

APPROXIMATION AND ANALYSIS FOR FIR BASED MULTIBAND PASS INTEGRATOR FOR FREQUENCY ω_m ; $0 < \omega_m < \pi$

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Abstract

A novel efficient approximation for linear phase FIR based Multiband Pass Integrator (MPI) of ideal magnitude response, $1/(j\omega)$ as well as for $|1/\omega|$, have been proposed without using fractional delays. The suggested approximations yield highly accurate response over a band of frequencies centered around different ω_m simultaneously. The points of maximal flatness ω_m can be chosen arbitrarily in the frequency range $0 < \omega_m < \pi$. As compared to previous single band integrators, relative error in Multiband Pass Integrator is reduced to a large extent. For $H_{n,m}(\omega)$ relative error observed is in the range of [2, 36]% and for $F_{n,m}(\omega)$ is [4.363, 30.636]%. Respective graphs for errors, are thus obtained for both $1/(j\omega)$ and for $|1/\omega|$.

Keywords: Finite impulse response, infinite impulse response, digital integrator, multiband pass integrator, relative error

Introduction

Single band integrators have many applications for summation of values defined over a given interval of time as in radars, sonar, vehicles etc. These are designed for a given frequency, the one at which the integrator operates (Kumar *et al.*, 1996). Many approximations have been done in relation to the single band (Al-Alaoui, 1993; Ngo, 2006), and involving Z transform. (Kumar and Kumar, 1999) gave the integrator for single order as well as for higher orders. Second order integrator by (Al-Alaoui, 1995) using interpolating Trapezoidal and Simpson integrator which works on widening of the desired frequency range for linear and constant

inputs for the given Nyquist interval. Using Simpson integration rule along with fractional delay filter (Tseng, 2006) found error less than Conventional Simpson integrator. Also based on Newton Cotes formula (Ngo, 2006), implemented a third order integrator approximation over the Nyquist interval. However, no such work, related to Multiband Pass Integrator has been carried out so far to best of the author's knowledge.

With the evolving demand for speed and accuracy, technology is moving towards the employment of multiband integrators which are not only cost effective but can also be employed to avail

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different services related to different frequencies at the same time with a hardware which is as compact as it is effective. Also, the error of multiband pass integrator, as we will observe later in this paper, will be less than different single band integrators employed for given set of frequencies. Taking inverse of these integrators will lead to the differentiator (Proakis and Manolakis, 2007; Mitra, 2011). Multiband Pass Integrators can be made using several methods such as- the conjunction of different single integrators working at different frequencies connected in parallel. The inclusion of multiplexer in the circuit is another viable option. Such digital MPIs are multifunctional. Approximation of the ideal integrator done by (Kumar and Kumar, 1997) and others (Minocha *et al.*, 1993; Luengo *et al.*, 2000; Deng, 2004; Upadhyay and Singh, 2011) make use of the Z transform and then the equation of the ideal integrator has been split into sine and cosine function for degree 1. However, for $\omega = 0$ and $\omega = \pi$, the above approximation is zero. Al-Alaoui (2011) extended his work on integrators and differentiators by using Newton-Cotes rules of numerical integration and gave very close approximation with the ideal one. Jain *et al.* (2013) ascending in IIR integrators, approximated first using minimax optimization technique and improved the response further using constant optimization and pole zero methods.

Priyanka and Upadhyay (2015) combined both the integrator and differentiator for the same frequency band using the Genetic Algorithm but only for second-order within Nyquist Range. Nair *et al.* (2016) worked to mimic the ideal integrator using the Cuckoo search algorithm focused upon getting the ideal DC frequency response for second-order within the Nyquist range. Many other literatures are there in which work has been done in Fractional Order Integrator, those having integral and derivative of orders with non-integer value. Mahata *et al.* (2016) proposed low and fractional-order IIR integrators for the same range within 0.2π and π , having Swarn intelligence dependent optimization algorithm. It generally involves mapping function for performing discretization and most of the coefficients used are dependent upon the sampling frequency. Kansal and Upadhyay (2017) working in the same league, developed integrator with the Genetic Algorithm and Binomial Expansion for matching the response with the Ideal integrator.

Ling *et al.* (2017) working for the IIR integrator made error related to phase is restricted within the constraint of magnitude relative error. Yuan (2017) designed an integrator for the VLSI application used for time register and adder circuits using gated delay lines, dual discharge paths, etc.

Busarello *et al.* (2019) worked on the experimental part of the integrator and obtained output for second-order integrator subjected to phase-locked loop. Similarly, Shan and Ren (2019) got experimental results for integrator comb filter, cascaded form under different modulation schemes. Previous integrators as discussed above are focused on one frequency band for a particular instance of operation. We in this paper will be proposing an efficient designs of FIR based linear Multiband pass integrator operable at separate multiple bands centered around at different frequencies, without using fractional delays.

In this paper, Section 2 depicts the methodology involved to designed the proposed Multiband pass integrator by determining the weights of the integrator at the respective frequencies. Both the equations of sine and cosine functions are taken into the consideration. Section 3 plots and discusses the magnitude response of the both $H_{n,m}(\omega)$ and for $F_{n,m}(\omega)$ for different lengths of the integrator. Section 4 involves the performance comparison the proposed Multiband Pass integrator with the existing integrators in terms of magnitude response and relative error. Section 5 concludes the paper.

Proposed Integrator

The frequency response for Ideal integrator, given in Equation (1). The same can be spilt using Z transform to obtain Equations (2) and (3).

$$H(\omega) = \frac{1}{D(\omega)} \quad (1)$$

(Kumar and Dutta Roy, 1989) designed and gave the equation for ideal differentiator, $D(\omega)$; the inverse of which is the integrator (Al-Alaoui, 2007). Transformation of the ideal function of integrator implementing Z transform gives us the comparative equations in terms of sine and cosine for the Multiband Pass Integrator.

$$H_{n,m}(\omega) = \sum_{i=1}^n w_{1,i}^{(n)} \sin i \omega, n = \frac{N-1}{2} \quad (2)$$

$$F_{n,m}(\omega) = \sum_{i=1}^n w_{2,i}^{(n)} \cos i \omega, n = \frac{N-1}{2} \quad (3)$$

The above equations can be further used for the calculation of weights $w_1^{(n)}$ and $w_2^{(n)}$. Value of $\omega_m = \frac{\pi}{2}$ is taken for the calculation of weights in the Butterworth sense. N depicts the length of the filter; the weights calculated are different for different values of N . Many algorithms to control the phase using fractional delay (Al-Alaoui, 1995; Johansson, 2013), Simpson Rule (Tseng, 2006) are there. For N and even integer, the realization of the integrator requires a fractional delay (a half delay of $z^{-\frac{1}{2}}$)

which is not acceptable in many digital systems. However, on the other hand, choosing an odd n ensures non-fractional delay in the realization. Hence we will be designing a linear FIR multiband pass integrator by using an odd order n , where n is calculated as given in Equation (2). Here both the imaginary and the real one, without the fractional delay, are approximated. There is no commonality between the weights of designs having two different orders. It implies that whenever the filter length N changes, all the weights have to be changed necessarily so as to maintain the optimality criteria. For Multiband Pass Integrator, Equation (2) will be differentiated for all the desired frequencies, keeping n constant.

The subscript m , of H and F in aforementioned Equations (2) and (3) represents multiband integrator. The j factor in the sine part ensures the constant phase delay of $\frac{\pi}{2}$. As mentioned, weights are independent of the differing orders of the filter used hence, n is kept constant for multiple bands at the instance when work is done upon different frequencies, as represented graphically in the results. Proceeding in a similar manner (Kumar and Choudhary, 1996) the coefficients or so called weights can be calculated using the recursive formula for the range of $\omega = 0$ to $\omega = \pi$, excluding 0 and π . Taking any frequency between the interval 0.1π to 0.9π , for instance 0.5π , and then deriving the Equations (2) and (3) will give the resulting weight equation which is recursive and dependent upon the previous values of $H_{n,m}(\omega)$ and $F_{n,m}(\omega)$. The values thus derived are

$$F_n^{(t)}(\omega)|_{\omega=\omega_m} = (-1)^t t! \left(\frac{1}{\omega_m}\right)^{t+1}, t = 0, 1, 2, \dots, n \quad (4)$$

$$F_{n+1}^{(t)}(\omega)|_{\omega=\omega_m} = (-1)^t t! \left(\frac{1}{\omega_m}\right)^{t+1}, t = 0, 1, 2, \dots, n+1 \quad (5)$$

$$F_{n+2}^{(t)}(\omega)|_{\omega=\omega_m} = (-1)^t t! \left(\frac{1}{\omega_m}\right)^{t+1}, t = 0, 1, 2, \dots, n+2 \quad (6)$$

The constant C_m , obtained after differentiation, is thus calculated by using simple manipulations and trigonometric identities to yield an equation which is dependent on the previous two values of $F_{n,m}(\omega)$. For this the first two values of $F_{n,m}(\omega)$ have been calculated manually and later the program will further calculate the values of $F_{n,m}(\omega)$ upto n . The value of C_m is given by Equation (7)

$$C_m = \left[\frac{-1}{(n+1) \sin \omega_m} \right] \frac{(-1)^{n+1} n+1! \left(\frac{1}{\omega_m}\right)^{n+2} - F_{n+1}^{(n+1)}(\omega_m)}{(-1)^n n! \left(\frac{1}{\omega_m}\right)^{n+1} - F_n^{(n)}(\omega_m)}, n = 2, 3, 4, \dots \quad (7)$$

Calculation of the weights after the differentiation is manipulated to be dependent upon

the main function of $H_{n,m}(\omega)$ and similarly the weights in Equation (2) upon $F_{n,m}(\omega)$,

$$w_i^{(n+2)} = w_i^{(n+1)} + \frac{\bar{B}_m}{2} [(w_{i+1}^{(n+1)} - 2 \cos \omega_0 w_i^{(n+1)} + w_{i-1}^{(n+1)}) - (w_{i-1}^{(n)} - 2 \cos \omega_0 w_i^{(n)} + w_{i+1}^{(n)})] \quad i = 1, 2, \dots, n+1 \quad (7)$$

Here, $w_k^{(r)}$ has been assumed to be 0, for $k = 0$ or for $k > r$. To begin recursion, $w_1^{(1)}$ and $w_1^{(2)}$ are calculated, same is done for $w_2^{(2)}$. Progressively, the values of $w_i^{(n)}$ for the approximation $H_{n,m}(\omega)$ have been computed by using mathematical formula as shown in Equation (7). This procedure holds good for any point of maximal flatness ω_0 , except for $\omega_m = 0$ or π . Magnitude response related to different values of n , for both $H_{n,m}(\omega)$ and for $F_{n,m}(\omega)$ are plotted and shown in Figure 1 through Figure 8.

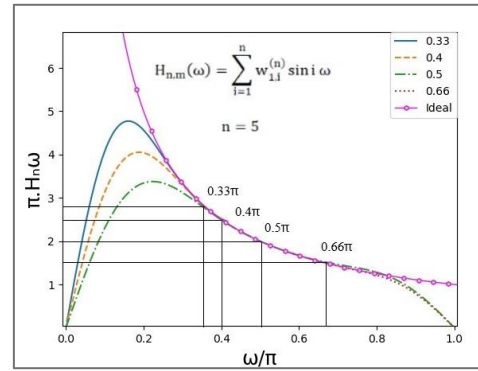


Figure 1. $H_{n,m}(\omega)$ with $n = 5$ and $\omega_m = 0.33\pi, 0.4\pi, 0.5\pi$, and 0.66π

Results and Observations

The Multiband Pass Integrator thus designed is maximally flat at desired frequencies, i.e. at $\omega_m = 0.33\pi, 0.4\pi, 0.5\pi$, and 0.66π , meaning that it gives approximately the same response as an ideal integrator for the specified frequencies at a particular time. Such a digital integrator is capable of performing multifold operations for any given instance and is operational at different bandwidths at any single moment. Authors are working to include the multiband for the extended range i.e. beyond π .

Figure 1 gives the graphical representation of magnitude response, $H_{n,m}(\omega)$ plotted along the Ideal response of the integrator, represented by the blue dotted line. It is clear from Figure 1 that the

designed MPI is flat or we can say is overlapping for the desired frequencies, i.e for 0.33π , 0.4π , 0.5π , and 0.66π . The desired frequencies are indicated with the help of black lines, whose coordinates on y axis are representing the overlapping selected frequencies. As we are increasing the value of n , which in turn related to N (length of integrator), the amplitude of the integrator is also increasing. Therefore, it is a challenge to control the error for higher value of n , or for higher degree of integrator, $H_{n,m}(\omega)$.

Figure 2 compares the magnitude response of the approximated Multiband Pass integrator $H_{n,m}(\omega)$ with n being 17 and giving flat response at

desired frequency, ω_m , 0.33π , 0.4π , 0.5π , and 0.66π . the response so obtained is compared with the magnitude response of the ideal integrator. For all the four frequencies the response is flat and is overlapping with the response of the ideal integrator. In Figure 3 the length of the integrator, n , is taken as 23 and for Figure 4 the length of the integrator is taken as 31. The frequency of interest, i.e $\omega_m = 0.33\pi$, 0.4π , 0.5π , and 0.66π , is considered. Next, in Figure 5, the plot of $F_{n,m}(\omega)$, the integrator without the phase delay is plotted with the magnitude response of the Ideal Integrator. The length of the approximated integrator is taken as $n = 5$ with the overlapping frequencies being taken

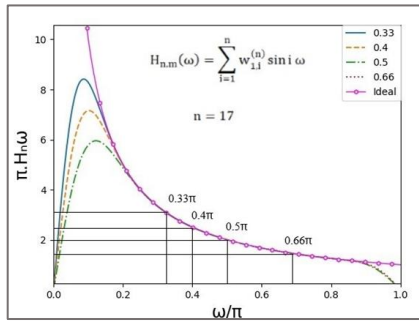


Figure 2. $H_{n,m}(\omega)$ with $n = 17$ at $\omega_m = 0.33\pi$, 0.4π , 0.5π , and 0.66π

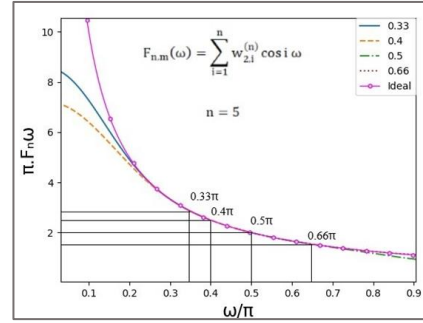


Figure 5. $F_{n,m}(\omega)$ with $n = 5$ and $\omega_m = 0.33\pi$, 0.4π , 0.5π , and 0.66π

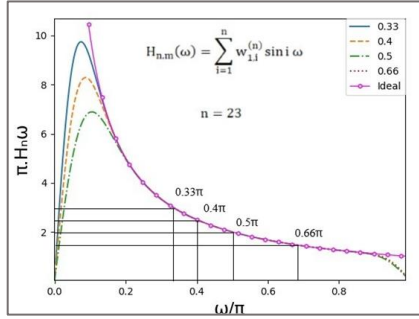


Figure 3. $H_{n,m}(\omega)$ with $n = 23$ and $\omega_m = 0.33\pi$, 0.4π , 0.5π , and 0.66π

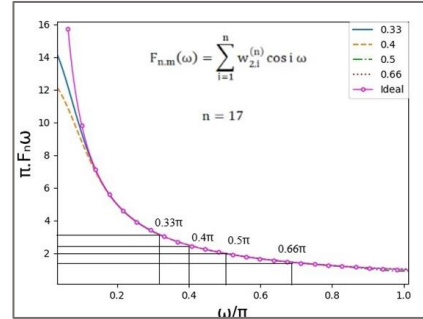


Figure 6. $F_{n,m}(\omega)$ with $n = 17$ and $\omega_m = 0.33\pi$, 0.4π , 0.5π , and 0.66π

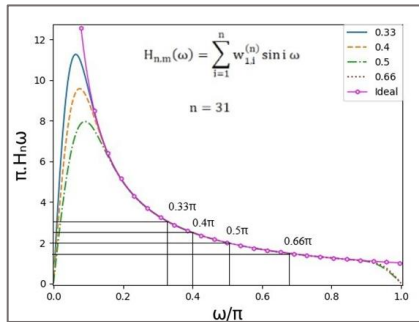


Figure 4. $H_{n,m}(\omega)$ with $n = 31$ at $\omega_m = 0.33\pi$, 0.4π , 0.5π , and 0.66π

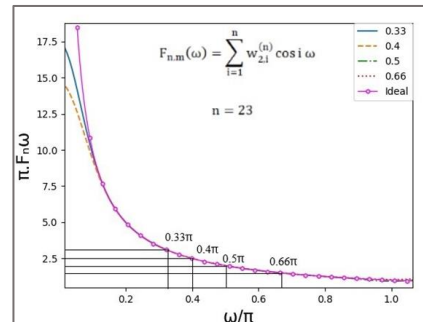


Figure 7. $F_{n,m}(\omega)$ with $n = 23$ and $\omega_m = 0.33\pi$, 0.4π , 0.5π , and 0.66π

as 0.33π , 0.4π , 0.5π , and 0.66π . Figure 6 represents the plot of approximate Multiband Pass integrator with the length of the integrator as $n = 17$ and ω_m being 0.33π , 0.4π , 0.5π , and 0.66π . Figures 7 and 8 depicts the magnitude response of the approximated multiband pass integrator $F_{n,m}(\omega)$ with $n = 23$ and 31.

The Relative error of approximated function is calculated by using the formula given below

$$E_R = \frac{F_{n,m}(\omega) - F_i(\omega)}{F_i(\omega)} * 100 \quad (8)$$

where $F_{n,m}(\omega)$ the value of multiband integrator is arrived at by approximation and $F_{n,m}(\omega)$ is the required value of Ideal Multiband Integrator. The graphs, in Figures 9 and 10 represent the error calculated for various value of n for different ω both for $H_{n,m}(\omega)$ and $F_{n,m}(\omega)$. The error thus calculated reveals that the error range for the value $H_{n,m}(\omega)$ is minimum at 2% and maximum at 36%; this range is quite small as compared to previous values of single band integrator. Similarly, for the approximated $F_{n,m}(\omega)$ the minimum error is at 4.363% and maximum is at 30.636%. As seen from the graph in Figure 9, the error is minimum for the value of $\omega_m = 0.6\pi$, being minimum for $n = 5$ and increasing steadily for $n = 31$, highest value being 28.396%.

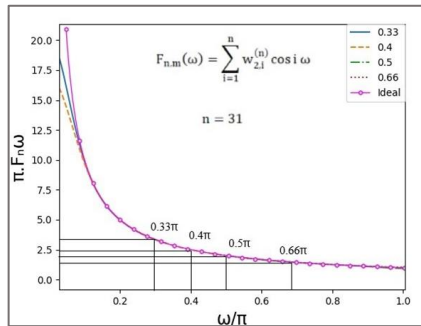


Figure 8. $F_{n,m}(\omega)$ with $n = 31$ and $\omega_m = 0.33\pi$, 0.4π , 0.5π , and 0.66π

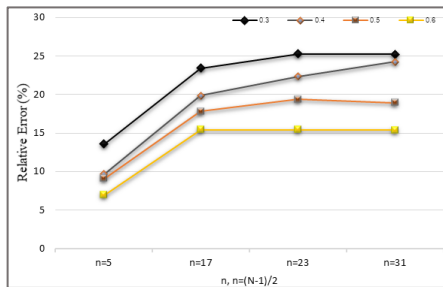


Figure 9. Relative Error for $H_{n,m}(\omega)$ at $\omega_m = 0.33\pi$, 0.4π , 0.5π , and 0.66π for $n = 5, 17, 23$, and 31

The trend can be seen clearly as we are moving more towards 0 or π in terms of frequency, error will be maximum. Reason for this is understandable, since the approximation is done using sine and cosine function, as soon as wave spikes will be encountered the error will be highest. Also, one important point to be noted is that the higher we go for the value of n , from 5 till 31, the value of relative error is also increasing. This is clear from the fact that n is directly proportional to the amplitude of the sine and cosine, increasing n , requires more area to be flat for the desired frequencies, hence higher the error. One important observation from the relative error graph is that on increasing the value of n , E_R is increasing proportionally but after a certain point, which is $n = 23$ in Figures 9 and 10, the E_R reaches to saturation. It can be seen clearly from both E_R graphs that first a linear rise is there but after a certain point E_R is constant.

Performance Comparison

In terms of magnitude response and relative error, a performance comparison of the proposed integrator with the existing integrator has been done. Figure 11 compare the magnitude response of the proposed integrator with existing integrators along with the magnitude response of the Ideal integrator. The comparison is done for the value of first 50

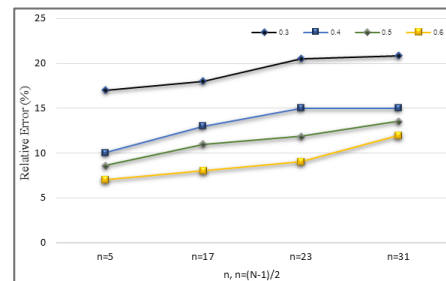


Figure 10. Relative Error for $F_{n,m}(\omega)$ at $\omega_m = 0.3\pi$, 0.4π , 0.5π , and 0.66π for $n = 5, 17, 23$, and 31

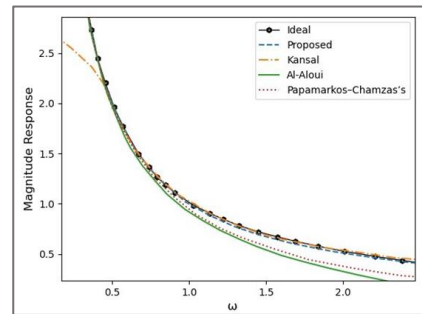


Figure 11. Magnitude Comparison of $F_n(\omega)$ with existing integrators

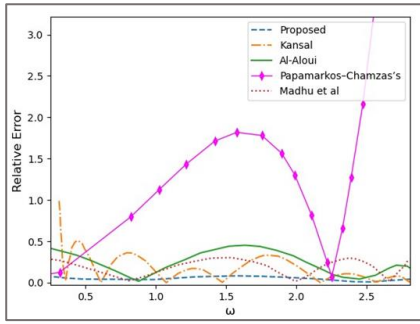


Figure 12. Relative Error comparison of $F_n(\omega)$ with existing integrators

coefficients of $F_n(\omega)$. It can be seen that for the range specified of normalised frequency, ω , proposed integrator imitate closely the ideal integrator throughout. The integrator designed by Kansal and Upadhyay (2017) also approximated the ideal integrator closely for the range greater than 0.5 radians. The integrator designed by Al-Alaoui (2011) fits the curve of ideal integrator for the value of ω less than 0.7 radians. Papamarkos and Chamzas, (1996) integrator is also close to ideal integrator response for the range $0.2 < \omega < 0.7$. However as the ω increases it delineates from the ideal integrator response.

In Figure 12, the relative error of the proposed integrator observed is compared with the previous integrators, the relative error E_R is calculated using the formula as mentioned in Equation (8). The proposed integrator gives the E_R for first 50 values of $F_n(\omega)$ within the range 0.03 to 0.23 which is the least when compared to previous integrators. The Relative error of integrator given by Kansal and Upadhyay (2017) is having the highest value of 1. Also the integrator designed by Al-Alaoui (2011) is having the relative error between 0.47 and 0.012 for $0 < \omega < 0.9$ radians, 0.5 to 0.18 for $0.9 < \omega < 2.4$ radians. Papamarkos and Chamzas, (1996) integrator is having the highest relative error. For value of ω greater than 2.3 radians, the relative error is higher than 3, and for 0 to 2.3 radians, the relative error is in the range 0.18 to 2. Jain *et al.* (2013) proposed an integrator which gives relative error in the range of 0.01 to 0.3 for $0.1 < \omega < 0.9$ radians and 0.4 to 0.01 for $0.9 < \omega < 1.9$ radians and so on. Hence it is evident from the relative error representation that the proposed integrator possesses minimum relative error and best matched magnitude response with ideal integrator. The suggested integrators yield highly accurate frequency response, particularly in the midband frequency regions from 0.1π to 0.9π radians.

Such type of integrators, as approximated in this paper, are useful for multiple operations at the

same time. These types of integrators reduce the time of operation, cost of hardware as they are working at multiple bands simultaneously and are hence are best suited for real time applications. The above approximated integrators are highly suitable for applications requiring fast calculations like for freezing in an IC, storing the weights vectors, and providing various output taps to simulate approximation of various lower order filters. Another practical implementation of these integrators can be in situations where the equiripple (Chebyshev type) approximations are generally preferred. These designs are accomplished by optimization techniques using iterations which are time-consuming and non-analytical. In such applications, our approximation will reduce the complexity by giving the exact mathematical formula that will help in designing the integrator required for equiripple passband of frequencies. The disadvantage of the proposed integrator although, is that, the suggested approximations for $H(\omega) = 1/(j\omega)$ meant to yield both optimum amplitude response and a constant phase of $\pi/2$ radians (i.e. a factor j) use $\sin \omega$ series. Such approximations are identically zero at $\omega = 0$ and $\omega = \pi$. It should be feasible to obtain approximations which are non-zero at $\omega = 0, \pi$ if we allow a fractional delay (half delay : $z^{-1/2}$) in the structure. Since after the Nyquist range there is a rise in relative error because of the approximation done using sine and cosine functions, hence this divergence from the ideal integrator response should be optimized further and is the area to be worked upon. Authors are working to develop novel FIR integrator for working smoothly at the edges of the band.

Conclusions

The FIR based linear multiband pass integrator which can operate at multiple frequencies is approximated, for both $F_{n,m}(\omega)$, and $H_{n,m}(\omega)$, where $F_{n,m}(\omega)$ is the real part of the obtained equation after Z-Transform. Highly accurate response over a band of frequencies centered around different ω_m , simultaneously has been achieved without using fractional delays. The operating range of frequency ($\omega_m = \omega_1, \omega_2, \omega_3, \omega_4 \dots$) can be chosen arbitrarily in the frequency range, $0 < \omega_m < \pi$. The integrator is operational for any frequency lying within the specified range, excluding 0 and π . Relative error in Multiband Pass Integrator is reduced, i.e. for $H_{n,m}(\omega)$ range is [2, 36]% and for $F_{n,m}(\omega)$ is [4.363, 30.636]%, for specified frequencies in comparison to single band integrator.

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