

LONDON PENETRATION DEPTH OF CaAlSi SUPERCONDUCTORS BY SEMI-CLASSICAL APPROACH

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Abstract

The temperature dependence of the London penetration depth in-plane (ab plane) and out-plane (c axis) of single band anisotropic S-wave superconductors was investigated by semi-classical theory in 3D spherical Fermi surface. The ellipse spherical and pancake spherical gap model were used as anisotropic model for energy gap. The London penetration depth was derived by analytical and numerical approach and applied to CaAlSi-I and CaAlSi-II superconductors. We found that our model can fit well with experimental data of CaAlSi-I and CaAlSi-II at low temperature scenery. CaAlSi-I showed anisotropic and isotropic properties in ab plane and c axis respectively and CaAlSi-II showed anisotropic properties both in-plane and out-plane.

Keywords: Semi-classical theory, London penetration depth, s-wave superconductors, anisotropic gap function, AlB_2

Introduction

The discovery of superconductivity in an intermetallic compound of magnesium diboride (MgB_2) at 39 K (Nagamatsu *et al.*, 2001) was attracted attention for both experimental and theoretical work. High T_c , simple crystal structure, large coherence lengths, high critical current densities (Eom *et al.*, 2001) and fields (Patnaik *et al.*, 2001), and transparency of grain boundaries to current flow promise that MgB_2 will be a good material for both large scale applications and electronic devices (Buzea and Yamachita, 2001). Hass and Maki (2001); Posazhennikova *et al.* (2002) have constructed a model of anisotropic s-wave superconductivity for single crystal MgB_2 . This simple theory appears to

account rather well for the thermodynamic properties, the optical measurements and for the upper critical field data.

MgB_2 possesses the simple hexagonal AlB_2 -type structure (space group $P6/mmm$), which is common among borides. Superconductors with an AlB_2 structure have received increased attention after the discovery of superconductivity in MgB_2 . AMSi family of pseudo-ternary layered silicides is a typical example, where A = Ca, Sr or Ba and M = Al or Ga (Imai *et al.*, 2001, 2002; Majumdar *et al.*, 2001; Kito *et al.*, 2002). Silicides have been important materials also for Si-based devices because they have appropriate compatibility with

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silicon and can be formed easily by silicidation in a metal-silicon couple. They are used for interconnection, gates in metal-oxide-semiconductor structure, ohmic contacts and Schottoky barriers in Si integrated circuits (Imai *et al.*, 2002).

The London penetration depth (λ) refers to the exponentially decaying magnetic field at the interior surface of a superconductor. It is related to the superfluid density (ρ) (Changjan *et al.*, 2019). Accurate and precise measurements of the absolute value of $\lambda(T \rightarrow 0)$ are very important for understanding the behavior of high temperature superconductors (Askerzade, 2003a). London penetration depth can be measured by several methods. Lin *et al.* (2015) developed methodology to determine the London penetration depth of FeSe_{0.3}Te_{0.8} thin film by scanning SQUID microscope. Shengelaya *et al.* (2005) reported the studies of the London penetration depth of Sr_{0.9}La_{0.1}CuO₂ infinite-layer superconductors by transverse- field Muon spin rotation (TF- μ SR) technique. Chen *et al.* (2014) developed technique for determine the London penetration depth by ac susceptibility measurements of a square film. Pang *et al.* (2016) reported measurements London penetration depth of K₂Cr₃As₃ single crystals and polycrystalline Rb₂Cr₃As₃ by using a tunnel diode oscillator based technique. Two-band Ginzburg-Landau (GL) theory were solved analytically to determine the temperature dependence of the London penetration depth of MgB₂ (Askerzade and Gencer 2002; Niyomsilpchai *et al.*, 2013), YNi₂B₂C (Askerzade, 2003b) and Ba(Fe_{0.93}Co_{0.07})₂As₂ (Changjan and Udomsamuthirun, 2014). Tongkhonburi and Udomsamuthirun (2019) applied anisotropic two-band Ginzburg Landau theory to investigate the London penetration depth of MgB₂ and CaAlSi.

The semi-classical approach of Onsager and Pippard has been very successful in relating the electronic properties of a normal metal to its band structure and fermi surface. Chandrasekhar and Einzel (1993) extend the method to a superconductor. The semi-classical theory were solved analytically to determine the temperature dependence of superfluid density of anisotropic superconductors, e.g. MgB₂ (Changjan *et al.*, 2019) and CaAlSi (Prozorov and Giannetta, 2006; Chanilkul and Udomsamuthirun, 2018; Changjan *et al.*, 2019).

In this paper, we investigated the London penetration depth of s-wave superconductors by semi-classical theory (Chandrasekhar and Einzel, 1993) and modified with 2 model anisotropic gap function, ellipse spherical (Hass and Maki, 2001) and pancake spherical (Posazhennikova *et al.*, 2002)

model. Finally, the temperature dependent London penetration depth was investigated and applied to experimental data CaAlSi Superconductors.

Materials and Methods

The semi-classical theory to the London penetration depth proposed by Chandrasekhar and Einzel (1993) was used as the main approach in our model. The relationship between normalized superfluid density components and the London penetration depth by semi-classical approach are given by $\rho = \frac{R_{ii}(T)}{R_{ii}(0)} = \left(\frac{\lambda_{ii}(0)}{\lambda_{ii}(T)} \right)^2$ (Prozorov and Giannetta, 2006).

The energy gap $\Delta(T)$ is obeyed the weak-coupling limit that electron-phonon interaction is the main mechanism of the Cooper pair occurrence (Bardeen *et al.*, 1957). In this study, we are interested in the temperature-dependent energy gap near zero temperature. The BCS gap equation with the anisotropic energy gap has been set up to calculate approximately for near zero-temperature energy gap (Chanilkul and Udomsamuthirun, 2014; Changjan *et al.*, 2019)

$$\Delta_k = \frac{1}{2} \sum_{k'} \frac{V_{kk'} \Delta_{k'}}{\sqrt{E_k^2 + \Delta_{k'}^2}} \tanh \left(\frac{\sqrt{E_k^2 + \Delta_{k'}^2}}{2T} \right) \quad (1)$$

Where $\Delta_k = \Delta(T) f(k)$ is anisotropic energy gap, $V_{kk} = V_0 f(k) f(k')$ denote electron interaction coefficient E_k reveal energy of cooper pair. Let coupling constant $\lambda = N_0 V_0$. For spherical Fermi surface, let the energy gap has the form as

$$\frac{1}{\lambda} = \langle f^2(\theta, \phi) \rangle \cdot \int_0^{\omega_D} \frac{\tanh \left(\frac{\sqrt{E_k^2 + (\Delta(T) f(\theta, \phi))^2}}{2T} \right)}{\sqrt{E_k^2 + (\Delta(T) f(\theta, \phi))^2}} dE_k \quad (2)$$

$$\text{Here } \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi \sin \theta d\theta d\phi f^2(\theta, \phi) = \langle f^2(\theta, \phi) \rangle.$$

Consider temperature- dependent energy gap near zero temperature. Then, we have been obtained the temperature dependent s-wave energy gap with the anisotropic function as

$$\Delta(T) = \Delta(0) \left(1 - \sqrt{\frac{2\pi T}{\Delta(0) f(\theta, \phi)}} \cdot \exp \left(-\frac{\Delta(0) f(\theta, \phi)}{T} \right) \right) \quad (3)$$

Here $\Delta(0)$ is the zero-temperature energy gap. $f(\theta, \phi)$ denote anisotropic function.

As derived from Maxwell's equations and all three spatial components, the superfluid density of a 3D spherical Fermi surface described as (Prozorov and Giannetta, 2006)

$$\rho_{ab} = 1 - \frac{3}{4\pi T} \int_0^1 (1 - z^2) \int_0^{2\pi} \left(\frac{\cos^2(\phi)}{\sin^2(\phi)} \right) \int_0^\infty \cosh^{-2} \left(\frac{\sqrt{\varepsilon^2 + \Delta^2(T, \theta, \phi)}}{2T} \right) d\varepsilon d\phi dz, \quad (4)$$

$$\rho_c = 1 - \frac{3}{2\pi T} \int_0^1 z^2 \int_0^{2\pi} \cos^2(\phi) \int_0^\infty \cosh^{-2} \left(\frac{\sqrt{\varepsilon^2 + \Delta^2(T, \theta, \phi)}}{2T} \right) d\varepsilon d\phi dz, \quad (5)$$

where $z = \cos \theta$, ρ_{ab} and ρ_c are the normalized superfluid density for ab plane and c axis respectively.

To include the effect of the anisotropic Fermi surface, let the energy gap has the form as

$$\Delta(T, \theta, \phi) = \Delta(T) f(\theta, \phi), \quad (6)$$

Where $f(\theta, \phi)$ is the anisotropic function of Fermi surface

Substituting Equation (3) into Equation (6), we get the superfluid density of a 3D spherical Fermi surface of anisotropic superconductor for the ab plane (ρ_{ab}) and c- spatial components (ρ_c) as

$$\rho_{ab} = 1 - \frac{3}{4} \int_0^1 (1 - z^2) dz \sqrt{\frac{8\pi\Delta f(\theta, \phi)}{T}} \cdot e^{-\frac{\Delta(0)f(\theta, \phi)}{T}} \quad (7)$$

and

$$\rho_c = 1 - \frac{3}{2} \int_0^1 z^2 dz \sqrt{\frac{8\pi\Delta f(\theta, \phi)}{T}} \cdot e^{-\frac{\Delta(0)f(\theta, \phi)}{T}} \quad (8)$$

To calculate especially on the shape of Fermi sphere, we use 2 model anisotropic gap functions. First, the spherical ellipse Fermi surface (Hass and Maki, 2001) that was given as $f(\theta) = \frac{1+a'z^2}{1+a'}$ where $z = \cos \theta$ and a' is the elliptical constant parameter that the coordinate is reduced to be dependent only on azimuthally angle. Second, the pancake spherical model (Posazhennikova *et al.*, 2002) that given as $f(\theta, \phi) = \frac{1}{\sqrt{1+a'z^2}}$.

Model I (Spherical Ellipse)

Substituting the spherical ellipse Fermi surface into Equations (7) and (8). Let $\left(\frac{1+a'z^2}{1+a'}\right) = 1 - \left(\frac{a'}{1+a'}\right)(1 - z^2)$ and $A = \frac{\Delta(0)}{T} \cdot \left(\frac{a'}{1+a'}\right)$. The near zero-temperature approximation of the London penetration depth for the ab plane (λ_{ab}) and c- spatial components (λ_c) can be found as

$$\left(\frac{\lambda_{ab}(0)}{\lambda_{ab}(T)}\right)^2 = 1 - \frac{3}{4} \sqrt{\frac{8\pi\Delta(0)}{T}} e^{-\frac{\Delta(0)}{T}} \cdot \left\{ \left(4A^2 \left(1 - \frac{1}{2} \left(\frac{a'}{1+a'} \right) \right) - 2A \left(1 - \left(\frac{a'}{1+a'} \right) \right) - \frac{3}{2} \left(\frac{a'}{1+a'} \right) \right) \right\} \quad (9)$$

and

$$\left(\frac{\lambda_c(0)}{\lambda_c(T)}\right)^2 = 1 - \frac{3}{2} \sqrt{\frac{8\pi\Delta(0)}{T}} e^{-\frac{\Delta(0)}{T}} \cdot \left\{ \left(2A \left(1 - \frac{1}{2} \left(\frac{a'}{1+a'} \right) \right) + \frac{3}{2} \left(\frac{a'}{1+a'} \right) \right) \frac{\sqrt{\pi} \operatorname{erf}(\sqrt{A}) e^A}{8A^2 \sqrt{A}} \right\} \quad (10)$$

Model II (Pancake Spherical)

Substituting the pancake spherical Fermi surface into Equations (7) and (8) in 2 cases, $a' = -a$ and $a' = a$. Let $B = \frac{\Delta(0)}{T} \cdot \left(\frac{a}{2}\right)$. For $a' = -a$, we get the near zero-temperature approximation of the London penetration depth as

$$\left(\frac{\lambda_{ab}(0)}{\lambda_{ab}(T)}\right)^2 = 1 - \frac{3}{4} \sqrt{\frac{8\pi\Delta(0)}{T}} e^{-\frac{\Delta(0)}{T}} \cdot \left\{ \left(4B^2 + \left(\frac{a}{4} - 1 \right) 2B - \left(\frac{3a}{4} \right) \frac{\sqrt{\pi} \operatorname{erf}(\sqrt{B})}{8B^2 \sqrt{B}} + \frac{e^{-B}}{2B} \cdot \left(1 + \frac{3B}{8A} \right) \right) \right\} \quad (11)$$

and

$$\left(\frac{\lambda_c(0)}{\lambda_c(T)}\right)^2 = 1 - \frac{3}{2} \sqrt{\frac{8\pi\Delta(0)}{T}} e^{-\frac{\Delta(0)}{T}} \cdot \left\{ \left(2B + \left(\frac{3a}{4} \right) \frac{\sqrt{\pi} \operatorname{erf}(\sqrt{B})}{8B^2 \sqrt{B}} - \frac{e^{-B}}{2B} \cdot \left(1 + \left(\frac{a}{4} \right) + \left(\frac{3a}{8B} \right) \right) \right) \right\} \quad (12)$$

For $a' = a$, we get

$$\left(\frac{\lambda_{ab}(0)}{\lambda_{ab}(T)}\right)^2 = 1 - \frac{3}{4} \sqrt{\frac{8\pi\Delta(0)}{T}} e^{-\frac{\Delta(0)}{T}} \cdot \left\{ \left(4B^2 + \left(\frac{a}{4} + 1 \right) 2B + \left(\frac{3a}{4} \right) \frac{\sqrt{\pi} \operatorname{erf}(\sqrt{B})}{8B^2 \sqrt{B}} - \frac{e^{-B}}{2B} \cdot \left(1 + \frac{3B}{8A} \right) \right) \right\} \quad (13)$$

and

$$\left(\frac{\lambda_c(0)}{\lambda_c(T)}\right)^2 = 1 - \frac{3}{2} \sqrt{\frac{8\pi\Delta(0)}{T}} e^{-\frac{\Delta(0)}{T}} \cdot \left\{ \left(- \left(2B + \left(\frac{3a}{4} \right) \right) \frac{\sqrt{\pi} \operatorname{erf}(\sqrt{B})}{8B^2 \sqrt{B}} + \frac{e^{-B}}{2B} \cdot \left(1 - \left(\frac{a}{4} \right) + \left(\frac{3a}{8B} \right) \right) \right) \right\} \quad (14)$$

We category the pancake spherical constant parameter (a') into 2 cases for consider the result of calculation in all part, real part and imaginary part.

Results and Discussion

Crystal structure analysis of CaAlSi has been carried out using synchrotron X-rays and neutron diffraction measurements. Two different stackings along the c axis -5-fold and 6-fold structures-with different values of the critical temperature (T_c) 6.0 and 8.2 K respectively are confirmed (Hajime *et al.*, 2006). Prozorov *et al.* (2006) using a tunnel-diode resonator measured the in-plane and out-plane London penetration depths in CaAlSi single crystals

($T_c = 6.2$ and 7.3 K). The gap anisotropy was found to significantly decrease for higher T_c samples (Prozorov and Giannetta, 2006). In this paper, we were interested in the effect of the anisotropic gap function to the temperature dependent London penetration depth of s-wave superconductors. We considered anisotropic gap function from 2 model, ellipse spherical (Hass and Maki, 2001) and pancake spherical (Posazhennikova *et al.*, 2002). The effect of spherical ellipse and pancake spherical Fermi surface on the London penetration depth of CaAlSi I ($T_c = 6.2K, \Delta(0) = 13.02K$) and CaAlSi II ($T_c = 7.3K, \Delta(0) = 15.33K$) (Prozorov and Giannetta, 2006), the graph of $\left(\frac{\lambda_{ab}(0)}{\lambda_{ab}(T)}\right)^2$ and $\left(\frac{\lambda_c(0)}{\lambda_c(T)}\right)^2$ versus reduce temperature $\left(\frac{T}{T_c}\right)$ of CaAlSi I and CaAlSi II were shown in Figures 1, 2, 3, and Figures 4, 5, 6 respectively.

Figure 1 showed the relationship during the London penetration depth by ellipse spherical model (Figure 1(a) ab plan and Figure 1(b) c axis) and reduce temperature for CaAlSi I superconductors. The dash line showed the result of calculation from

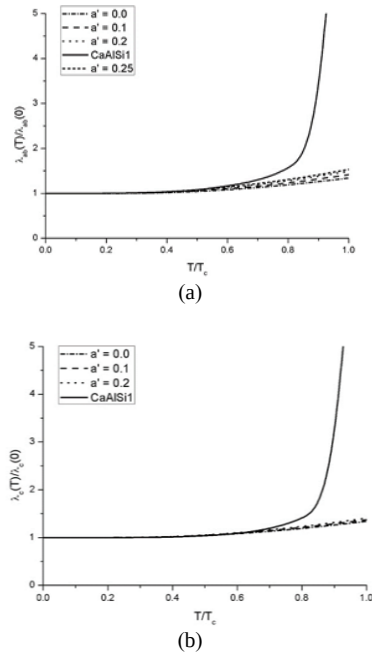


Figure 1. (a) The comparison between results of calculations for ab plane (Equation 9, $T_c = 6.2K$, $\Delta(0) = 13.02K$) by ellipse spherical model (dash line) with experimental data of CaAlSi I (solid line) (Prozorov and Giannetta, 2006). (b) The comparison between results of calculations for c axis (Equation 10, $T_c = 6.2K$, $\Delta(0) = 13.02K$) by ellipse spherical model (dash line) with experimental data of CaAlSi I (solid line) (Prozorov and Giannetta, 2006)

Equation (9) for $\left(\frac{\lambda_{ab}(0)}{\lambda_{ab}(T)}\right)^2$ and Equation (10) for $\left(\frac{\lambda_c(0)}{\lambda_c(T)}\right)^2$ in Figures 1(a) and 1(b) respectively. The solid line showed the experimental data of CaAlSi I (Prozorov and Giannetta, 2006). The effect of the elliptical constant parameter on superfluid density in ab plan and c axis were shown in Figures 1(a) and 1(b). The result of calculation showed the London penetration depth was exponentially increase when the reduce temperature increase. Figures 2 and 3 showed the relationship during the London penetration depth by pancake spherical model (case $a' = -a$, Figure 2(a) ab plan and Figure 2(b) c axis; case $a' = a$, Figure 3(a) ab plan and Figure 3(b) c axis) and reduce temperature $\left(\frac{T}{T_c}\right)$ for CaAlSi I superconductors respectively. In case $a' = -a$, the result of calculations of the London penetration depth for c axis in accordance with the experimental data of CaAlSi I rather than the result of calculations of the London penetration depth for ab plan. In case $a' = a$, the result of calculation showed the London

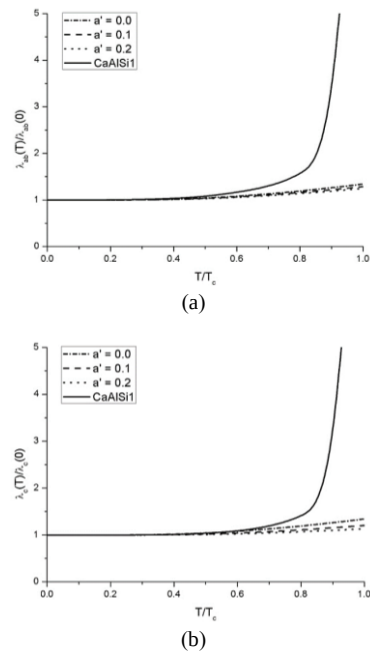


Figure 2. (a) The comparison between results of calculations for ab plane (Equation 11, $a' = -a$, $T_c = 6.2K$, $\Delta(0) = 13.02K$) by pancake spherical model (dash line) with experimental data of CaAlSi I (solid line) (Prozorov and Giannetta, 2006). (b) The comparison between results of calculations for c axis (Equation 12, $a' = -a$, $T_c = 6.2K$, $\Delta(0) = 13.02K$) by pancake spherical model (dash line) with experimental data of CaAlSi I (solid line) (Prozorov and Giannetta, 2006)

penetration depth was exponentially increase when the reduce temperature increase both ab plan and c axis. We could find that our numerical calculation both ab plan and c axis Equations (13) and (14) correspondence with the experimental data of CaAlSi I superconductor (Prozorov and Giannetta, 2006) for low temperature scenery. We found that the ab plan and c-spatial components fit well at $a' = 0.2$ and $a' = 0.0$, respectively.

Figure 4 showed the relationship during the London penetration depth by ellipse spherical model (Figure 4(a) ab plan and Figure 4(b) c axis) and reduce temperature for CaAlSi II superconductors. The solid line showed the experimental data of CaAlSi II (Prozorov and Giannetta, 2006). The result of calculation showed the London penetration depth was exponentially increase when the reduce temperature increase and elliptical constant parameter were 0.25 and 0.35 in ab plan and c axis. Figures 5 and 6 showed the relationship during the London penetration depth by pancake spherical model (case $a' = -a$, Figure 5(a)

ab plan and Figure 5(b) c axis; case $a' = a$, Figure 6(a) ab plan and Figure 6(b) c axis) and reduce temperature $\left(\frac{T}{T_c}\right)$ for CaAlSi II superconductors respectively. From Figure 5(b) , the London penetration depth could be fit well with experimental data in range $0 < \frac{T}{T_c} < 0.6$. In case $a' = a$ (Figures 6(a) and 6(b)) our numerical agreement with the experimental data of CaAlSi II superconductor (Prozorov and Giannetta, 2006) for low temperature scenery. We found that the elliptical constant parameter were 1.5 for ab plan and 0.5 for c axis.

The elliptical constant parameter of CaAlSi I and CaAlSi II were show in Tables 1 and 2 respectively. From Table 1 the London penetration depth of CaAlSi I showed anisotropic properties ($a' \neq 0$) in ab plane and isotropic properties ($a' = 0$) in c axis. Similarly, the elliptical constant parameter of CaAlSi II from Table 2 showed anisotropic properties both in ab plane and c axis.

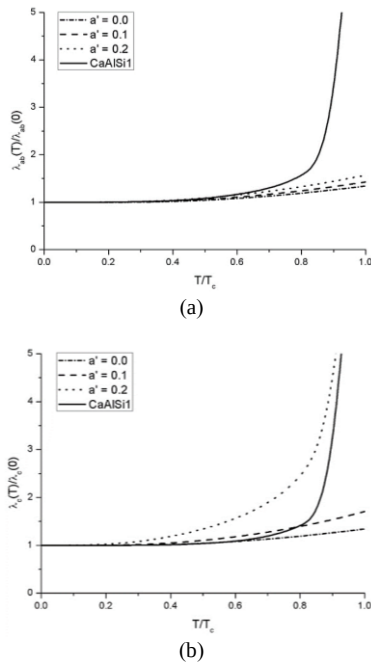


Figure 3. (a) The comparison between results of calculations for ab plane (Equation 13, $a' = a$, $T_c = 6.2K$, $\Delta(0) = 13.02K$) by pancake spherical model (dash line) with experimental data of CaAlSi I (solid line) (Prozorov and Giannetta, 2006). (b) The comparison between results of calculations for c axis (Equation 14, $a' = a$, $T_c = 6.2K$, $\Delta(0) = 13.02K$) by pancake spherical model (dash line) with experimental data of CaAlSi I (solid line) (Prozorov and Giannetta, 2006)

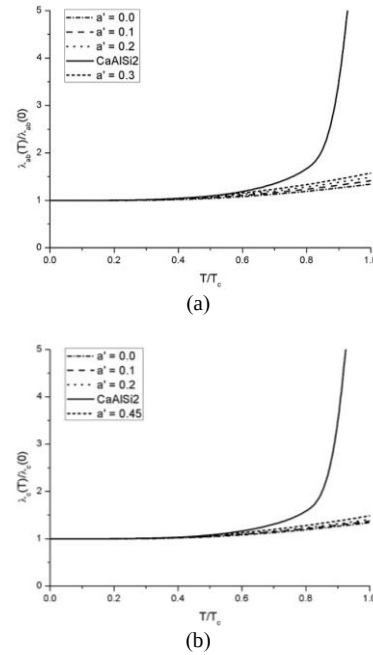


Figure 4. (a) The comparison between results of calculations for ab plane (Equation 9, $T_c = 7.3K$, $\Delta(0) = 15.33K$) by ellipse spherical model (dash line) with experimental data of CaAlSi II (solid line) (Prozorov and Giannetta, 2006). (b) The comparison between results of calculations for c axis (Equation 10, $T_c = 7.3K$, $\Delta(0) = 15.33K$) by ellipse spherical model (dash line) with experimental data of CaAlSi II (solid line) (Prozorov and Giannetta, 2006)

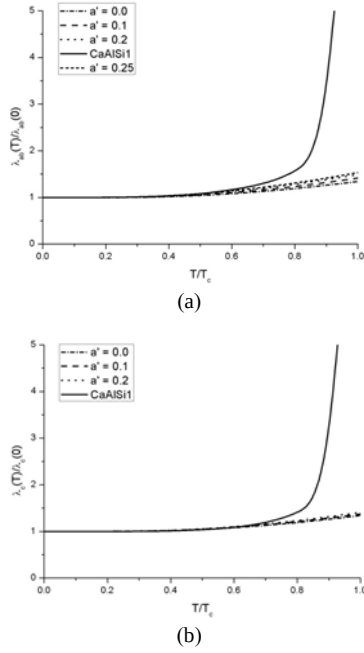


Figure 5. The comparison between results of calculations for ab plane (Equation 11, $a' = -a$, $T_c = 7.3K$, $\Delta(0) = 15.33K$) by pancake spherical model (dash line) with experimental data of CaAlSi II (solid line) (Prozorov and Giannetta, 2006). (b) The comparison between results of calculations for c axis (Equation 12, $a' = -a$, $T_c = 7.3K$, $\Delta(0) = 15.33K$) by pancake spherical model (dash line) with experimental data of CaAlSi II (solid line) (Prozorov and Giannetta, 2006)

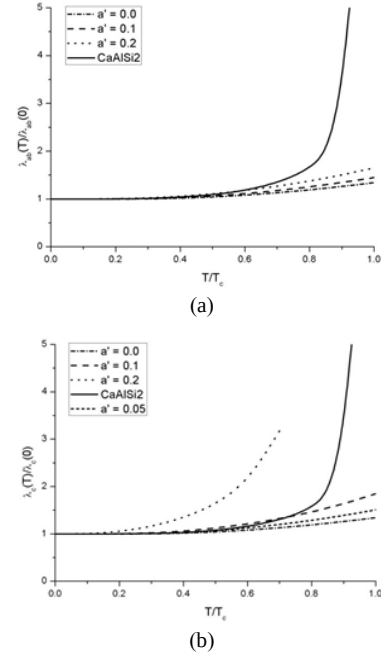


Figure 6. (a) The comparison between results of calculations for ab plane (Equation 13, $a' = a$, $T_c = 7.3K$, $\Delta(0) = 15.33K$) by pancake spherical model (dash line) with experimental data of CaAlSi II (solid line) (Prozorov and Giannetta, 2006). (b) The comparison between results of calculations for c axis (Equation 14, $a' = a$, $T_c = 7.3K$, $\Delta(0) = 15.33K$) by pancake spherical model (dash line) with experimental data of CaAlSi II (solid line) (Prozorov and Giannetta, 2006)

Table 1. Elliptical constant parameter of CaAlSi I

Anisotropic model		*Superfluid density (ρ)		London penetration depth (λ)	
		ab plane	c axis	ab plane	c axis
Ellipse	analytical approach	0.2	0	0.2	0
spherical	numerical approach	0.2	0	0.25	0
Pancake	analytical approach	1.5	0	1.5	0
spherical	numerical approach	0.16	0	0.2	0

* Changjan *et al.* (2019)

Table 2. Elliptical constant parameter of CaAlSi II

Anisotropic model		*Superfluid density (ρ)		London penetration depth (λ)	
		ab plane	c axis	ab plane	c axis
Ellipse	analytical approach	0.2	0.3	0.25	0.35
spherical	numerical approach	0.3	0.4	0.3	0.45
Pancake	analytical approach	1.7	0.4	1.5	0.5
spherical	numerical approach	0.16	0.05	0.2	0.05

* Changjan *et al.* (2019)

Conclusions

The semi-classical theory (Chandrasekhar and Einzel, 1993) was used for investigated the London

penetration depth of anisotropic s-wave superconductors. The London penetration depth was a sensitive tool to study multi-band superconductivity and it has provided several important insights to the behavior of Fe-based superconductors (Prozorov and Kogan, 2011). We

Table 3. Range of reduce temperature $\left(\frac{T}{T_c}\right)$ that our model be fit well with experimental data

Anisotropic model	CaAlSi I		CaAlSi II	
	ab plane	c axis	ab plane	c axis
Ellipse spherical	< 0.4	< 0.6	< 0.3	< 0.4
Pancake	$a' = a$	< 0.3	< 0.3	< 0.3
spherical	$a' = -a$	< 0.3	< 0.4	< 0.6

<0.3 = less than 0.3, <0.4 =less than 0.4, <0.6 =less than 0.6

derived the temperature dependent of the London penetration depth for the ab plane and c axis analytically and applied to CaAlSi I and CaAlSi II superconductors (Prozorov and Giannetta, 2006). Our model was included the spin-singlet superconducting state with the anisotropic gap function; ellipse spherical (Hass and Maki, 2001) and pancake spherical (Posazhennikova *et al.*, 2002) Fermi surface. For pancake spherical model, we category the pancake spherical constant parameter into 2 cases for consider the result of calculation in all part; real part and imaginary part (Changjan *et al.*, 2019). For CaAlSi I ($T_c = 6.2K$, $\Delta_0 = 13.02K$) and CaAlSi II ($T_c = 7.3K$, $\Delta_0 = 15.33K$) superconductor, our model accordingly with the experimental data for low temperature scenario can be presented by Table 3. CaAlSi-I showed anisotropic and isotropic properties in ab plane and c axis and CaAlSi-II showed anisotropic properties both in- plane and out-plane. It demonstrated that CaAlSi show the properties of anisotropic one band S-wave superconductors at low temperature scenery.

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