

MONITORING THE PROCESS MEAN OF A MODIFIED EWMA CHART FOR ARMA(1,1) PROCESS AND ITS APPLICATION

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Abstract

The performance of a control chart monitoring a statistical process is regularly measured in terms of the average run length (ARL) which guarantees that a comparative study can be conducted between several monitoring and detecting procedures. Explicit formulas are derived and evaluated for an exact ARL based on a first-order autoregressive moving average (ARMA(1,1)) process with exponential white noise through a modified exponentially weighted moving average (EWMA) control chart. The analytical results are more accurate in comparison with the numerical integral equation method as it compares the performance of the control chart between the EWMA and modified EWMA. The superiority of the modified EWMA control chart is illustrated by applying it to a dataset of changes in Earth's rotation.

Keywords: Modified EWMA, average run length, ARMA process, explicit formulas

Introduction

Autocorrelation is characteristic of a dataset in which there is correlation among the values of the same variables, and the basic assumption of instance independence, which underlies most conventional models, is violated. It frequently exists in those types of datasets in which the data is from the same source, and since it occurs mostly due to dependencies

within the data, the observations collected periodically need to be checked by a statistical analysis for a time series. There are various processes of time series modeling which have been used in many applications. The popular structures of autocorrelation are autoregressive (AR), moving average (MA), and joint AR and MA (ARMA). ARMA models are a very broad

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and flexible family of stationary stochastic processes that are useful in representing time series data. One of the main advantages is the usage of various applications to develop models to estimate or predict the parameter for the value of interest.

Various phenomenological problems often involve autocorrelation to which ARMA models are often applied, such as pricing on stock or velocity of money (Wang *et al.*, 2012), estimating parameters to develop a predictive model of the neuromuscular system (Agarwal *et al.*, 1982), a runoff forecasting model for asphalt planes (Chua and Wong, 2011), establishing a forecast model for a rotating speed signal (Liu *et al.*, 2011), modeling tree crown profiles for California hardwood species (Gill and Biging, 2002), developing a model for electricity demand loads (Pappas *et al.*, 2008), providing a comprehensive approach to high-frequency velocity profile data cleaning (Dilling and MacVicar, 2017), and fitting a model for earthquake prediction (Kozin, 1988; Mobarakeh *et al.*, 2002). In addition, an ARMA process has been generally applied to predict the accuracy of Earth's rotation parameters (Yokoyama, 2002; Bogusz *et al.*, 2015; Kim and Kim, 2015; Lei *et al.*, 2016). Moreover, comparative studies of ARMA with other processes have also been investigated for several applications (Ubeyli and Guler, 2004; Chattopadhyay *et al.*, 2011; Valipour *et al.*, 2013; Sivavaraprasad and Ratnam, 2017). These reports have touched on ARMA models outperforming others for a variety of problems involving serially correlated processes and real-life datasets.

In general, traditional monitoring methodology for process deviation is a widely used practical application. One of the popular detection tools in statistical process control is the control chart. The first was introduced by Shewhart in 1931, namely the Shewhart control chart, which is based on the standard assumption of random independently and normally distributed data and is sensitive to detecting a large change in a process. Many control charts have been developed to improve monitoring performance and detecting relatively small changes in a process. The cumulative

sum (CUSUM) and exponentially weighted MA (EWMA) control charts were proposed by Page (1954) and Roberts (1959), respectively. They are good alternatives to the original control chart for monitoring and detecting small shifts in the mean and variance of processes and observations that violate the assumption. There are plenty of such studies, such as those by Mastrangelo and Montgomery (1995), Reynolds and Lu (1997), Jiang *et al.* (2000), Fu *et al.* (2002), and Zhang (2012).

Later, Patel and Divecha (2011) presented a new control chart which is a modification of the EWMA procedure. It is very effective for detecting small and abrupt changes and its advantage lies in its applicability to any free inertia problem. They applied the modified EWMA control chart to various real-life applications, such as data from a highly autocorrelated normal process collected every minute for chemical temperature measurements and independent normal manufacturing process measurements for detecting process shifts. Their findings show that the modified EWMA chart exceeded the performance of the original EWMA chart. Monitoring of the coefficient of variation to further enhance the efficiency of the EWMA control scheme where the process mean and standard deviation are not constant was proposed by Zhang *et al.* (2014). A sintering process to manufacture mechanical parts data was used to compare the modified EWMA control chart with existing ones, the results of which show that the modified chart was sensitive and performed better than most of the others. Herdiani *et al.* (2018) investigated the performance of the modified EWMA chart when the observations were autocorrelated in the case of a first-order AR model. The numerical results indicate that the proposed procedure was more sensitive than the others for weighted parameters of 0.05, 0.1, and 0.2.

Many studies of control charts have been universally based on numerical computation through the average run length (ARL), which is a traditional measurement of a control chart's ability. The ARL is the expected value of the number of observations of the in-control

state before the first observation signal is at the out-of-control limit. It can be divided into 2 states: ARL_0 for the in-control state and ARL_1 for the out-of-control state. There are various methods which are available for determining the ARL, such as the numerical integral equation (NIE), the Markov chain approach, Monte Carlo simulation, and explicit formulas to find and determine the exact value of the ARL. Sukparungsee and Areepong (2009) derived an explicit formula for the EWMA chart ARL in cases of a Weibull distribution by using a simple transformation technique. The accuracy of the numerical results was compared via Monte Carlo simulations. Mititelu *et al.* (2010) used the explicit formulas to evaluate the ARL for an EWMA control chart with a Laplace distribution and a CUSUM control chart with a hyperexponential distribution. Busaba *et al.* (2012) analyzed the NIE method for the ARL of a CUSUM control chart for the case of a stationary first-order AR process and compared it with explicit formulas. Suriyakit *et al.* (2012) used the NIE method to solve the ARL and derived an explicit expression for a first-order AR procedure for an EWMA control chart process. Areepong (2012) proposed explicit formulas for an MA control chart for observations that are binomially distributed. Petcharat *et al.* (2013) derived explicit formulas for the ARL of EWMA control charts for an MA process. The ARL for CUSUM and EWMA control charts using the NIE method for an ARMA process with exponential white noise was presented by Phanyaem *et al.* (2014), who showed that the performance of the EWMA chart was better than the CUSUM chart when there were small shifts in the process mean. Petcharat *et al.* (2015) analyzed an explicit formula for the ARL of a CUSUM control chart when the observations comprised an MA order q process. Phantu *et al.* (2016) proposed an explicit formula for evaluating the ARL of an MA control chart for the first-order AR of a serially dependent Poisson process. Peerajit *et al.* (2017) derived the explicit expression for the ARL using the NIE on a CUSUM control chart for a long memory process with an AR integrated MA (ARIMA) (p,d,q) process and

compared the efficiency in terms of the ARL between the explicit formulas and the NIE. Sunthornwat *et al.* (2018) investigated the ARL of an EWMA control chart for a long memory AR fractionally integrated MA (ARFIMA) process via analytical and numerical methods, the results of which suggest the analytical EWMA chart was the best.

The studies mentioned previously indicate that explicit formulas are easy to derive and save a great deal of computational time. Therefore, in this study, an ARL derived by explicit formulas on a modified EWMA control chart where the observations are a first-order ARMA (ARMA(1,1)) process with exponential white noise is proposed. The NIE is a good method that provides accuracy in the calculation of the proposed approach. Application of the method to compare the performance of the modified and original EWMA control charts is also reported.

The Control Charts

Let $X_i, i = 1, 2, 3, \dots$ be the i^{th} autocorrelated process for monitoring the mean shift of the white noise that follows an exponential distribution with mean β_0 . Suppose that at some period τ , which is usually called the stopping time, the mean of the distribution changes from β_0 to β_1 . It can be said that the mean distribution undergoes a persistent one-sided change to $\beta_1 = (1+\delta)\beta_0$ from the period onwards of τ , where δ is the shift size and τ is unknown.

The EWMA control chart

The first change point outside the one-sided control limit l for the EWMA chart can be written as

$$\tau(l) = \inf \{n \geq 1: Y_t \geq l\}, \quad (1)$$

where

$$Y_t = \lambda X_t + (1-\lambda)Y_{t-1} \quad (2)$$

and λ is a weighting parameter ($0 < \lambda \leq 1$). It is convenient to set $Z_0 = \mu_0$ where μ_0 is the center line of the control chart with the standard deviation of Z_t being $\sigma_{z_t} = \sigma \sqrt{\frac{\lambda}{2-\lambda} [1 - (1-\lambda)^{2t}]}$.

The limiting value for the increase in t is $\sigma_z = \sigma \sqrt{\frac{\lambda}{2-\lambda}}$ where σ is the process standard deviation. The control limit of the EWMA chart is given by

$$\mu_0 \pm H_1 \sigma \sqrt{\frac{\lambda}{2-\lambda}}, \quad (3)$$

where H_1 is the optimal width of the control limit.

The Modified EWMA Control Chart

According to Patel and Devicha (2011), the first change point outside the one-sided control limit b for the modified EWMA control chart can be written as

$$\tau(b) = \inf \{n \geq 1: Z_t \geq b\}. \quad (4)$$

The statistic of the modified EWMA control chart can be defined as

$$Z_t = \lambda X_t + (1-\lambda)Z_{t-1} + (X_t - X_{t-1}), \quad (5)$$

where λ is a weighting parameter ($0 < \lambda \leq 1$) and μ_0 is the center line of the control chart with the standard deviation of Z_t being $\sigma_{z_t} = \sigma \sqrt{\frac{\lambda}{2-\lambda} + \frac{2\lambda(1-\lambda)}{2-\lambda}}$, where σ is the process standard deviation. The control limit of the modified EWMA chart is as follows

$$\mu_0 \pm H_2 \sigma \sqrt{\frac{\lambda}{2-\lambda} + \frac{2\lambda(1-\lambda)}{2-\lambda}}, \quad (6)$$

where H_2 is a suitable control width limit.

The practical study of the EWMA and modified EWMA control charts with their control limits is reported in the Application section.

The evaluation of the ARL

For the convenience of the derivation, ARMA(1,1) is focused. It is described on which can be in the form

$$X_t = \mu + \varepsilon_t + \phi X_{t-1} + \theta \varepsilon_{t-1}, \quad (7)$$

where ϕ and θ are ARMA coefficients, respectively, and ε_t is a white noise process which is assumed to be an exponential distribution with starting values $\varepsilon_0 = S$, $Z_0 = u$ and $X_0 = v$.

Therefore, the statistic for the modified EWMA chart can be written as

$$Z_t = (1-\lambda)Z_{t-1} + (\lambda+1)\varepsilon_t + (\lambda+1)\mu + (\lambda\phi + \phi - 1)X_{t-1} - (\lambda\theta + \theta)\varepsilon_{t-1}.$$

Let $h(u)$ be denoted by the ARL of the modified EWMA control chart. The integral equation can be written as follows:

$$h(u) = 1 + \frac{\int_0^b \int_0^1 \frac{b - (1-\lambda)u - (\lambda+1)y - (\lambda\phi + \phi - 1)v + (\lambda\theta + \theta)s}{(1-\lambda)u - (\lambda+1)y - (\lambda\phi + \phi - 1)v + (\lambda\theta + \theta)s} h \left[\frac{(1-\lambda)u + (\lambda+1)y + (\lambda+1)\mu}{(\lambda\phi + \phi - 1)v - (\lambda\theta + \theta)s} \right] f(y) dy ds}{\lambda + 1}. \quad (8)$$

Changing the variables, the integral equation is obtained as

$$h(u) = 1 + \frac{1}{\lambda + 1} \int_0^b h(k) f \left(\frac{k - (1-\lambda)u - (\lambda\phi + \phi - 1)v + (\lambda\theta + \theta)s}{\lambda + 1} \right) dk. \quad (9)$$

$h(u)$ is a Fredholm integral equation of the second kind with an exponential distribution, thus,

$$h(u) = 1 + \frac{1}{\lambda + 1} \int_0^b h(k) \cdot \frac{1}{\beta} \cdot e^{-\frac{k}{\beta}} \cdot e^{-\frac{(1-\lambda)u}{\beta(\lambda+1)}} \cdot e^{-\frac{(\lambda\phi + \phi - 1)v}{\beta(\lambda+1)}} \cdot e^{-\frac{(\lambda\theta + \theta)s}{\beta(\lambda+1)}} \cdot e^{-\frac{\mu}{\beta}} dk. \quad (10)$$

Suppose that

$$C(u) = e^{\frac{(1-\lambda)u}{\beta(\lambda+1)}} \cdot e^{\frac{(\lambda\phi + \phi - 1)v}{\beta(\lambda+1)}} \cdot e^{\frac{(\lambda\theta + \theta)s}{\beta(\lambda+1)}} \cdot e^{\frac{\mu}{\beta}} \quad \text{and} \\ D = \int_0^b h(k) \cdot e^{-\frac{k}{\beta(\lambda+1)}} dk,$$

then the explicit formulas of the *ARL* of the modified EWMA control chart for the (1,1) process with exponential white noise is defined by

$$ARL = 1 - \frac{\lambda e^{\frac{(1-\lambda)u}{\beta(\lambda+1)}} \left[e^{-\frac{b}{\beta(\lambda+1)}} - 1 \right]}{\lambda e^{\frac{-\mu}{\beta}} \cdot e^{\frac{(\lambda\theta+\phi)s}{\beta(\lambda+1)}} \cdot e^{\frac{(\lambda\phi+\phi-1)v}{\beta(\lambda+1)}} + e^{\frac{-\lambda b}{\beta(\lambda+1)}} - 1}, \quad (11)$$

where $\beta = \beta_0 = 1$ for an in-control process and $\beta = \beta_1 > \beta_0$ for an out-of-control process.

The Comparison Criteria

For comparing the numerical results, the absolute percentage relative error (*APRE*) is measured to compare between the analytical formulas and the NIE method for the *ARL*. It can be represented as

$$APRE = \frac{|h(u) - \tilde{h}(u)|}{h(u)} \times 100\%, \quad (12)$$

where $\tilde{h}(u)$ is the NIE method for evaluating the *ARL* based on the Gauss-Legendre quadrature rule by following as Equation (9) with 1,000 nodes. Each node is weighted by $w_j = \frac{b}{m}$ on the interval $[0, b]$ which is partitioned by $a_j = \frac{b}{m} \left(j - \frac{1}{2} \right)$. The integral equation obtained can be written in the form

$$\tilde{h}(u) = 1 + \frac{1}{\lambda + 1} \sum_{j=1}^m w_j h(a_j) f \left(\frac{a_j - (1-\lambda)u - (\lambda\phi + \phi - 1)v + (\lambda\theta + \theta)s}{\lambda + 1} - \mu \right). \quad (13)$$

This is the linear equation system attained for solving the approximate solution of the *ARL*.

Existence and Uniqueness of Solution

In this section, the integral equation approach is used to prove the existence and uniqueness of the solutions for the average run length of

the first order autoregressive moving average process. Consider that the right-hand side of Equation (9) is a continuous function; the solution of the average run length is also continuous.

According to the complete metric space (X, p) where X denotes the space of all continuous functions on $[0, b]$ with the norm $\|h\|_\infty = \sup_{u \in [0, b]} |h(u)|$ and every Cauchy's

sequence $\{h_n\}_{n \geq 0}$ of a point of X converges to a point. Let T be an operation in the class of all continuous functions on $[0, b]$ defined by

$$T(h(u)) = 1 + \frac{1}{\lambda + 1} \int_0^b L(k) \cdot \frac{1}{\beta} \cdot e^{\frac{-k}{\beta(\lambda+1)}} \cdot e^{\frac{(1-\lambda)u}{\beta(\lambda+1)}} \cdot e^{\frac{(\lambda\phi+\phi-1)v}{\beta(\lambda+1)}} \cdot e^{\frac{(\lambda\theta+\theta)s}{\beta(\lambda+1)}} \cdot e^{\frac{\mu}{\beta}} dk. \quad (14)$$

Recall that an operation T is a contraction such that the fixed point equation $T(h(u)) = h(u)$ has a unique solution; we use the integral equation method to prove the existence and uniqueness of the *ARL* for ARMA(1,1) by the following theorem.

Theorem 1 (Banach Fixed Point Theorem)

Let T be a contraction mapping from a non-empty complete metric space (X, p) into itself such that there is a real number c with $0 \leq c < 1$, satisfying $\|T(h_1) - T(h_2)\| \leq c \|h_1 - h_2\|$ for all $h_1, h_2 \in X$. Then there exists a unique fixed point in X .

Proof (Existence)

Firstly, to show part of the existence of the solution of the integral equation, let $C[0, b]$ be a set of all continuous functions on a complete metric space (X, p) , and $h_0 \in C[0, b]$ and $\{h_n\}_{n \geq 0}$ be a Cauchy's sequence such that $h_{n+1} = T(h_n)$, $h(0) = h_0$. To show that the sequence $\{h_n\}_{n \geq 0}$ converges to a point h , consider

$$\begin{aligned} p(h_2, h_1) &= p(T(h_1), T(h_0)) \leq cp(h_1, h_0); \quad 0 \leq c < 1 \\ p(h_3, h_2) &= p(T(h_2), T(h_1)) \leq cp(h_2, h_1) \leq c^2 p(h_1, h_0) \\ &\vdots \\ p(h_n, h_{n-1}) &= p(T(h_{n-1}), T(h_{n-2})) \leq cp(h_{n-1}, h_{n-2}) \leq \\ &\quad c^{n-1} p(h_1, h_0) \\ p(h_{n+1}, h_n) &= p(T(h_n), T(h_{n-1})) \leq cp(h_n, h_{n-1}) \leq \\ &\quad c^n p(h_1, h_0); \\ &\quad \forall n \geq 0 \end{aligned}$$

By the triangular inequality, for $n \geq m$

$$\begin{aligned} p(h_n, h_m) &\leq p(h_n, h_{n-1}) + p(h_{n-1}, h_{n-2}) + \dots + p(h_{m+1}, h_m) \\ &\leq c^{n-1} p(h_1, h_0) + c^{n-2} p(h_1, h_0) + \dots + c^m p(h_1, h_0) \\ &\leq (c^{n-1} + c^{n-2} + \dots + c^m) p(h_1, h_0) \\ &\leq \frac{c^m (1 - c^{n-m})}{1 - c} p(h_1, h_0) = \frac{c^m - c^n}{1 - c} p(h_1, h_0) \end{aligned}$$

So, $p(h_1, h_0)$ exists and $\lim_{n \rightarrow \infty} c^n = 0$; $0 \leq c < 1$. Hence, $\lim_{n \rightarrow \infty} p(h_n, h_m) = 0$ and the sequence h_n converge in a Cauchy sequence.

Since the sequence h_n is an element of complete metric space, then h_n will converge to a limit h in this space $\lim_{n \rightarrow \infty} h_n = h$. The limit point h is actually the fixed point of the operator T which is $T(h) = h$, that is $p(h, T(h)) = 0$. Therefore, the solution of the integral equation is existing.

Proof (uniqueness)

First, we have to show T is a contraction for $u \in [0, b]$ and $h_1, h_2 \in X$. The inequality $\|T(h_1) - T(h_2)\| \leq c \|h_1 - h_2\|$ for all $h_1, h_2 \in X$

with $0 \leq c < 1$. Consider equation (14), then $\|T(h_1) - T(h_2)\|_\infty = \sup_{u \in [0, b]} |h(u)|$

$$\begin{aligned} &= \sup_{u \in [0, b]} \left| (h_1(u) - h_2(u)) \frac{e^{\frac{(1-\lambda)u}{\beta(\lambda+1)} + \frac{(\lambda\phi+\phi-1)v}{\beta(\lambda+1)} - \frac{(\lambda\theta+\theta)s}{\beta(\lambda+1)} - \frac{\mu}{\beta}}}{\beta(\lambda+1)} \int_0^b h(k) \cdot e^{-\frac{k}{\beta(\lambda+1)}} dk \right| \\ &\leq \sup_{u \in [0, b]} \|h_1 - h_2\|_\infty \frac{e^{\frac{(1-\lambda)u}{\beta(\lambda+1)} + \frac{(\lambda\phi+\phi-1)v}{\beta(\lambda+1)} - \frac{(\lambda\theta+\theta)s}{\beta(\lambda+1)} - \frac{\mu}{\beta}}}{\beta(\lambda+1)} (-\beta(\lambda+1)) \left(e^{-\frac{b}{\beta(\lambda+1)}} - 1 \right) \\ &= \|h_1 - h_2\|_\infty \left| 1 - e^{-\frac{b}{\beta(\lambda+1)}} \right| \sup_{u \in [0, b]} \left| e^{\frac{(1-\lambda)u}{\beta(\lambda+1)} + \frac{(\lambda\phi+\phi-1)v}{\beta(\lambda+1)} - \frac{(\lambda\theta+\theta)s}{\beta(\lambda+1)} - \frac{\mu}{\beta}} \right| \\ &\leq c \|h_1 - h_2\|_\infty \end{aligned}$$

where

$$c = \left| 1 - e^{-\frac{b}{\beta(\lambda+1)}} \right| \sup_{u \in [0, b]} \left| e^{\frac{(1-\lambda)u}{\beta(\lambda+1)} + \frac{(\lambda\phi+\phi-1)v}{\beta(\lambda+1)} - \frac{(\lambda\theta+\theta)s}{\beta(\lambda+1)} - \frac{\mu}{\beta}} \right|; \quad 0 \leq c < 1.$$

Triangular inequality was implemented and implied that $|h_1 - h_2| \leq \sup_{u \in [0, b]} |h_1 - h_2| = \|h_1 - h_2\|_\infty$

Therefore, the existent and uniqueness of the solution of average run length is guaranteed by the Banach fixed-point theorem.

Table 1. Comparison of ARL for ARMA(1,1) on modified EWMA control charts by using explicit formulas versus numerical integral equation for $ARL_0 = 370$, $\mu = 2$ and $(\phi, \theta) = (0.3, 0.1)$ with $\lambda = 0.05, 0.1$

λ	Shift size	Explicit	NIE	APRE
0.05 ($b=0.301950105$)	0.00	370.0000281653	370.0000263514	4.902×10^{-7}
	0.005	126.6531233766	126.6531228854	3.878×10^{-7}
	0.01	76.3881866474	76.3881863704	3.626×10^{-7}
	0.03	29.5209067279	29.5209066321	3.245×10^{-7}
	0.05	18.3131222921	18.3131222368	3.020×10^{-7}
	0.07	13.2944636359	13.2944635983	2.828×10^{-7}
	0.09	10.4539988838	10.4539988561	2.650×10^{-7}
	0.10	9.4523220648	9.4523220405	2.571×10^{-7}
	0.20	4.9262437657	4.9262437563	1.908×10^{-7}
	0.50	2.3043173373	2.3043173353	8.679×10^{-8}
	1.00	1.5362305669	1.5362305665	2.604×10^{-8}
	2.00	1.2190632313	1.2190632312	8.203×10^{-9}
0.10 ($b=0.305078073$)	0.00	370.0000353347	370.0000319774	9.074×10^{-7}
	0.005	119.2888541047	119.2888535073	5.008×10^{-7}
	0.01	71.1377154106	71.1377151127	4.188×10^{-7}
	0.03	27.2708107215	27.2708106316	3.297×10^{-7}
	0.05	16.9227005269	16.9227004768	2.961×10^{-7}
	0.07	12.3057213119	12.3057212784	2.722×10^{-7}
	0.09	9.6967403037	9.6967402791	2.537×10^{-7}
	0.10	8.7773090152	8.7773089938	2.438×10^{-7}
	0.20	4.6249649028	4.6249648946	1.773×10^{-7}
	0.50	2.2150661680	2.2150661662	8.126×10^{-8}
	1.00	1.5040867729	1.5040867725	2.659×10^{-8}
	2.00	1.2077595056	1.2077595055	8.280×10^{-9}

Numerical Results

The purpose of this section is to present the evaluation results of the ARL of one side of a modified EWMA control chart based on ARMA(1,1) observations with exponential white noise. The numerical results of the the ARL s were obtained by explicit formulas and the NIE method with 1,000 nodes by using the Mathematica program. The results in Tables 1 and 2 can be used to compare the

computational results for various values of the mean shift β process by setting the shift size to 0.005, 0.01, 0.03, 0.05, 0.07, 0.09, 0.10, 0.20, 0.50, 1.00, and 2.00 with the constant of every process being 2 given $ARL_0 = 370$. When the coefficients of the ARMA(1,1) process are arbitrary (0.3,0.1), the upper control limits of the control chart for smoothing parameter λ of 0.05 and 0.10 are 0.301950105 and 0.305078073, respectively (Table 1), while when the coefficients of the ARMA(1,1)

Table 2. Comparison of ARL for ARMA(1,1) on modified EWMA control charts by using explicit formulas versus numerical integral equation for $ARL_0 = 370$, $\mu = 2$ and $(\phi, \theta) = (-0.3, -0.1)$ with $\lambda = 0.05, 0.1$

λ	Shift size	Explicit	NIE	APRE
0.05 ($b=0.452229145$)	0.00	370.0000481671	370.0000440189	1.121×10^{-6}
	0.005	138.1702709260	138.1702696983	8.885×10^{-7}
	0.01	84.9542988239	84.9542981216	8.267×10^{-7}
	0.03	33.4700538252	33.4700535790	7.356×10^{-7}
	0.05	20.8756576302	20.8756574874	6.841×10^{-7}
	0.07	15.1969499704	15.1969498728	6.422×10^{-7}
	0.09	11.9704858840	11.9704858116	6.048×10^{-7}
	0.10	10.8301625974	10.8301625338	5.872×10^{-7}
	0.20	5.6536143761	5.6536143510	4.440×10^{-7}
	0.50	2.6107395752	2.6107395697	2.107×10^{-7}
0.10 ($b=0.458429542$)	1.00	1.6916368840	1.6916368827	7.685×10^{-8}
	2.00	1.2960148644	1.2960148641	2.315×10^{-8}
	0.00	370.0000443766	370.0000364988	2.129×10^{-6}
	0.005	130.7573016990	130.7573001297	1.200×10^{-6}
	0.01	79.4618156533	79.4618148644	9.928×10^{-7}
	0.03	31.0221832356	31.0221829977	7.669×10^{-7}
	0.05	19.3442326668	19.3442325344	6.844×10^{-7}
	0.07	14.0996712106	14.0996711218	6.298×10^{-7}
	0.09	11.1252108818	11.1252108166	5.861×10^{-7}
	0.10	10.0747763694	10.0747763124	5.658×10^{-7}
	0.20	5.3097182249	5.3097182017	4.369×10^{-7}
	0.50	2.5048183425	2.5048183376	1.956×10^{-7}
	1.00	1.6520049754	1.6520049742	7.264×10^{-8}
	2.00	1.2815480483	1.2815480481	1.561×10^{-8}

Table 3. Comparison ARL for ARMA(1,1) on EWMA and modified EWMA control charts by using explicit formulas for $ARL_0 = 370$, $\mu = 2$ and $(\phi, \theta) = (-0.3, 0.1)$ with $\lambda = 0.05, 0.1$

Shift size	$\lambda = 0.05$		$\lambda = 0.1$	
	EWMA ($l = 2.087033 \times 10^{-8}$)	Modified ($b = 0.553811072$)	EWMA ($l = 8.83092 \times 10^{-4}$)	Modified ($b = 0.562675871$)
0.00	370.00	370.00	370.00	370.00
0.005	332.40	144.72	349.11	137.35
0.01	298.94	89.98	329.58	84.39
0.03	197.61	35.85	263.25	33.31
0.05	132.77	22.43	212.05	20.83
0.07	90.61	16.36	172.19	15.20
0.09	62.79	12.90	140.89	12.01
0.10	52.56	11.67	127.79	10.88
0.20	10.93	6.10	52.74	5.73
0.50	1.26	2.80	8.00	2.69
1.00	1.01	1.79	1.89	1.75
2.00	1.00	1.35	1.10	1.33

process are $(-0.3, -0.1)$ the upper control limits for both smoothing parameters are 0.452229145 and 0.458429542, respectively (Table 2). It can be seen in Tables 1 and 2 that the ARL obtained by NIE is slightly smaller than the explicit formulas. When the shift sizes are larger, the ARLs of both methods are also in unison, and as a result, the relative percentage errors are very small.

Table 3 summarizes the performance of the modified EWMA chart versus the original one in parallel situations of the process mean given $ARL_0 = 370$. The results illustrate that the modified EWMA control chart is better than the original one except when detecting mean shifts of 1.00 and 2.00 only when the smoothing parameter is 0.05. The rest of the cases indicate that the modified EWMA chart performed excellently. It shows that the performance of the modified EWMA is depreciated for detecting the large shift when the smoothing parameter is 0.05. Consequently, the smoothing parameter 0.10 is appropriate to determine the most efficient of the modified EWMA control chart.

Application

In general accordance with the earth's rotation, there are actually 2 different types of rotation that need to be considered. First, the earth revolving around the Sun takes around 365 days. Second, the amount of time it takes for the earth to turn once on its axis results in the

day changing to the night and back again. It is known that it takes 24 h for the earth to rotate once on its axis. However, this is not certain as it may be longer or shorter than 24 h; for instance, the global effect of a change in the earth's rotation or polar motion on the length of a day such as the shortened day length caused by the earthquake in the Indian Ocean and the tsunami in 2004 with its epicenter off the west coast of Sumatra, Indonesia (Changyi and Wenke, 2012; Zhou *et al.*, 2014). The maximum Mercalli intensity of 9.0 magnitude of the undersea megathrust triggered a series of huge tsunamis of up to 30 m. Over 230,000 people in 14 countries have been killed so far by deadly natural disasters in the 21st century (Chadha *et al.*, 2005; Candella *et al.*, 2008; Monecke *et al.*, 2008). Therefore, perceiving changes in the Earth's rotation is very important to prepare for mitigation against the deleterious effects of a future natural disaster when the planet's rotation varies.

The annual changes in Earth's rotation by day length (unit: $\text{second} \times 10^{-5}$) from 1827–1970 is employed for the time series model. The data provides a fit for an ARMA(1,1) process with no mean and the coefficient parameters of AR and MA are 0.084 and -0.531, respectively, whereas the white noise process follows an exponential distribution with a mean of 19.76. The performance of the EWMA and modified EWMA control charts in terms of the ARL when the process mean is changed (Table 4), representing the capabilities for detecting changes in the process mean, are

Table 4. Application of changes in the earth's rotation to comparatively determine ARL for ARMA(1,1) on EWMA and modified EWMA control charts by using explicit formulas for $ARL_0 = 370$, $\mu = 0$ and $(\phi, \theta) = (0.984, -0.531)$, $\beta_0 = 19.76$ with $\lambda = 0.05, 0.1$

Shift size	$\lambda = 0.05$		$\lambda = 0.1$	
	EWMA ($l = 0.932462045$)	Modified ($b = 20.63306493$)	EWMA ($l = 1.914768403$)	Modified ($b = 22.129644$)
0.00	370.00	370.00	370.00	370.00
0.005	350.05	325.25	339.23	326.60
0.01	332.14	290.19	313.20	292.34
0.03	275.71	202.89	239.70	206.07
0.05	235.67	156.11	194.21	159.24
0.07	205.79	126.94	163.28	129.84
0.09	182.64	107.02	140.88	109.67
0.10	172.91	99.25	131.84	101.78
0.20	112.83	57.75	80.49	59.44
0.50	55.28	26.13	37.47	26.95
1.00	29.93	14.11	20.14	14.55
2.00	15.69	7.77	10.79	8.00

considered through ARL_1 , which should be as small as possible. The results indicate that the modified EWMA chart yields a lower ARL_1 than the original one for all magnitudes of changes and smoothing parameters.

Moreover, the ability of the modified EWMA control chart is also indicated by the prominent feature in Figure 1. For the statistical process of monitoring and detecting the change in the earth's rotation dataset, the EWMA control chart detects the change at the 41st observation, while the modified EWMA chart detects the change in the 9th observation, thereby demonstrating the sensitive performance of the latter when the process mean changes quickly.

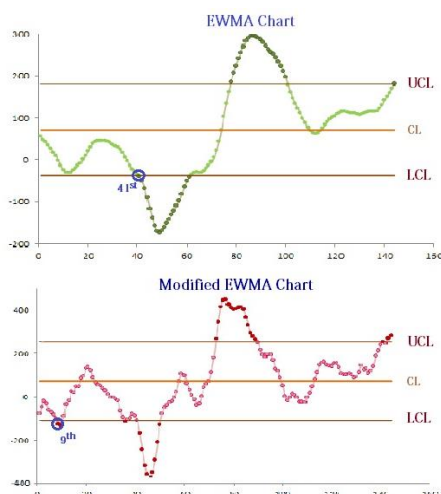


Figure1. Comparative performance of EWMA and modified EWMA charts

Conclusion and Discussion

In general, we expect to be able to monitor and detect process changes in a timely manner to maintain the high standard and quality of a process. Therefore, several control charts must be used in a variety of areas. The proposed procedure was shown to exceed the existing one (NIE) on geometric MAs in terms of the ARL evaluated through explicit formulas. The exact ARL obtained by explicit formulas demonstrated improved accuracy over the NIE

method. Although the theoretically derived ARL of the modified EWMA control chart was mostly superior to the original EWMA control chart, it was very effective when applied to real-life data for all cases of the smoothing parameter of the control chart. This was clearly visible when the observations were plotted on the control chart with its upper and lower control limits.

The results of the study allow us to recommend our method as a suitable procedure for control charts for a variety of observation characteristics in many situations. In addition, the selection of the optimal parameter value of the control chart to further enhance the sensitivity of the chart makes it even more accurate.

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