

THE RELATIONSHIP BETWEEN SPEED DIFFERENTIAL AND TIME TO COLLISION OF MALAYSIAN EXPRESSWAYS

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Abstract

Lane change (LC) manoeuvre has been recognized as an important aspect of driving behavior that significantly impacts traffic operation and management. Incorrect LC execution, particularly incorrect gap distance selection before the LC movement, can lead to traffic accidents, most often in the form of a rear-end, sideswipe, or angled collision. This paper investigates the relationship between time to collision and the speed differential between the leading and following vehicles when lane change occurs. Using an instrumented vehicle method, data was gathered along a typical length of the Kuala Lumpur-Seremban expressway. A VBox (Video Velocity Box) is an on-board data collection device that is used to videotape traffic incidents on the road, was installed in a passenger car. In a three-day period, a total of 175 instances of lane changing were documented. Following gap distance was used to calibrate the VBox equipment as a measure of efficiency. A simple linear regression was conducted between time to collision (TTC) and speed differential. It was found that 60% and 75% of drivers have TTC fewer than 5 sec and 10 sec, separately, with 6.10 sec average TTC. The time to collision (TTC) has a negative linear relationship with speed differential ($R^2 = 84.47\%$). The finding shows that the higher the speed differential between vehicles, the lower the TTC value, which indicates a higher probability of collisions. It can be concluded that the speed differential between the test car and following vehicles is affecting the TTC, which is to be utilised as a risk indicator throughout lane-changing operation.

Keywords: Lane-changing, TTC, speed differential, instrumented vehicle, risk indicator

Introduction

Recently, there are many research studies have reported high statistics numbers of severe crashes causing due to Lane Change (LC) maneuver on the road (Sen *et al.*, 2003; Naranjo *et al.*, 2008; Suh *et al.*, 2018; Yang *et al.*, 2019; Shawky, 2020).

According to Shawky, abrupt lane changes were responsible for 17.0 percent of all severe collisions between 2010 and 2017 (Shawky, 2020). A total of 13939 fatal collisions occurred during an overtaking maneuver in the USA from 1994 to 2005, according

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to another research by Naranjo *et al.* (2008). According to the Traffic Management Bureau of the Public Security Ministry, 4.9 % of all traffic crashes were reported in China in 2015 were caused by lane change maneuver on the road (Yang *et al.*, 2019). Traffic Management Bureau Based on the National Highway Traffic Safety Administration (NHTSA) crash database in the USA, the lane change crashes consist of 539,000 crashes. This constitutes about 9% of the 6.3 million crashes recorded in 1999 (Sen *et al.*, 2003).

On the other hand, Möbus (2017) reported that accidents during lane change movements have remained at a reasonably steady level of 5-7 % over the previous 20 years, indicating the need for safety time between cars after they change lanes.

In parallel with this, the analysis of executing LC on the road, the Time To Collision (TTC), represents the essential part that raised the crash severity during this maneuver on the road (Hayward, 1972; Schwarz, 2014). It was worth mentioning that the TTC itself was a crucial reason to cause crashes (Horst and Hogema, 1993; Svensson, 1998; Vogel, 2003; Xu and Qu, 2014). "The time needed for two vehicles to collide assuming they remain at their current speed and on the same trajectory," Hayward (1972) defined TTC. Hayward's equation is shown in Equation 1.

$$TTC = \frac{V_r}{D_0} \quad (1)$$

Where V_r donate the distance range, D_0 the ratio of the speed range.

TTC is calculated as the ratio of the speed range (D_0) between vehicles and the rate of change of the distance range (V_r) (Hayward, 1972; Julia *et al.*, 2000). TTC is the amount of time it takes for two objects to collide if their trajectory and speed stay constant.

In two-lane highways, TTC is defined as the gap time between the opposing vehicle and the subject vehicle when the subject vehicle and the lead vehicle in the opposing lane pass each other (Toledo and Farah, 2011). TTC is a crucial part of a driver's trajectory management decision-making process when driving on the expressway (McLeod and Ross, 1983; Horst, 1991; Vogel, 2003). As a result, TTC determines the degree of vehicle contact. If the TTC is to be utilized as a decision-making tool, it must take into account car lane changes.

Many studies have considered the parameters that affect the TTC, such as driving volatility, road characteristics, road environment, congestion, vehicle types, driver behavior, speed differential between the leading and trailing cars, and braking

process (McLeod and Ross, 1983; Horst, 1991; Horst and Hogema, 1993; Xu and Qu, 2014; Wali *et al.*, 2020). Horst discovered, for example, that TTC information was used in both the decision to begin braking and the regulation of the braking operation itself (Horst, 1991). McLeod and Ross found that sex difference implies a significant difference in estimating TTC which males giving higher and more accurate estimation than females (McLeod and Ross, 1983). The findings of Wali *et al.* study confirmed that the greater driving volatility in TTC increases the severity of crashes (Wali *et al.*, 2020). Xu and Qu's primary findings indicated that road surroundings (weaving segments or various lanes) appear to have a substantial impact on TTC methods (Xu and Qu, 2014). Horst found that in fog road's environment, the TTC significantly increased (Horst and Hogema, 1993). Another research found that a TTC criterion in the range of 4.5 to 5 sec for triggering a Collision Avoidance System CAS is appropriate to investigate under unfavorable visibility conditions. Free-driving speeds are excessively high, especially in the visibility region of 40 to 120 m; only encounters with a TTC of less than 1.5 sec are regarded required, and experienced observers tend to apply this threshold value rather consistently in actual traffic (Horst and Hogema, 1993).

It is found by Yang *et al.* (2019) that more than 70% LC shows that the smallest TTC occurs between beginning and cross-lane points. This suggests that braking of the connected autonomous vehicles (CAV) can be designed in the safer phase. The findings of Julia *et al.* (2000) looked into how to avoid accidents including rear-end collisions, single-vehicle road departure accidents, side-wipe collisions, and angle collisions by establishing a minimum longitudinal distance between cars.

The TTC has frequently been utilized as a risk assessment indicator (Xu and Qu, 2014; Svensson, 1998; Vogel, 2003; Horst and Hogema, 1993; Wali *et al.*, 2020). Although Hydén (1987) focused on the high probability of collision, Laureshyn (Laureshyn *et al.*, 2010) searched to assess drivers' immediate risk and discussed surrogate events focusing on crash nearness, assuming that the speeds of the interrelating vehicles warrant a sufficiently severe danger. To achieve this aim, additional indicators such as the speed of both the leading and following cars and the time taken by the latter vehicle to arrive at the probable accident area must be added. The aim is to compute a TTC that can influence the reaction movements of the subject vehicles. The focus isn't only on calculating the expected or actual TTC. The purpose is to assess the driver's immediate hazard (the probability of an accident occurring) if he/she stays on the present trajectory. To determine

the effects of TTC as a surrogate indicator, St-aubin and Candidate (2014) presented a research looks at and compares the definitions and interpretations of TTC, one of the most widely used and least context-specific surrogate safety indicators, to see if it can be used to predict dangerous traffic occurrences.

In general, the value of TTC has a significant meaning; when TTC is negative (i.e., two cars are separating) implies that even if no action is taken (the following vehicle speed is less than the leading vehicle speed), a collision will not occur. A positive TTC, on the other hand, indicates that if neither the trailing vehicle nor the subject vehicle changes speed, a collision is likely. Small positive TTC values might indicate a risky move, and the lower the TTC, the more dangerous the activity (Peng *et al.*, 2015).

The time to collision TTC and the merging gap acceptance determine whether the driving situation satisfies the parameters for conducting a lane change (Yang *et al.*, 2019). To guarantee the safety of the lane change manoeuvre, both the merging gap acceptability and the TTC must be suitably large. The study presented by Peng *et al.* (2015) on parameters that affect lane changing behavior indicated that the prediction accuracy of the developed model in his study reaches 85.54% till 1.5 sec before changing lanes if we consider 85 % to be a high-accuracy criterion. Huo *et al.* (2014) considered acceleration in addition to position and speed to calculate the new TTC algorithm (Huo *et al.*, 2014). Since the speeds of proceeding and following vehicles moving in the same lane and both adjacent lanes have a major influence on driver lane changing decision (Horst and Hogema, 1993; Moridpour *et al.*, 2012; Hou *et al.*, 2014), the relationship between speed and lane change has to be investigated further. This paper aims to evaluate the relationship between TTC and speed differential between the leading and the trailing vehicles when changing lanes on the Malaysian expressway.

Methodology

Field Data Collection

The data for this study was gathered on a designated sample portion of the Seremban and Kuala Lumpur expressway in Malaysia, which is roughly 50.2 km long, as shown in Figure 1. The highway content of three lanes in each direction of traffic, each with a 3.70 m lane width and a 2.0 m paved shoulder. The test car was travel in the center lane for the purposes of this study. Due to the existence of many on- and off-ramps within the selected study stretch, the posted speed along the expressway is generally 110 km/h. The

expressway's traffic volume includes a diverse range of vehicle classes with frequent lane changing maneuvers. This ensures that a representative sample of lane changing occurrences occurs on the field.

On three days of a typical weekday, data was gathered during the daylight hours under different traffic circumstances for a total of 4.5 h. Data were collected in two directions within the designated area. Because heavy trucks are widely recognized for being linked with low travel speeds and typically utilize the outermost lane, this study focuses on LC events, including cars and heavy trucks. As a result, their LC behavior is undoubtedly impacting traffic flow. The LC manoeuvres of motorbikes, on the other hand, were not investigated in this study since distinct LC manoeuvre behavior distinguishes them (Ataelmanan *et al.*, 2021b). The data was collected using an instrumented car in the middle lane fitted with V-Box, which recorded the test and following vehicles' speeds, following gap distance, TTC, and lane changing frequency. The instrumented vehicle's V-Box camera was positioned backward on the instrumented vehicle to catch LC occurrences as the vehicle traveled through the study segment. Figure 2 depicts a typical traffic view from behind the test vehicle.

Calibration of VBox Camera for Estimation of Following Gap Distance

The V-Box camera was calibrated prior to data collection on site in order to predict the following gap distance, as illustrated in Figure 3. During the calibration procedure, Figure 3 illustrates the placement of the test car and the vehicle immediately after it. The calibration process was performed as follows: (1) The test vehicle was set and equipped with a V-Box system. For the setting requirement, the rear and front cameras were fixed on the front and back frames of the test car.

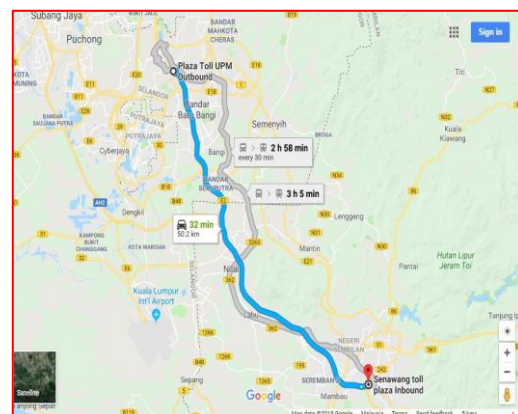


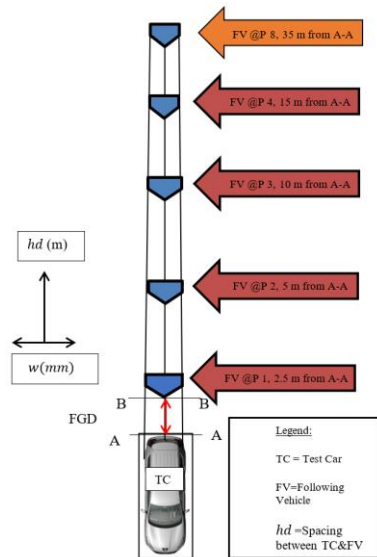
Figure 1. Study Site Location

Table 1. Relationship between the gap and width of the following vehicle

hd (m)	2.5	5	10	15	20	25	30	35
w (mm)	45.7	23.4	12.05	8.167	6.193	4.99	4.19	3.616

Table 2. Total number of vehicles

	Left Lane	Right Lane	Total (%)
Cars	21	120	141 (80.8%)
Heavy vehicles	33	1	34 (19.4%)

**Figure 2. During a field observation, a view of traffic from a test car****Figure 3. During the calibration of the VBox, the test and following vehicles are set up**

Then, the test vehicle stopped in appropriate open place to let the following vehicle move freely in a long enough distance behind it. (2) The following vehicle is fixed on the same aligned and on distance behind the test vehicle. This gap distance, hd , was first selected as 2.5 m exactly from the rear of the

test car to the front of the following vehicle (from Line A_A to Line B_B in Figure 3). (3) Then, the rear camera on the test vehicle recorded the view in the video tab of V-Box. This video was replayed later in the computer to measure the width w of the front in the rear camera (B_B) in Figure 3. The millimetre ruler could be used to measure the value w on the screen window of the video, then with the aid of the AutoCAD software program. (4) This process was repeated many times after the gap distance, hd from the rear test vehicle to the front bumper of the following vehicle (from Line A_A to Line B_B) increased systematically 5.0 m accordingly. In each time, the width w of the front bumper of the following car was measured accordingly using the video recording of VBox of the test vehicle. When the gap distance is becoming 35 m, this repetition process ends. It is worthy to mention that, for each time the gap distance, hd increased, the w decreased with the systematic rate (5) Finally, the calibration factor could be extracted as the decrement rate of the width, w divided by each increment value of the gap distance, hd .

Table 1 listed these parallel values between width, w and gap distance, hd for each increment. The data was determined to be best matched by a negative power function, with the generic form given by Equation 2 representing these values (Toledo *et al.*, 2003; Ibrahim, 2015). This calibration Equation could be multiplied by any random value of the front following vehicle w to find easily the gap distance value between the test vehicle and the following vehicle. It was noticeable that the value of the calibration factor was adopted as the absolute value. This method was used to evaluate the following gap distance FGD (distance between the back of the leading car and the front of the trailing car) and was found to be adequate (Ataelmanan *et al.*, 2021a).

hd = Gap distance, w = Displayed front width of the following car

$$h_d = 133.24704 (w)^{-1.039975} \quad (2)$$

Where h_d , the gap distance, w displayed front width of the following car.

Data Extraction

Data extraction from the video revealed that 175 vehicles were involved in lane-change behaviour. Table 2 shows the total number of

vehicles concerning vehicle composition and lane position of the following vehicle movement while the test car travels in the middle lane.

The data were manually retrieved from recording video that was played on the computer monitor. When the following car began changing lanes, the test vehicle's speed displayed on the V-Box speedometer was recorded. Meanwhile, the trailing car's speed was measured before it changed lanes in the same direction as the test car. The speed of the following vehicle was measured by calibration process on the video player VLC application as the following: (1) The movement of the following vehicle was recorded on camera and replayed in the monitor using VLC media player. (2) The length time of the frame on the screen has been predetermined by the VLC media player as one second consists of 24-frames. As a result, a single frame on the screen takes 1/24 second to complete. (3) Then, if we can count the number of frames required for the following vehicle to pass over one "paint white mark line," the speed may be simply estimated as the length of this point dividing by the duration time of these frames (it is predefined that the length from the begin to the end of one paint white mark line). Because of the nature of this type of data processing, this location data can only be calculated to a precision of 1.0 m from video frames (Lv *et al.*, 2013).

The speed differential was calculated by subtracting the leading vehicle speed from the following vehicle speed. For each time of Lane Change (LC) operation, the test vehicle speed, following vehicle speed, and the speed differential was measured accordingly overall 175-times. Table 3 shows the descriptive statistics over all the population of LC operation for these three speeds' values. The values in each column in this are independent of each other. In other words, the values in each column show the statistical indicator value for the population of 175 LC cases, such as minimum, maximum, mean, and standard deviation for the speed of test vehicle, following vehicle, and speed differential.

As shown in Table 3, the lowest value of speed differential during LC operation was -7.63 (the following vehicle speed was 89.99 km/h, and the leading vehicle speed was 97.62 km/h). This value

implied that in this LC manoeuvre, the speed of the leading vehicle (test vehicle) was greater than the speed of the following vehicle by 7.63 km/h. The maximum value of speed differential, 53.59, was obtained when the following vehicle speed was 134.99 km/h and the leading vehicle speed was 81.40 km/h. The pattern of the leading and following vehicles' speed for the lane change behaviour is shown in Figure 4.

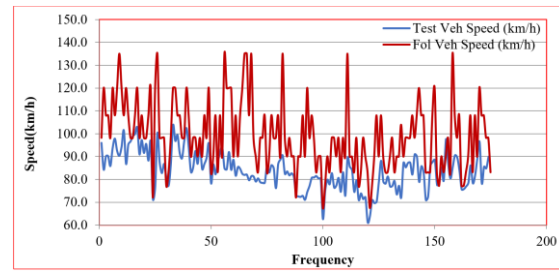


Figure 4. Speed trend for the test and following vehicles in (km/h)

As shown in Figure 4, in most LC cases, the following vehicle speed is greater than the leading vehicle speed.

TTC Measurement

TTC calculates the urgency of a lane change. It will take two cars to crash if the trailing car does not execute a move to avoid colliding, i.e., TTC is calculated by dividing the longitudinal distance between the leading and following vehicles by their speed differential. The TTC of a vehicle driver combination i at instant t in the case of a leading vehicle $i - 1$ on the same path is calculated using Equation 3.

$$TTC = \frac{X_i(t) - X_{i-1}(t) - l}{V_i(t) - V_{i-1}(t)} \quad (3)$$

Where V_i means the speed, X_i the location, and l the vehicle length.

From Equation 2, the TTC is generally computed for a given path, as may be shown. The following vehicle response was measured using speed change rate, braking timestamp, and time-to-collision (TTC) to better understand the influence of

Table 3. Descriptive of speed data for the observation days (N = 175)

Speed characteristics (km/h)	Test Vehicle Speed	Following Vehicle Speed	Speed Differential
Minimum	61.19	67.49	-7.63
Maximum	103.78	134.99	53.59
Mean	83.87	99.44	15.57
Standard Deviation	8.23	14.93	12.83

lane changes or the following car (Yang *et al.*, 2019). The driver's perceptions of ground speed, proximity to their intended headway, and the interaction dynamics with the preceding vehicle all impact this constant adjustment process (Brackstone, 2010).

TTC is calculated by dividing the distance between two vehicles by their relative velocity or distance range. Consider the example of two 100 feet-distance vehicles. The speed range is 100 feet/second minus 120 feet/second, or -20 feet/second if the front vehicle travels at 100 feet/second and the following vehicle moves at 120 feet/second. The TTC is calculated by dividing 100 feet by -20 feet/second. Therefore, the TTC is 5.0 s. In other words, assuming the following car's velocity remained constant, it would take 5.0 sec for the following car to crash with the leading car. On the other hand, the TTC parameter assumes constant speed and ignores vehicle acceleration (Smith *et al.*, 2002).

For the specific objective, regression analysis was conducted to find the relationship between TTC and speed differential of lane changing. Regression is a statistical approach that uses the values of one or more other variables to predict the value of a continuous variable. A prediction line is fitted in simple linear regression between a dependent variable, TTC, and a single independent speed difference.

Data Analysis and Results

Figure 5 illustrates cumulative values of TTC for the lane-change behaviour.

From the data in Figure 5, it is apparent that about 60% of cars change lanes in less than 5 sec, for a total of more than 75% of drivers changing lanes in less than 10 sec with an average of 6.10 sec. The lower the TTC, the fewer times drivers have to observe and react before colliding, and hence the greater the probability of a collision (Hydén, 1987; Nobukawa *et al.*, 2016). This high proportion, when compared to TTC's low, shows a degree of danger and risk that might result in accidents between following and leading cars. Unless there is a shift in the current lane in which the following vehicle is going, an increase in the leading vehicle's speed, or a deceleration (braking) by the following vehicle to avoid any risk on the road that may impact even other adjacent vehicles. Figure 6 shows a linear regression between speed differential and TTC.

As shown in Figure 6, there is a negative linear relationship exists between speed differential and TTC. The correlation determination ($R^2 = 0.8447$) reveals that the TTC and speed differential has

a good statistical association. It is noted that the speed differential was calculated by subtracting the leading vehicle speed from the following vehicle speed ($\Delta V = VFV - VLV$). As the speed differential increases, the following cars will be travelling faster than the leading vehicles, increasing the chances of a collision. The higher probability of collision is corresponding to the lower TTC. This finding is in agreement with other studies (Peng *et al.*, 2015; Li *et al.*, 2020). It is also shown in Figure 6 that the majority of the points occur at 10 sec and above of TTC value, and this TTC is corresponding to the speed differential less than 20 km/h. This is implied that hazard crashes due to small TTC can be avoided as long we can keep the speed differential less than 20 km/h. There is a negligible quantity of negative speed differential in the chart, which is corresponding to the TTC value of 25 sec.

Conclusions

Lane change (LC) manoeuvre has been recognized as an important aspect of driving behavior that significantly impacts traffic operation and management. This study used an instrumented

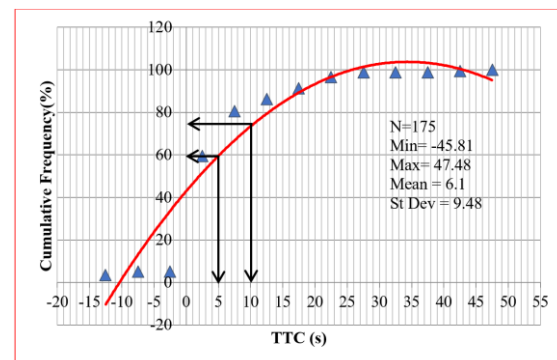


Figure 5. Cumulative frequency of TTC values

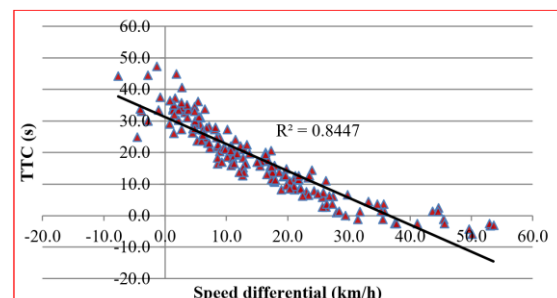


Figure 6. Relationship between TTC and speed differential

vehicle equipped with a V-Box device for lane-changing behavior data collection. The data involved the tested vehicle's speed, following vehicle speed, following gap distance, and following vehicle type. Prior to the data collection, The V-Box device was calibrated to measure the distance between the leading and trailing cars, allowing the distance between the two cars to be measured while they were on the road.

From the recorded videos, 175 lane-changing incidents were observed, 141 involving cars and 34 involving heavy vehicles. The speed differential between the leading vehicle and the following vehicle was measured for all the 175 lane-changing incidents. It was found that the mean speed differential is 15.57 km/h with a 12.83 standard deviation. According to the findings of this investigation, the tiny difference in speed, indicating a lower degree of hazard and risk. This finding is in agreement with other literature which indicated that the majority of lane-change crashes occurred in a case of little or no longitudinal gap and small TTC between vehicles (Winsum *et al.*, 1999; Julia *et al.*, 2000; Sen *et al.*, 2003; Zhou and Itoh, 2016; Zhao *et al.*, 2017). In addition to the estimation of the speed differential, the study also estimated the time to collision. It was found that 75% of vehicles have TTC less than 10 sec with an average of 6.10 sec.

In this paper also, the relationship between TTC and speed differential was investigated. The finding of this study confirmed that there is a negative relationship between the speed differential and the time to collision, TTC (R square = 0.8447). It was found that the TTC value decreased as the difference in the speed between the leading vehicle and the following vehicle increased. The higher the speed differential, the lower the TTC, which indicates a higher probability of collision. In general, these findings may aid in identifying and quantifying the impact of important traffic characteristics on lane change for safety (Bascunana, 1997; Winsum *et al.*, 1999; Weber, 2017; Zhou and Itoh, 2016). Also, these results are consistent with those of other studies and suggest that TTC evaluation has a significant indicator on traffic safety (Hydén, 1987; Horst and Hogema, 1993; Talmadge *et al.*, 1997; Saffarzadeh *et al.*, 2013). It is important to note that the results of this analysis were based on preliminary research and that more extensive sample data should be gathered over numerous days, remarkably field data variability related to traffic safety, mainly on expressways, and that because this study was limited to one segment, more research is recommended to investigate some other Malaysian highways.

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