v_r^2}{c}$ and $\frac{v_t^2}{c}$ are less than 1. Therefore, the causality condition is satisfied. Similarly, we obtain $0 \leq |v_t^2 - v_r^2| \leq 1$. Therefore, the stability condition is satisfied. The redshift and adiabatic index are given by:

$$z = \frac{1}{\sqrt{B}(1+x)^3} - 1; \gamma = \left(\frac{P_r + \rho}{P_r} \right) \frac{dP_r}{d\rho}. \quad (25)$$

We observe from equation (25) the gravitational redshift is non-zero and it has finite positive value at the center. Figure 25-30 show the variation in the redshift for EXO 1785-248, SMC X-1, Her X-1, 4U 1538-52, LMC X-4, and Cen X-3. The variation in the redshift along the radial coordinate shown in Figure 25-30 are analogous to the figure shown by Errehymy (Errehymy *et al.*, 2021). Figure 31-36 show that variation of speed of sound in the radial and tangential direction along the radial coordinate for EXO 1785-248, SMC X-1, Her X-1, 4U 1538-52, LMC X-4, and Cen X-3. It is clear from Figure 31-36 that the speed of sound is

monotonically decreasing towards the surface which is a well-behaved condition (Kumar and Bharti, 2022). Figure 31-36 are analogous to the figure shown by Malaver (Malaver, 2018). Figure 37-42 show the variation in adiabatic index along the radial coordinate for EXO 1785-248, SMC X-1, Her X-1, 4U 1538-52, LMC X-4, and Cen X-3. The variation shown in Figure 37-42 in adiabatic index is analogous the figure shown by Maurya (Maurya and Tello-Ortiz, 2019). Figure 43-48 show the variation of compression moduli under the anisotropy of EXO 1785-248, SMC X-1, Her X-1, 4U 1538-52, LMC X-4, and Cen X-3, taken along the radial coordinate. The product of adiabatic index and radial pressure is compression moduli $K_e = \gamma P_r$, which is used to determine the density of any neutron star. It should be decreasing from the inner core the crust of a neutron star.

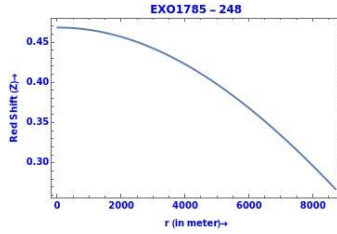


Figure 25. Plot of variation in the redshift along radial coordinate r for EXO 1785-248

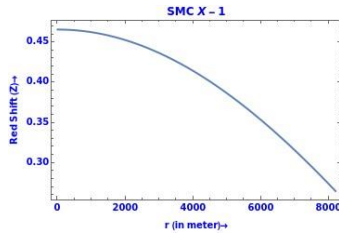


Figure 26. Plot of variation in the redshift along radial coordinate r for SMC X-1

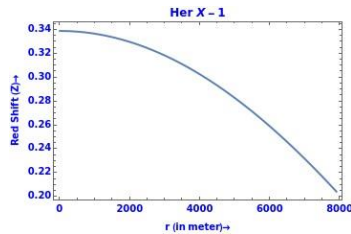


Figure 27. Plot of variation in the redshift along radial coordinate r for Her X-1

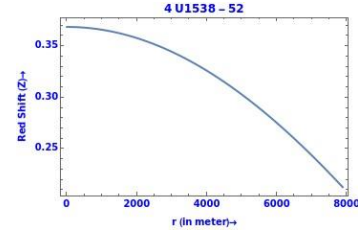


Figure 28. Plot of variation in the redshift along radial coordinate r for 4U 1538-52

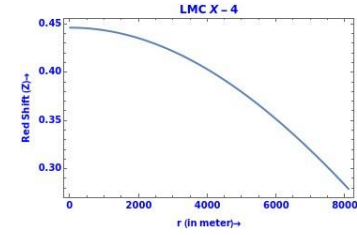


Figure 29. Plot of variation in the redshift along radial coordinate r for LMC X-4

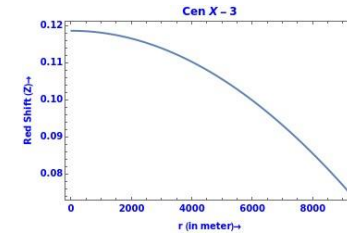


Figure 30. Plot of variation in the redshift along radial coordinate r for Cen X-3

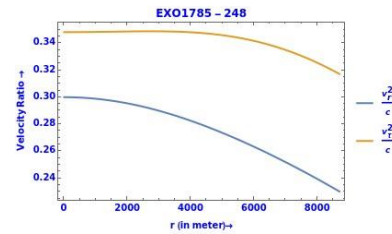


Figure 31. Plot of variation of radial and tangential velocity of sound along radial coordinate r for EXO 1785-248

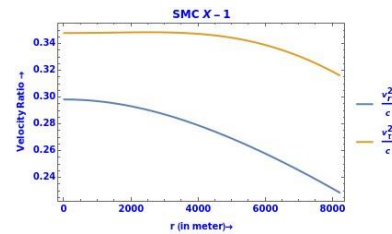


Figure 32. Plot of variation of radial and tangential velocity of sound along radial coordinate r for SMC X-1

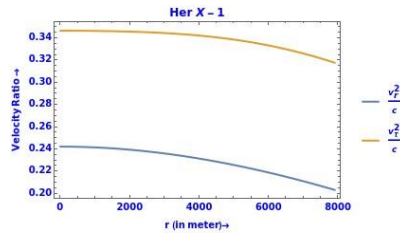


Figure 33. Plot of variation of radial and tangential velocity of sound along radial coordinate r for Her X-1.

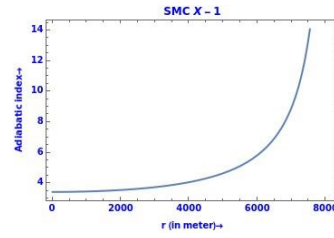


Figure 38. Plot of adiabatic index in SMC X-1 along the radial coordinate r .

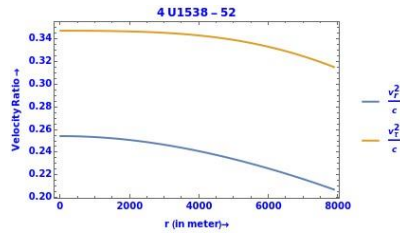


Figure 34. Plot of variation of radial and tangential velocity of sound along radial coordinate r for 4U 1538-52.

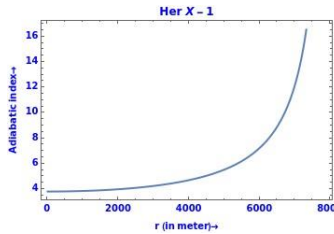


Figure 39. Plot of adiabatic index in Her X-1 along the radial coordinate r .

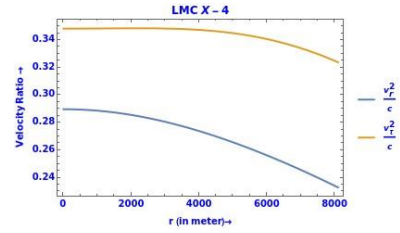


Figure 35. Plot of variation of radial and tangential velocity of sound along radial coordinate r for LMC X-4.

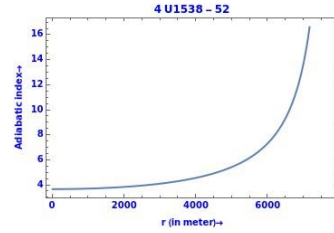


Figure 40. Plot of adiabatic index in 4U 1538-52 along the radial coordinate r .

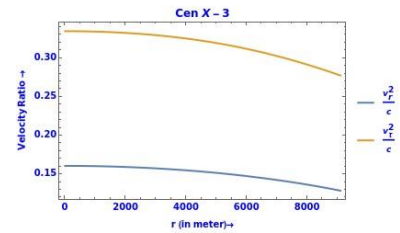


Figure 36. Plot of variation of radial and tangential velocity of sound along radial coordinate r for Cen X-3.

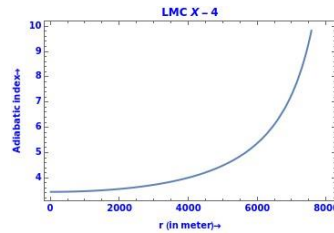


Figure 41. Plot of adiabatic index in LMC X-4 along the radial coordinate r .

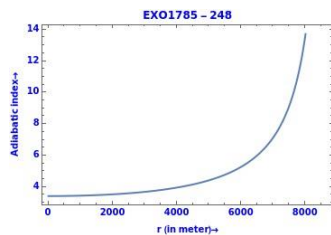


Figure 37. Plot of adiabatic index in EXO 1785-248 along the radial coordinate r .

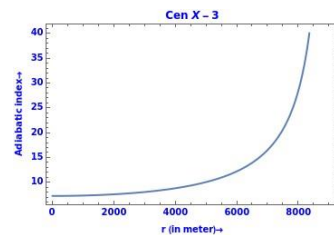


Figure 42. Plot of adiabatic index in Cen X-3 along the radial coordinate r .

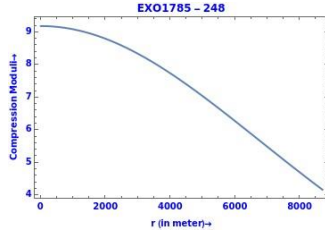


Figure 43. Plot of the variation of compression moduli in EXO 1785-248 along radial coordinate.

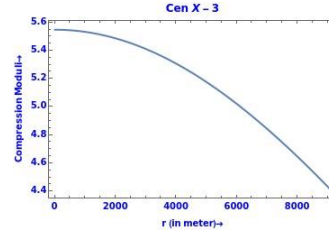


Figure 48. Plot of the variation of compression moduli in Cen X-3 along radial coordinate.

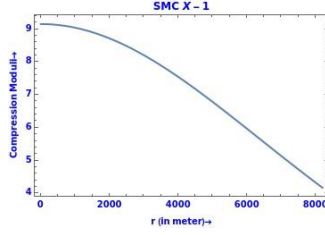


Figure 44. Plot of the variation of compression moduli in SMC X-1 along radial coordinate.

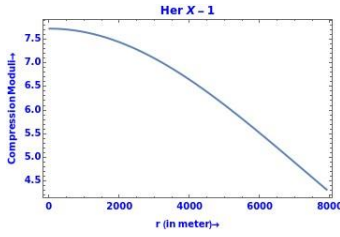


Figure 45. Plot of the variation of compression moduli in Her X-1 along radial coordinate.

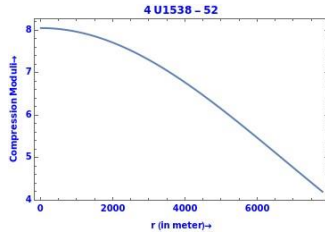


Figure 46. Plot of the variation of compression moduli in 4U 1538-52 along radial coordinate.

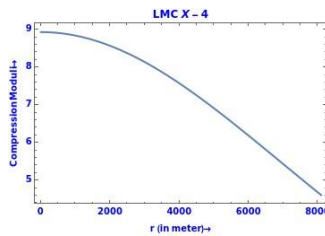


Figure 47. Plot of the variation of compression moduli in LMC X-4 along radial coordinate.

Matching Condition at the Boundary

All the solutions obtained for the neutron stars should be matched with the Schwarzschild metric at the boundary, the well-known Schwarzschild metric is given as,

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2. \quad (26)$$

Having matched the metric potential across the boundary that is $r = r_b$ we obtain the following equations,

$$e^{\nu_b} = \left(1 - \frac{2GM}{c^2 r}\right) = B(1 + X)^6, \quad (27)$$

$$e^{-\lambda_b} = \left(1 - \frac{2GM}{c^2 r}\right) = Y, \quad (28)$$

Where $X = Cr_b^2$. The radial pressure at the boundary is zero, therefore, we can take,

$$[P_r]_{r=r_b} = 0 \quad (29)$$

Using the condition of equation (29) we calculate the value of the constant A as follows:

$$A = \ln \left[\frac{12}{(1+X)^5} \right], \quad (30)$$

from the equations (11), (27), and (29) constant B can be calculated as

$$B = \frac{1}{(1+X)^5(1+13X)}, \quad (31)$$

Mass of a neutron star can be evaluated as,

$$M = \frac{c^2 r_b}{2G} \frac{12X}{(1+13X)}, \quad (32)$$

and the surface density is expressed as,

$$\frac{8\pi G}{\chi c^2} \rho r_b^2 = \frac{12(3+26X+11X^2)}{(1+X)(1+13X)^2}. \quad (33)$$

In Table 1, we have shown the radial pressure, tangential pressure, the density, at the center of EXO 1785-248, SMC X-1, Her X-1, 4U 1538-52, LMC X-4, and Cen X-3. We observed that all these parameters were showing the well-behaved nature. In Table 2, we have shown the redshift, the square of speed of sound in radial and tangential direction, stability factor, and anisotropy. In Table 3, the computed value of mass and radius along with the Buchdahl limit of the neutron stars EXO 1785-248, SMC X-1, Her X-1, 4U 1538-52, LMC X-4, and Cen X-3 have been shown.

Conclusions

In this paper, we have proposed a new model of anisotropic uncharged dense star inserting class I condition (Karmakar, 1948) in Durgapal metric (Durgapal, 1982). The model fulfills all the real physical conditions and free from central singularity in the spacetime considered for the study. We took $n = 6$ in Durgapal model going beyond the fifth model proposed. We obtained a well behaved simple solution for the constant values A in $[1.5760, 2.295]$, B in $[0.0602, 0.5577]$, and C in $[7.492 \times 10^{-10}, 3.397 \times 10^{-9}]$ for EXO 1785-248, SMC X-1, Her X-1, 4U 1538-52, LMC X-4, and Cen X-3.

Table 1. Shows the value of the constants A and C . The central density, the surface density, and the central pressure, for EXO 1785-248, SMC X-1, Her X-1, 4U 1538-52, LMC X-4, and Cen X-3.

Compact Stars	A	C	Central Density (kg/m^3)	Surface Density (kg/m^3)	Central Pressure (N/m^2)	References
EXO 1785-248	2.2316	6.634×10^{-10}	0.99×10^{18}	7.91×10^{17}	8.38×10^{33}	Özel <i>et al.</i> (2009)
SMC X-1	2.2332	7.492×10^{-10}	1.10×10^{18}	9.05×10^{17}	9.41×10^{33}	Singh <i>et al.</i> (2016)
Her X-1	2.2995	5.757×10^{-10}	0.91×10^{18}	6.12×10^{17}	3.65×10^{33}	Singh <i>et al.</i> (2016)
4U 1538-52	2.2841	6.620×10^{-10}	1.03×10^{18}	7.04×10^{17}	6.80×10^{33}	Rawls <i>et al.</i> (2011)
LMC X-4	2.2429	6.358×10^{-10}	0.95×10^{18}	6.46×10^{17}	7.7×10^{33}	Rawls <i>et al.</i> (2011)
Cen X-3	2.4196	5.570×10^{-10}	3.24×10^{18}	6.47×10^{17}	9.57×10^{33}	Rawls <i>et al.</i> (2011)

Table 2. Shows the value of the constant B , redshift, v_r^2/c , v_t^2/c , and anisotropy for EXO 1785-248, SMC X-1, Her X-1, 4U 1538-52, LMC X-4, and Cen X-3.

Compact Stars	B	Redshift (Z)	v_r^2/c	v_t^2/c	Anisotropy (Δ)	References
EXO 1785-248	0.4636	0.2618	0.4777	0.6478	0.8874	Özel <i>et al.</i> (2009)
SMC X-1	0.4653	0.2606	0.4772	0.6472	0.8864	Singh <i>et al.</i> (2016)
Her X-1	0.5577	0.1983	0.4489	0.6228	0.8141	Singh <i>et al.</i> (2016)
4U 1538-52	0.5338	0.2132	0.5657	0.7159	0.8512	Rawls <i>et al.</i> (2011)
LMC X-4	0.4779	0.2513	0.4732	0.6425	0.6785	Rawls <i>et al.</i> (2011)
Cen X-3	0.7990	0.0754	0.4149	0.5810	0.4813	Rawls <i>et al.</i> (2011)

Table 3. Shows the calculated mass and radius with Buchdahl condition for EXO 1785-248, SMC X-1, Her X-1, 4U 1538-52, LMC X-4, and Cen X-3.

Compact Stars	Mass ($M_\odot$)	Radius (Km)	Buchdahl Condition ($2M/R \leq 8/9$)	References
EXO 1785-248	1.295	8.7	0.2977	Özel <i>et al.</i> (2009)
SMC X-1	1.024	8.2	0.2497	Singh <i>et al.</i> (2016)
Her X-1	0.818	7.9	0.2097	Singh <i>et al.</i> (2016)
4U 1538-52	1.877	7.8	0.4812	Rawls <i>et al.</i> (2011)
LMC X-4	1.162	8.1	0.2869	Rawls <i>et al.</i> (2011)
Cen X-3	2.122	9.4	0.4514	Rawls <i>et al.</i> (2011)

We selected EXO 1785-248, SMC X-1, Her X-1, 4U 1538-52, LMC X-4, and Cen X-3 to show the physical properties of the model graphically. Figure 1-6 show that the metric potential is regular throughout the interior of the compact star and free from physical or geometrical singularity. Figure 7-30 show the variation in the physical parameters ρ , P_r , P_t , Δ , and Z are positive finite in the astral model and decreasing function of radial coordinate from inner core to the surface. We further observe that the radial pressure is vanishing at the boundary and the tangential pressure is non-negative. We observe a similar behavior of the velocity of sound. The anisotropic factor $\Delta > 0$ and increases from the inner core to the crust of the compact star and it vanishes at the center, which is a realistic feature of an anisotropic compact star. Figure 31-36 are showing the comparative variation of $\frac{v_r^2}{c^2}$ and $\frac{v_t^2}{c^2}$ in the anisotropy of EXO 1785-248, SMC X-1, Her X-1, 4U 1538-52, LMC X-4, and Cen X-3 along r . The adiabatic index is increasing with respect to the increasing radial coordinate as shown in Figure 37-42. So, we can infer that it is a physically feasible configuration. We observe from Figure 43-48 that the compression moduli is decreasing from center to the boundary, which shows that the star is denser at the inner core as compared to the crust. All these evidences are enough to infer the stability of our model. The radial and tangential velocity of sound in the matter is monotonically decreasing and satisfies the condition $\frac{v_r^2}{c^2}$ and $\frac{v_t^2}{c^2}$ are less than 1 throughout. Therefore, the causality condition is satisfied. The Buchdahl condition is also well satisfied by the model. Among all the compact stars taken by us the mass and radius of Cen X-3 is highest. The lowest mass is possessed by Her X-1 while the lowest radius is of 4U 1538-52. We observed the highest anisotropy 1.3813 in Cen X-3 and the lowest in Her X-1. Cen X-3 exhibits the highest redshift 0.9159 and Her X-1 shows the lowest redshift 0.1983. SMC X-1 has the highest central density $1.108 \times 10^{18} \text{Kg/m}^3$ Her X-1 has the lowest central density $0.91 \times 10^{18} \text{Kg/m}^3$. The highest surface density is $9.05 \times 10^{17} \text{Kg/m}^3$ for SMC X-1 while the lowest surface density is $6.12 \times 10^{17} \text{Kg/m}^3$ for Her X-1. SMC X-1 Possesses highest central pressure $9.41 \times 10^{33} \text{N/m}^2$ and Her X-1 possesses the lowest central pressure $3.65 \times 10^{33} \text{N/m}^2$. We have developed a simple compact star model embedding Karmakar class I reducibility condition in Durgapal metric. The model well behaves for all the physical parameter of the compressed stars and satisfies the stability and causality conditions.

Acknowledgment

I am thankful to my parent organization Birla Institute of Technology, Mesra, Ranchi, India for allowing me to complete this work at Amity Institute of Applied Sciences, Amity University Noida, India.

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