

FINITE ELEMENT METHOD FOR NATURAL CONVECTION PROBLEM INSIDE PARABOLIC CROSS-SECTION ENCLASURE

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Abstract

Numerical solution procedure for natural convection in parabolic cross-section enclosure is proposed in this study. The governing equations in term of finite element equations are solved by using iterative method. According to solution procedure is performed on computational domain, transformations $X = \xi + (\eta/4)(1-2\xi)$ and $Y = (\eta/4) + (1-\eta)(\xi - 1/2)^2$ are defined to transform solutions back to physical domain which is original problem. Numerical tests involving natural convection heat transfer are carried out to display the results in forms of flow circulation and temperature distribution. Two cases of temperature boundary conditions are considered. First, the top of the enclosure is cooled and the parabolic curve is uniformly heated. Second, the top of the enclosure is adiabatic and the parabolic curve is nonuniformly heated. The interested parameter is Rayleigh number ($5 \times 10^4 \leq Ra \leq 1 \times 10^6$) with constant Darcy number ($Da = 10^{-2}$) and Prandtl number ($Pr = 0.71$). The solution for natural convection problem shows that the increase in Ra results in stronger flow intensity and affects the characteristic of temperature distribution indicating the change from conduction mode to convection mode. The variation of heat transfer along heated curve for case 1 is maximum at the left and the right sides and becomes constant at the middle portion for $Ra = 5 \times 10^4$ and $Ra = 10^5$ while it is slightly increased near the center for higher Ra . For case 2, heat transfer is increased along the curve and symmetric with respect to vertical center line. The maximum heat transfer occurs for two points when $Ra = 5 \times 10^5$ and $Ra = 10^6$. Graphs of relative error values are also plotted to show the convergence of numerical solution obtained from the proposed method. From the numerical results, it is demonstrated that the proposed method is efficient and suitable for natural convection problem in the enclosure with parabolic curve.

Keywords: Finite element method, natural convection, parabolic enclosure, coordinate transformation

Introduction

There are many studies of natural convection problem in an enclosure due to the applications associated with heat transfer process. The objective of these studies is to propose numerical method for

solving the problem. The numerical results are often displayed in the forms of streamlines and isotherms used to explain the effects of interested parameters on characteristics of flow and temperature

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distribution inside the enclosure. Nusselt number is also presented to illustrate heat transfer rate along heated boundary. Natural convection in regular geometries such as square, rectangle or triangle can be found in early work (Baytas and Pop, 2002; Corcione, 2003; Roy and Basak, 2005; Basak *et al.*, 2006a, 2006b; Basak *et al.*, 2007; Koca *et al.*, 2007; Das and Basak, 2016). In addition, the effects of various shapes on natural convection have been studied. Das *et al.* (2016) investigated the natural convection heat transfer in porous enclosure with various shapes, square, rhombus, concave and convex by using Galerkin finite element method. Das *et al.* (2018) analysed the entropy generation in square and triangular enclosures involving the different positions of heaters along the side walls. Triangular enclosure is designed for 2 types, regular isosceles triangle and inverted isosceles triangle. The numerical solution were obtained by employing finite element method. The combination of finite element and finite difference methods for viscoelastic flow can be found in Rasheed and Anwar, 2018a, 2018b. Dutta *et al.* (2019) used finite element method to studied magnetohydrodynamic natural convection heat transfer and entropy generation in rhombic enclosure filled with Cu-water nanofluids.

The complexity of natural convection problem is increased when the geometry is irregular. Das and Morsi (2001) analyzed heat transfer and free convective motion in dome shaped enclosure represented by using a general conic equation. The solution method is based on finite element technique. Oztop (2005) examined heat transfer and fluid flow in shaped duct geometries by using SIMPLE algorithm with non-staggered grid arrangement. Elliptic grid generation method was used to obtain grid points in curvilinear surfaces. Chen and Cheng (2011) investigated the steady laminar buoyancy-driven and convection heat transfer characteristics for different across-shape concave enclosures in terms of curvilinear coordinates using finite difference method. The flow behavior and heat transfer characteristics of fluid contained in an inclined arc-shaped enclosure were studied by Chen and Cheng (2012). Numerical grid generation method was applied to obtain coordinate transformation function and finite volume method was used to solve the governing equations. Oztop *et al.* (2011) studied natural convection in wavy-walled enclosure with volumetric heat sources. An algebraic coordinate transformation was used to transform the wavy enclosure (or physical domain) into a rectangular geometry (or computational domain). The governing equations were solved by finite volume method. Rahman *et al.* (2012) used Galerkin finite element method to

analyze the effects of buoyancy ratio on double diffusive natural convection in a triangular cavity with zig-zag shaped bottom wall. The solution procedure was performed in triangle such that grid distribution was used near the boundaries. Natural convection in rectangular enclosure with sinusoidal corrugated top surface was investigated numerically by Hasan *et al.* (2012). The numerical results were obtained by using finite element method adapted to triangular non-uniform mesh element. Natural convection heat transfer process inside two-dimensional quarter circular cavity with two sinusoidal wavy walls and two straight wall was investigated by Tang *et al.* (2017). Finite volume and respond surface methods were applied to study the effects of phased difference between the inner and outer wall for finding optimal cavity shape. Morsli *et al.* (2017) was interested in the influence of aspect ratio on the natural convection in rectangular enclosure with wavy wall and finite volume method was used to solve the governing equations. Hussein and Mustafa (2018) investigated the natural convection in concave and convex parabolic enclosure placed an internal heat source. The body-fitted coordinates system was utilized to transfer parabolic enclosure to rectangle and numerical investigation was carried out by using finite volume method. Roy (2019) applied finite difference method for the flow characteristics due to natural convection induced by exothermic reaction in an annulus bounded by wavy wall cylinders. Masoumi *et al.* (2019) dealt with the laminar natural convection inside annular spaces between concentric cylinders. Finite element method with a non-uniform unstructured triangular grid was applied to solve the governing equations. From the literatures, the effective numerical techniques were applied to solve the natural convection problem inside various shapes of the enclosures for the predictions of flow behaviour and temperature distribution. One of those techniques is finite element method. Finite element method has been widely used because it can be modelled the various shapes which are complex or irregular. The domain is discretized into elements and the governing equations of each element are formulated. These finite element equations are assembled to form the global equations representing the original shape of the problem and boundary conditions are then applied prior to solving for the solutions.

The main purpose of this study is to present numerical solution procedure based on finite element and iterative methods to solve natural convection problem inside the enclosure which is cross-section of paraboloid. Mathematical formulations, solution procedure and numerical results derived from the proposed method are presented in the next section.

Materials and Methods

Governing Equations

The governing equations for steady two-dimensional natural convection flow in a porous enclosure in term of dimensionless are used in this study and can be expressed by equations of conservation of mass, conservation of momentum and conservation of energy as follows (Basak *et al.*, 2006b),

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \text{Pr} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - \frac{\text{Pr}}{Da} U$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \text{Pr} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \frac{\text{Pr}}{Da} V + Ra \text{Pr} \theta \quad (2)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \quad (3)$$

where X and Y are dimensionless distance in horizontal and vertical directions respectively, U and V are dimensionless velocity components in X and Y directions, respectively, P is dimensionless pressure, θ is dimensionless temperature, and Pr , Da , and Ra are Prandtl, Darcy and Rayleigh numbers, respectively.

Substitution of P such that $P = -\gamma (\partial U / \partial X + \partial V / \partial Y)$ and $\gamma = 10^7$ introduced in Reddy (1993) into Equation (2) yields

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \gamma \frac{\partial}{\partial X} \left(\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right) + \text{Pr} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - \frac{\text{Pr}}{Da} U,$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = \gamma \frac{\partial}{\partial Y} \left(\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right) + \text{Pr} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \frac{\text{Pr}}{Da} V + Ra \text{Pr} \theta. \quad (4)$$

In this study, the velocities in X and Y directions are $U = 0$ and $V = 0$ for all boundaries because of no slip condition.

The fluid motion inside the enclosure is displayed by using the relationship between stream function and velocity components defined as $U = \partial \psi / \partial Y$ and $V = \partial \psi / \partial X$ (Batchelor, 2000). To derive stream function, U and V are differentiated. Then, $\partial U / \partial Y = \partial^2 \psi / \partial X^2$ and $\partial V / \partial X = -\partial^2 \psi / \partial Y^2$ are obtained. By subtraction of these two equations, the stream function used to study flow characteristic is

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = \frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X} \quad (5)$$

The boundary conditions $\psi = 0$ for all boundaries.

The local and average Nusselt numbers applied to measure heat transfer rate along heated boundary are defined by (6) and (7), respectively.

$$Nu = -\sum_{i=1}^9 \theta_i \frac{\partial \phi_i}{\partial Y} \quad (6)$$

$$\overline{Nu} = \int_0^1 Nu dx \quad (7)$$

Finite Element Equations

A unit square or computational domain in $\xi\eta$ -plane is considered to define shape functions. It is divided into equal subdomains, called element, as shown in Figure 1(a). By applying the quadratic Lagrange interpolation to define shape functions with nine nodes for each element, shape functions ϕ_i for $i = 1, 2, \dots, 9$ can be obtained by the product $L(\xi_j)L(\eta_k)$, for $j, k = 1, 2, 3$ such that

$$L(\xi_j) = \prod_{m=1, m \neq j}^3 \frac{(\xi - \xi_m)}{(\xi_j - \xi_m)}$$

$$L(\eta_k) = \prod_{m=1, m \neq k}^3 \frac{(\eta - \eta_m)}{(\eta_k - \eta_m)}$$

These shape functions are satisfied the property that $\phi_i = 1$ at node i , otherwise $\phi_i = 0$. Due to biquadratic shape function, the elements called biquadratic element are used and node numbering is given as shown in Figure 1(b). In addition, there are unknown variables U_i , V_i , θ_i and ψ_i for $i = 1, 2, \dots, 9$ at the nodes of each element approximated by

$$U = \sum_{i=1}^9 \phi_i(\xi, \eta) U_i,$$

$$V = \sum_{i=1}^9 \phi_i(\xi, \eta) V_i,$$

$$\theta = \sum_{i=1}^9 \phi_i(\xi, \eta) \theta_i,$$

$$\psi = \sum_{i=1}^9 \phi_i(\xi, \eta) \psi_i.$$

In order to obtain finite element equations, Galerkin weighted residual method is applied to Equations (3)-(5) which are changed to the integral forms. Substitution of (8) into these forms yields the finite element equations of energy, momentum and streamfunction equations in the matrix form as follows,

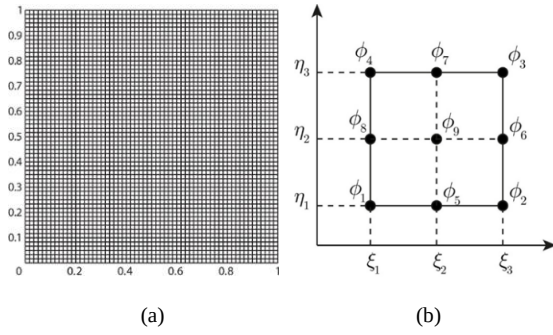


Figure 1. (a) Sketch of unit square divided into equal element (b) Biquadratic element with 9 nodes

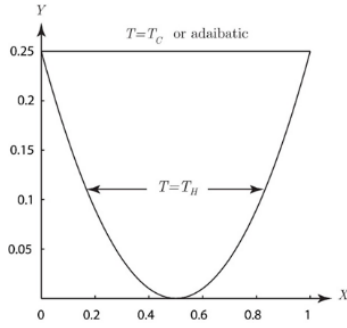


Figure 2. Physical model of parabolic enclosure

$$\left\{ \int_{\Omega_e} \left(\left[\frac{\partial \phi}{\partial X} \right] \left[\frac{\partial \phi}{\partial X} \right]^T + \left[\frac{\partial \phi}{\partial Y} \right] \left[\frac{\partial \phi}{\partial Y} \right]^T \right) d\Omega_e \right\} [\theta] + \left\{ \int_{\Omega_e} [\phi] [\phi]^T \left([U] \left[\frac{\partial \phi}{\partial X} \right]^T + [V] \left[\frac{\partial \phi}{\partial Y} \right]^T \right) d\Omega_e \right\} [\theta] = \int_{\Gamma_e} [\phi] \frac{\partial \theta}{\partial n} d\Gamma_e, \quad (9)$$

$$\left\{ \int_{\Omega_e} [\phi] [\phi]^T \left([U] \left[\frac{\partial \phi}{\partial X} \right]^T + [V] \left[\frac{\partial \phi}{\partial Y} \right]^T \right) d\Omega_e \right\} [U] + \gamma \left\{ \int_{\Omega_e} \left(\left[\frac{\partial \phi}{\partial X} \right] \left[\frac{\partial \phi}{\partial X} \right]^T \right) d\Omega_e \right\} [U] + \gamma \left\{ \int_{\Omega_e} \left(\left[\frac{\partial \phi}{\partial X} \right] \left[\frac{\partial \phi}{\partial Y} \right]^T \right) d\Omega_e \right\} [V] + \text{Pr} \left\{ \int_{\Omega_e} \left(\left[\frac{\partial \phi}{\partial X} \right] \left[\frac{\partial \phi}{\partial X} \right]^T + \left[\frac{\partial \phi}{\partial Y} \right] \left[\frac{\partial \phi}{\partial Y} \right]^T \right) d\Omega_e \right\} [U] + \frac{\text{Pr}}{Da} \left\{ \int_{\Omega_e} ([\phi] [\phi]^T) d\Omega_e \right\} [U] = 0, \quad (10)$$

$$\left\{ \int_{\Omega_e} [\phi] [\phi]^T \left([U] \left[\frac{\partial \phi}{\partial X} \right]^T + [V] \left[\frac{\partial \phi}{\partial Y} \right]^T \right) d\Omega_e \right\} [V] + \gamma \left\{ \int_{\Omega_e} \left(\left[\frac{\partial \phi}{\partial Y} \right] \left[\frac{\partial \phi}{\partial X} \right]^T \right) d\Omega_e \right\} [U] + \gamma \left\{ \int_{\Omega_e} \left(\left[\frac{\partial \phi}{\partial X} \right] \left[\frac{\partial \phi}{\partial Y} \right]^T \right) d\Omega_e \right\} [V] + \text{Pr} \left\{ \int_{\Omega_e} \left(\left[\frac{\partial \phi}{\partial X} \right] \left[\frac{\partial \phi}{\partial X} \right]^T + \left[\frac{\partial \phi}{\partial Y} \right] \left[\frac{\partial \phi}{\partial Y} \right]^T \right) d\Omega_e \right\} [V] + \frac{\text{Pr}}{Da} \left\{ \int_{\Omega_e} ([\phi] [\phi]^T) d\Omega_e \right\} [V] - \text{Ra Pr} \left\{ \int_{\Omega_e} ([\phi] [\phi]^T) d\Omega_e \right\} [\theta] = 0, \quad (11)$$

$$\left\{ \int_{\Omega_e} \left(\left[\frac{\partial \phi}{\partial X} \right] \left[\frac{\partial \phi}{\partial X} \right]^T + \left[\frac{\partial \phi}{\partial Y} \right] \left[\frac{\partial \phi}{\partial Y} \right]^T \right) d\Omega_e \right\} [W] + \left\{ \int_{\Omega_e} ([\phi] \left[\frac{\partial \phi}{\partial Y} \right]^T) d\Omega_e \right\} [U] - \left\{ \int_{\Omega_e} ([\phi] \left[\frac{\partial \phi}{\partial X} \right]^T) d\Omega_e \right\} [V] = 0, \quad (12)$$

where $[\bullet]$ represents column matrix.

Solution Procedure

The enclosure used in this study is cross-section of elliptic paraboloid. The cross-section results from cutting the surface of elliptic paraboloid $Y = (X-1/2)^2 + Z^2$ by a plane $Z = 0$, which is parabola $Y = (X-1/2)^2$, and it is enclosed by line $Y = 1/4$ for $0 \leq X \leq 1$ as shown in Figure 2. Due to the enclosure with parabolic curve, $X = g(\xi, \eta)$ and $Y = h(\xi, \eta)$ are defined on the computational domain such that their images are the points X and Y in physical domain. By concept of transfinite interpolation introduced in Farrashkhalvat and Miles (2003), $X = g(\xi, \eta)$ and $Y = h(\xi, \eta)$ are given by

$$X = \xi + \frac{\eta}{4}(1 - 2\xi),$$

$$Y = \frac{\eta}{4} + (1 - \eta) \left(\xi - \frac{1}{2} \right)^2 \quad (13)$$

By employing transformations (13), the computational domain shown in Figure 1(a) are transformed into parabolic enclosure and grid points (ξ, η) are changed to grid points (X, Y) as seen from Figure 3.

It can be seen that the integrations of Equations (9)-(12) are performed over the entire physical domain (XY-plane). These equations will be evaluated in computational domain $(\xi\eta\text{-plane})$ by using substitutions in double integrals such that $\partial \phi / \partial X$ and $\partial \phi / \partial Y$ can be computed by the following relations,

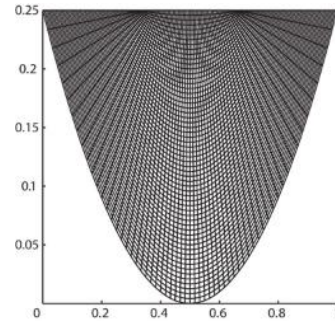


Figure 3. Sketch of parabolic enclosure with grid points

$$\begin{aligned}\frac{\partial \phi_j}{\partial X} &= \frac{1}{J} \left(\frac{\partial \phi_j}{\partial \xi} \frac{\partial Y}{\partial \theta} - \frac{\partial \phi_j}{\partial \eta} \frac{\partial Y}{\partial \xi} \right), \\ \frac{\partial \phi_j}{\partial Y} &= \frac{1}{J} \left(-\frac{\partial \phi_j}{\partial \xi} \frac{\partial X}{\partial \eta} + \frac{\partial \phi_j}{\partial \eta} \frac{\partial X}{\partial \xi} \right),\end{aligned}\quad (14)$$

where J is Jacobian of transformation given by

$$J = \frac{\partial(X, Y)}{\partial(\xi, \eta)} = \begin{vmatrix} \frac{\partial X}{\partial \xi} & \frac{\partial X}{\partial \eta} \\ \frac{\partial Y}{\partial \xi} & \frac{\partial Y}{\partial \eta} \end{vmatrix}$$

Prior to solving for the solutions, finite element equations addressed previously are assembled to form global equations. Boundary conditions are applied and iterative method is then used to solve for the solutions of unknown variables at each iteration. The solution procedure can be explained by the following steps.

Step 1. Solve the energy equation (9) by using the initiated values of velocities U and V to obtain the temperature values, θ .

Step 2. Substitute the temperature values obtained from step 1 into momentum equations (10)-(11) and then solve the equations system to obtain new values U and V .

Step 3. The new values U and V in step 2 are set to be the initial values in step 1 for the next iteration. The procedures step 1 and step 2 are continued until the relative error between consecutive iterations is satisfied the following criteria

$$\frac{\sum_{j=1}^m |\Phi_j^{(i)} - \Phi_j^{(i-1)}|}{\sum_{j=1}^m |\Phi_j^{(i)}|} \leq 10^{-5} \quad (15)$$

where $\Phi^{(i)}$ represents the unknown variables U , V , and θ at nodal point for i th iteration.

Step 4. When the process is terminated with criteria (15), the values of U , V , and θ are substituted into Equation (12) and the equations system is then solved to derive the values of ψ which is used to display the fluid flow inside the enclosure.

Results and Discussion

Validations

In order to validate the accuracy of proposed method, simple transformation $X = \xi$ and $Y = \eta$ are defined such that their images are the point in a unit square. The proposed method is tested for two cases

of natural convection problem in a square enclosure filled with porous media studied by Sathiyamoorthy *et al.* (2007). Two side walls are linearly heated for the first case and the left wall is linearly heated while the right wall is cooled for the second case. The bottom wall is uniformly heated and the top wall is adiabatic for both cases. The computational domain consists of 60×60 elements corresponding to 121×121 grid points. The results obtained from proposed method are presented by streamlines and isotherms for $Pr = 0.71$, $Da = 10^{-4}$ and $Ra = 10^6$. Local Nusselt numbers along heated bottom wall are also plotted for the same parameters. As seen from Figures 4 and 5, the results reveal good agreement. Graphs of relative error at each iteration for both cases are also shown in Figure 6. By the convergence criteria (15), it can be concluded that numerical solution is convergent and proposed method is suitable for natural convection problem with governing equations mentioned previousl.

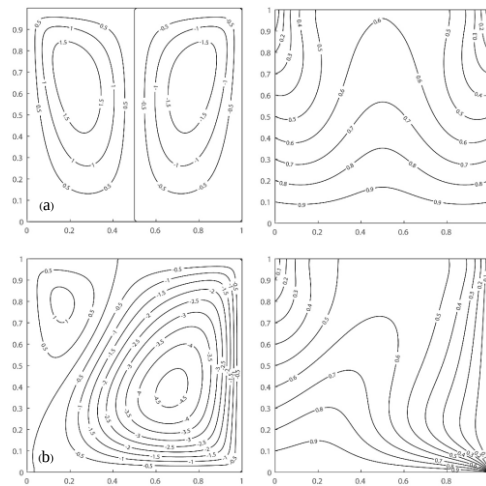


Figure 4. Streamlines (left) and isotherms (right) for $Pr = 0.71$, $Da = 10^{-4}$, and $Ra = 10^6$ obtained from proposed method; (a) case 1 and (b) case 2 (for comparison, see Sathiyamoorthy *et al.* (2007))

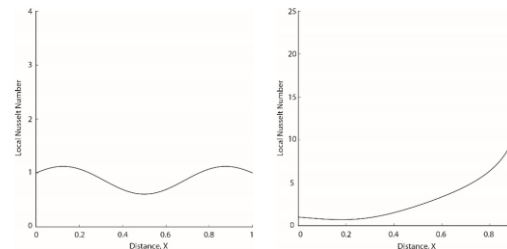


Figure 5. Local Nusselt number along heated bottom wall for $Pr = 0.71$, $Da = 10^{-4}$, and $Ra = 10^6$; (a) case 1 (b) case 2 (for comparison, see Sathiyamoorthy *et al.* (2007))

Natural Convection in the Enclosure with Parabolic Curve

Natural convection in the enclosure with parabolic curve and filled with fluid saturated porous media is studied. The fluid considered incompressible and Newtonian is contained inside the enclosure and its properties are constant except the density variation. In this study, the flow is assumed to be laminar and the fluid in the cavity is air. Values of Prandtl and Darcy numbers are kept constant at $Pr = 0.71$ and $Da = 10^{-2}$, respectively. Two cases of boundary conditions of temperature are considered. The top line is cooled (T_C) and parabolic curve is uniformly heated (T_H) for the first case while the top line is adiabatic and parabolic curve is nonuniformly

heated (T_H) for the second case. The boundary conditions of temperature in non-dimensional form are as follows,

- case 1: $\theta = 0$ for $0 \leq X \leq 1$ and $Y = 1/4$
 $\theta = 1$ for $0 \leq X \leq 1$ and $Y = (X-1/2)^2$,
 case 2: $\partial\theta/\partial n = 0$ for $0 \leq X \leq 1$ and $Y = 1/4$
 $\theta = (1/2)(1-\cos(2\pi X))$ for $0 \leq X \leq 1$ and $Y = (X-1/2)^2$,

where n represents the normal direction on a plane.

Figures 7 and 8 displays streamlines resulting from the increase in Ra for case 1 and case 2, respectively. It can be seen from the flow pattern that two circulations occur inside the enclosure and they are symmetric with respect to the vertical center line. As seen from the values of streamlines, the flow circulations rotate in different directions. Due to hotter fluid along parabolic curve, fluid near the curve moves into the center of the enclosure while colder fluid moves downwards. This situation results in the circulations in clockwise and anticlockwise directions of the left and right halves, respectively, for case 1. When the curve is nonuniformly heated for case 2, the maximum temperature is occurred at the middle curve. The heated fluid move upwards the top of the enclosure and then flow down along the curve forming two symmetric circulations. The left half rotates in anticlockwise direction and the right half rotates in clockwise direction. The shape of the flow fits the curve for both cases. The effects of increasing Ra from 5×10^4 to 10^5 on flow intensity is distinguishable but there is a little difference. The strength of flow circulation is very low as seen from the value of $|\Psi_{\max}|$ shown in Table 1 because the conduction mode becomes dominant. When Ra is increased to 5×10^5 and 10^6 , the intensity of fluid flow is higher due to the increase of convection mode. The flow configuration is smooth curve but it

is distorted when $Ra = 10^6$ for case 2 as shown in Figure 8(d).

The effects of variation in Ra on temperature distribution are displayed by isotherms. As seen

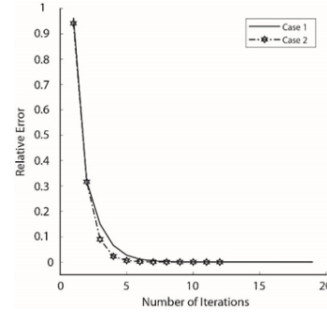


Figure 6. Relative error values obtained from proposed method for the validation

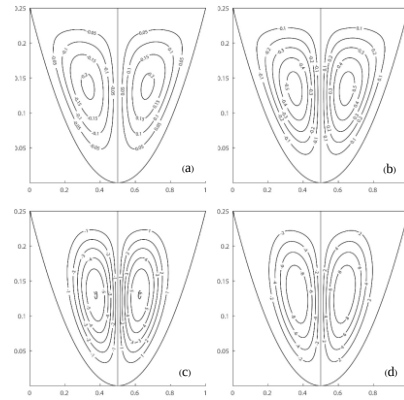


Figure 7. Streamlines for case 1; a) $Ra = 5 \times 10^4$, b) $Ra = 1 \times 10^5$, c) $Ra = 5 \times 10^5$, and d) $Ra = 1 \times 10^6$ with constant $Da = 10^{-2}$

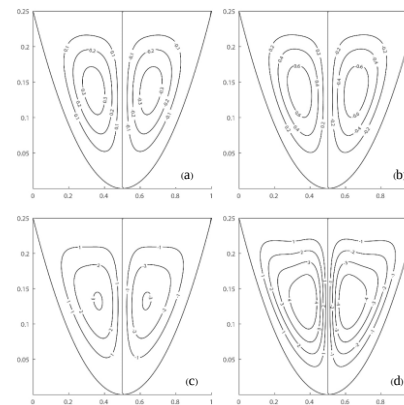


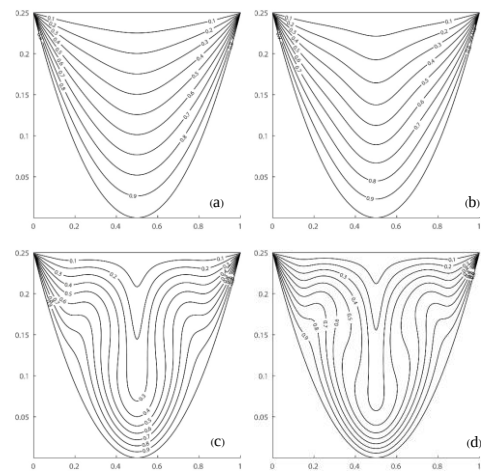
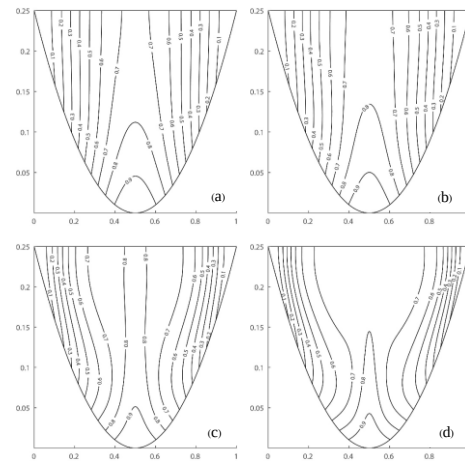
Figure 8. Streamlines for case 2; a) $Ra = 5 \times 10^4$, b) $Ra = 1 \times 10^5$, c) $Ra = 5 \times 10^5$, and d) $Ra = 1 \times 10^6$ with constant $Da = 10^{-2}$

Table 1. Maximum value of streamlines and average Nusselt number at different Ra with $Da = 10^{-2}$ and $Pr = 0.71$ for case 1 and case 2

Ra	Case 1		Case 2	
	$ \Psi_{\max} $	$\overline{Nu}$	$ \Psi_{\max} $	$\overline{Nu}$
5×10^4	0.2109	2.8459	0.3560	0.2517
1×10^5	0.5458	2.8609	0.7351	0.2541
5×10^5	6.0256	3.4374	3.0692	0.2876
1×10^6	9.8407	3.7891	4.9667	0.3193

from Figure 9, isothermal lines are raised from the corners and the compressed stratification can be found here indicating that heat transfer is higher. Isotherms are smooth curve and disperse in the entire domain. For $Ra = 5 \times 10^4$ and 10^5 , isotherms are parallel to each other showing that the conduction is dominating. As Ra is higher, the heated fluid moves into the center of the enclosure resulting in outer temperature lines are shifted near center vertical line. Simultaneously, inner temperature lines are pushed towards the lower portion because of the motion of the fluid with lesser temperature toward the curve. Figure 10 shows temperature distribution with various Ra for case 2. Isotherms are raised from the parabolic curve and disperse towards the top of the enclosure. Temperature distribution show similar trend with increasing Ra from 5×10^4 to 10^5 . The stratifications are found to be flat shape on the left and right sides of the enclosure. When Ra is higher, the convection heat transfer is dominant. Due to stronger flow intensity, the temperature lines are dense near the upper portion resulting from the direction of fluid motion and the area of $\theta = 0.7$ is larger when $Ra = 10^6$.

The heat transfer characteristics along heated curve for various Ra are displayed by local Nusselt number in Figure 11. For case 1, it is found that the variation of heat transfer is maximum at the left and the right sides of curve because of the temperature difference between heated curve and cooled top wall. For all Ra , heat transfer is decreased along the curve. It can be seen that heat transfer rate is constant between $X = 0.2-0.8$ for $Ra = 5 \times 10^4$ and $Ra = 10^5$ while it is slightly increased at the middle portion for higher Ra . Due to nonuniformly heated curve and the symmetry of temperature distribution in case 2, heat transfer is increased along the curve and the profiles of local Nusselt number are symmetric with respect to vertical center line for all Ra . The maximum heat transfer occurs for two points at $Ra = 5 \times 10^5$ and $Ra = 1 \times 10^6$. The overall heat transfer rate resulting from the increase in Ra is presented in term of average Nusselt number ($\overline{Nu}$). The increase in Ra means that heat is added to the system. As seen from Table 1, the values of $\overline{Nu}$ is

**Figure 9. Isotherms for case 1; a) $Ra = 5 \times 10^4$, b) $Ra = 1 \times 10^5$, c) $Ra = 5 \times 10^5$, and d) $Ra = 1 \times 10^6$ with constant $Da = 10^{-2}$** **Figure 10. Isotherms for case 2; a) $Ra = 5 \times 10^4$, b) $Ra = 1 \times 10^5$, c) $Ra = 5 \times 10^5$, and d) $Ra = 1 \times 10^6$ with constant $Da = 10^{-2}$**

enhanced with increasing Ra for both cases. Figure 12 shows the convergence of numerical solutions for case 1 and case 2 obtained from the proposed method. As seen from the graphs, the solutions become steady state faster for smaller Ra .

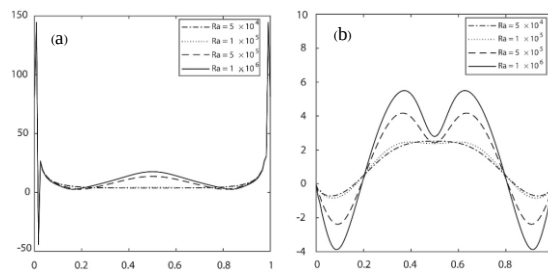


Figure 11. Local Nusselt number along heated curve for different Ra ; a) case 1, and b) case 2

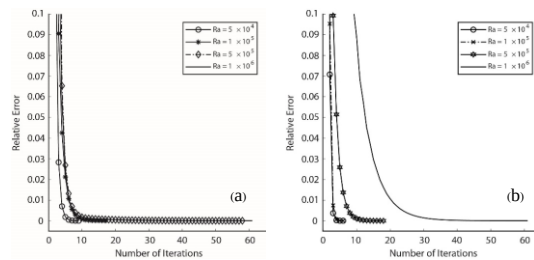


Figure 12. Graphs of relative error indicating the convergence of numerical solutions for (a) case 1 and (b) case 2

Conclusions

Finite element and iterative methods are proposed to solve the natural convection problem inside parabolic cross-section enclosure. Two cases of temperature boundary conditions are considered. The top of the enclosure is cooled and the parabolic curve is uniformly heated for case 1 while the top of the enclosure is adiabatic and the curve is nonuniformly heated for case 2. From the study results, it is found that there is a little effect on flow intensity and temperature distribution for small Ra . Stronger flow intensity and different characteristic of temperature distribution can be seen for higher Ra because the increase in Ra means the increase in heat inside the enclosure resulting in convection dominance. The heat transfer rate along heated curve is maximum at the left and right sides of curve and it is reduced along the curve for case 1. The maximum heat transfer rate for case 2 is maximum at middle portion and there are two maximum points when Ra is increased. The overall heat transfer rate along the curve is increased with increasing Ra . Numerical results obtained from the study reveal the efficiency of proposed method which is useful for natural convection problem in parabolic enclosure.

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