

THERMO-ELASTICITY OF SATURATED POROUS MEDIA

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Abstract

This paper presents the derivation of the thermal expansion of an elastic porous media skeleton, which has not been intensively investigated in the past. The theoretical explanation shows that the volumetric thermal expansion of the entire porous media is solely controlled by and is equal to the coefficient of thermal expansion of the solid phase. Moreover, the constitutive relations for an undrained heating case are derived. It has been shown that the simulated pore water pressure induced by heating agrees well with the experimental results obtained from the literature.

Keywords: Thermal expansion, porous media, thermo-elasticity, pore water pressure

Introduction

The thermo-mechanical behavior of elastic porous media has become a subject of interest in many engineering areas including, geothermal energy extraction, oil recovery from a porous media and radioactive waste disposal. Over the past three decades, several models have been developed for non-isothermal porous media by extending the existing knowledge of isothermal porous media (Campanella and Mitchell, 1968; Bear and Corapcioglu, 1981; Palciauskas and Domenico, 1982; McTigue, 1986; Lewis *et al.*, 1986; Baldi *et al.*, 1988; Hueckel and Borestto, 1990; Lingnau *et al.*, 1995; Hueckel *et al.*, 1998; Cui *et al.*, 2000; Masters *et al.*, 2000; Graham *et al.*, 2001). However, there have been some controversies concerning the thermal expansion of the elastic porous media body and pore.

Campanella and Mitchell (1968) have suggested that if all solid grains were in contact, the porous media body would experience the same thermal volumetric strain as that in solid grains. That means the coefficient of thermal expansion of the porous media body (α_b) is equal to that of the solid grains (α_s). However, they have not shown any theoretical proof or experimental evidence to support their suggestion.

Palciauskas and Domenico (1982) expressed α_b in terms of averages over the porous media volume. Based on the comparison of coefficient of thermal expansion for polycrystalline computed from theoretical bounds and individual values for several minerals presented by Walsh (1973), Palciauskas

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and Domenico (1982) have suggested that α_b is approximately equal to α_s . McTigue (1986) also used this approximation to simplify his governing equations. However, there has not been any appropriate explanation from the theoretical point of view.

Moreover, there has been an attempt to calculate the thermo-elastic volumetric strain of porous media body by using a number of parameters, which are interpolated from the experimental data (Baldi *et al.*, 1988; Hueckel *et al.*, 1998; Hueckel and Boreotto, 1990; Cui *et al.*, 2000). Furthermore, Lingnau *et al.* (1995), Tanaka (1995) and Graham *et al.* (2001) calculate the thermo-elastic volumetric strain from a change of compressibility of porous media with temperature. Although there has been some experimental evidence to support these concepts, the thermo-elastic volumetric strain is still poorly defined.

Therefore, the main objective of this paper is to derive the elastic governing equations for porous media and provide an understanding of the thermal expansion coefficient of the solid skeleton of porous media. Furthermore, a link between the porous media and the thermal expansion coefficient of the solid grains is developed. The undrained heating condition is raised as a special case. The simulated pore

water pressure is compared with the experimental results obtained from the literature.

Thermo-Elasticity of Saturated Porous Media

Consider representative planar element of a porous media subjected to external mean stress of p , pore fluid pressure of u_f and temperature increase of T respectively. The porous media is assumed to be homogeneous and thermal equilibrium among each phase. Following and extending the concepts presented by Nur and Byerlee (1971), the stresses and temperature applying to the element can be decomposed into two components as shown in Figure 1.

The first component corresponds to an equal pore and external hydrostatic pressure of u_f and a temperature increase of T both inside the pore and outside the media. The volumetric strain due to this component (ϵ'_v) is

$$\epsilon'_v = c_s u_f - \alpha_b T \quad (1)$$

where c_s is the compressibility of the solid grains and α_b is the thermal expansion coefficient of the porous media body.

The second component corresponds to an external mean stress of $(p - u_f)$ without any

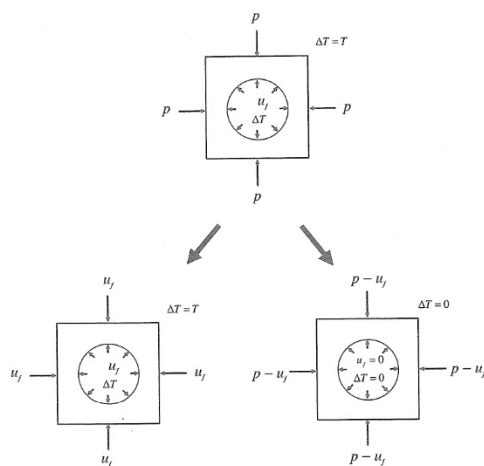


Figure 1. Stress and temperature decomposition of a representative porous element

further change of pore pressure and temperature. The volumetric strain due to this component (ϵ_v'') is

$$\epsilon_v'' = c(p - u_f) \quad (2)$$

where c is the effective compressibility of porous media. The total volumetric strain of a linear elastic body can be calculated by combining the volumetric strains due to both components, giving

$$\epsilon_v = c_s u_f - \alpha_b T + c(p - u_f) \quad (3)$$

According to the definition of the effective stress concept, the total volumetric strain can be written as,

$$\epsilon_v = c \left[p - \left(1 - \frac{c_s}{c} \right) u_f \right] - \alpha_b T \quad (4)$$

Considering changes in fluid mass of porous media (θ), the total change in the fluid mass due to both components (Figure 1) can be calculated using the following relationship

$$\theta = \theta^I + \theta^{II} \quad (5)$$

in which,

$$\theta^I = n u_f c_s - n u_f c_f + n(\alpha_f - \alpha_p) T \quad (6)$$

$$\theta^{II} = (c - c_s)(p - u_f) \quad (7)$$

where α_f and α_p are coefficients of thermal expansion of fluid and pores respectively, n is porosity, and c_f is compressibility of water. Substituting Eqns. (6) and (7) into Eqn. (5) and rearranging gives,

$$\theta = [c - c_s] p - [(c - c_s) + n(c_f - c_s)] \quad (8)$$

$$u_f + n(\alpha_f - \alpha_p) T$$

Thermal Expansion of Elastic Porous Media Skeleton

To define α_b and α_p in terms of coefficient of thermal expansion of solid (α_s), consider two identical Porous Media *A* and *B*, as shown in Figure 2. Suppose all the pores in Medium *A* (hereafter referred to as Solid *A*) are filled with solid and the inscribed surface *S* corresponds to the boundaries of the filled cavities within the medium. For both media, the solid is assumed to be homogeneous, isotropic, stress free and free to expand.

Now an increase in temperature (ΔT) surrounding Solid *A* and Porous Medium *B* results in a uniform increase in temperature in Solid *A*, and at every point within the solid skeleton and the boundary of the pores in Porous Medium *B*. In Solid *A*, the volumetric

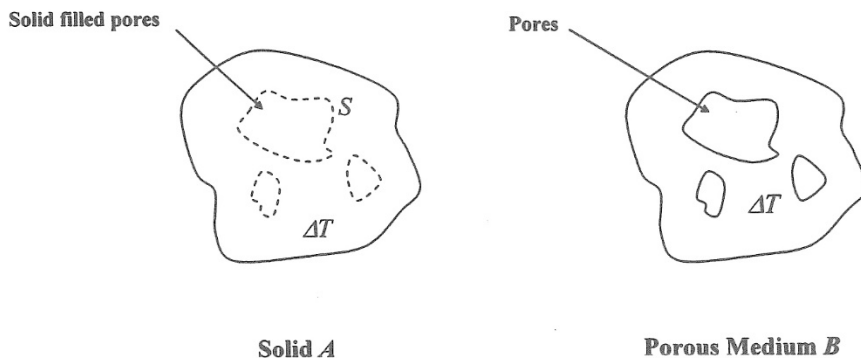


Figure 2. Homogeneous Solid *A* and Porous Medium *B* Subjected to a Temperature Change of ΔT

strain, $\varepsilon_{vT(A)}$, due to the increase in temperature, ΔT , is,

$$\varepsilon_{vT(A)} = -\alpha_s \Delta T \quad (9)$$

In Porous Medium B , the volumetric strain due to the increase in temperature is $\varepsilon_{vT(B)}$. Suppose we now remove material from inside the inscribed region S in Solid A while maintaining the change in temperature at the inscribed boundary at ΔT . We now invoke the uniqueness theorem for a boundary value elastic problem (Muskhelishvili, 1963), which states that the strain field in an elastic medium is uniquely related to the changes in its boundaries. Since by removing the inclusions from Solid A the temperature and the stresses have remained unchanged resulting in an immediate flow that, $\varepsilon_{vT(A)} = \varepsilon_{vT(B)}$, or that, $\alpha_b = \alpha_p = \alpha_s$. In other words, the volumetric thermal expansion of the entire porous media is solely controlled by, and is equal to, the coefficient of thermal expansion of the solid phase. This theoretical outcome agrees with Campanella and Mitchell (1968) observations that if the soil grains were in mineral to mineral contact and the temperature is changed, then the whole soil mass undergoes the same volumetric strain as the individual solid grains.

It should be noted that a direct consequence of this important assertion is that the porosity and the void ratio of porous media become independent of temperature.

Thus, Eqns. (4) and (8) become,

$$\varepsilon_v = c[p - (1 - \frac{c_s}{c})u_f] - \alpha_s T \quad (10)$$

$$\theta = (c - c_s)p - [(c - c_s) + n(c_f - c_s)]u_f + n(\alpha_f - \alpha_s)T \quad (11)$$

Undrained Response under Constant External Stresses

In the case of undrained heating, there are no changes in pore fluid mass. If the compressibility of porous media, solid and fluid are independent of temperature, Eqns. (10) and (11) can be expressed in partial differential equation form as,

$$\frac{\partial \varepsilon_u}{\partial T} = -c(1 - \frac{c_s}{c}) \frac{\partial u_f}{\partial T} - \frac{\partial}{\partial T}(\alpha_s T) \quad (12)$$

$$[(c - c_s) + n(c_f - c_s)] \frac{\partial u_f}{\partial T} = n \frac{\partial}{\partial T}[(\alpha_f - \alpha_s)T] \quad (13)$$

where ε_u is undrained volumetric strain.

If the compressibility of the solid and water are considerably less than the compressibility of the porous media, the solid phase will be assumed to be incompressible and Eqns. (12) and (13) will become,

$$\frac{\partial \varepsilon_u}{\partial T} = -c \frac{\partial u_f}{\partial T} - \frac{\partial}{\partial T}(\alpha_s T) \quad (14)$$

$$c \frac{\partial u_f}{\partial T} = n \frac{\partial}{\partial T}[(\alpha_f - \alpha_s)T] \quad (15)$$

It has been found that the compressibility of porous media depends on the mean effective stress level and can be expressed as (Wood, 1990),

$$c = \frac{\kappa}{vp'} \quad (16)$$

where κ is the slope of the unloading-reloading line in the semi-logarithmic compression, p' is mean effective stress, and v is the specific volume of the porous media. Since the solid phase is assumed to be incompressible, an increase in pore pressure will be equal to a reduction in effective stress. Therefore, Eqn. (15) becomes,

$$\frac{\kappa}{vp'} \frac{\partial p'}{\partial T} = -n \frac{\partial}{\partial T}[(\alpha_f - \alpha_s)T] \quad (17)$$

Simulation Results and Discussion

To validate the assumption made in this paper, the change in pore water pressure due to heating obtained from the literature is predicted using the constitutive relations derived in the previous section. The parameters used in predictions are shown in Table 1. The experiments were conducted on illitic clay under both normally consolidated and overconsolidated states (Tanaka, 1995), normally consolidated kaolin sample (Chiu, 1996) and overconsolidated silt samples obtained from Bourke, NSW Australia with preconsolidation pressure of 200 kPa

(Uchaipichat, 2005).

The predictions were conducted by solving the Eqn. (17) as a series of initial value problems. The fluid phase in the porous media sample is water and its coefficient of thermal expansion is considered to be dependent of both temperature and pressure, which can be expressed as (Baldi *et al.*, 1988),

$$\alpha_w = \alpha_0 + (\alpha_1 + \beta_1 T) \ln mp + (\alpha_2 + \beta_2 T) (\ln mp)^2 \quad (18)$$

in which

$$\begin{aligned} \alpha_0 &= 4.505 \times 10^{-4} \text{ } ^\circ\text{C}^{-1} \\ \alpha_1 &= 9.156 \times 10^{-5} \text{ } ^\circ\text{C}^{-1} \\ \alpha_2 &= 6.381 \times 10^{-6} \text{ } ^\circ\text{C}^{-1} \\ \beta_1 &= 1.2 \times 10^{-6} \text{ } ^\circ\text{C}^{-2} \\ \beta_2 &= -5.766 \times 10^{-8} \text{ } ^\circ\text{C}^{-2} \\ m &= 1 \text{ Mpa} \end{aligned}$$

It can be seen that the predictions using Eqn. (17) show good agreement with the experimental results obtained from the literature as shown in Figures 3 to 5. The scatter of data can be considered as errors of measurement due to testing at high temperature particularly at the testing temperature over 60°C. Moreover, Figure 3 shows a good prediction of a change in pore pressure with temperature for both normally consolidated and overconsolidated samples. This can also be observed in Figure 4 and 5, in which the samples are in normally consolidated and overconsolidated states, respectively.

Conclusion

A rigorous theory of thermo-elasticity for porous media has been derived. The theoretical explanation shows that volumetric thermal

Table 1. Parameters used in predictions

Parameters	Tanaka (1995)	Chiu (1996)	Uchaipichat (2005)
Soil Type	Illitic clay	Kaolin	Silt
n	0.52	0.33	0.36
α_s	2.1×10^{-5}	3.5×10^{-5}	3.4×10^{-5}
κ	0.044	0.021	0.006
ν	2.10	1.50	1.58

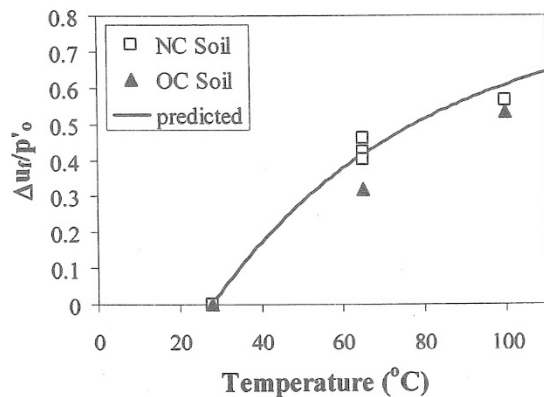


Figure 3. Comparison of predicted pore pressure induced by heating and data extracted from Tanaka (1995)

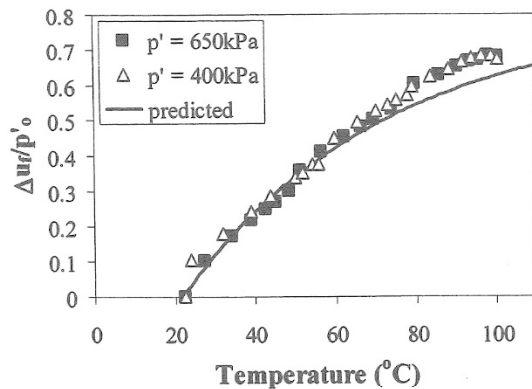


Figure 4. Comparison of predicted pore pressure induced by heating and data extracted from Chui (1996)

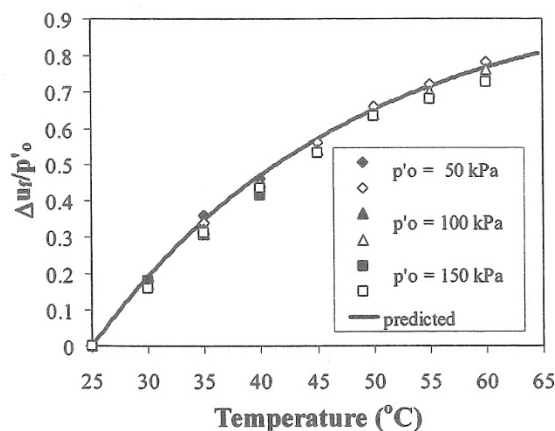


Figure 5. Comparison of predicted pore pressure induced by heating and data extracted from Uchaipichat (2005)

expansion of the entire porous media is solely controlled by and is equal to the coefficient of thermal expansion of the solid phase. The constitute relations have been derived for the case of undrained heating. The increase in pore pressure depends on the porosity and the thermal expansion coefficients of the fluid and the solid. Furthermore, it has been proven that the pore pressure predicted from the theory agrees well with the experimental results obtained from the literature.

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