

SEPARATED, DISCONNECTED AND COMPACT SETS IN BIGENERALIZED TOPOLOGICAL SPACES

Supunnee Sompong^{1,*}, Kannika Khompungson² and Benjawan Rodjanadid³

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Abstract

In recent year, the study of bigeneralized space, which is provided with two arbitrary generalized topologies, have received increasing attention in general topology. Several mathematician have studied this concept in many direction. In this paper we introduced the concept of separated, disconnected and compact sets in bigeneralized topological spaces.

Keywords: Bigeneralized topological spaces, separated sets, connected sets, disconnected sets, compact sets

Introduction

In recent year, the study of generalized topological space, which was introduced by Császár (2002) have received increasing attention in General Topology. Specifically, the generalized topology is a subset $\mathcal{g}_X$ of the power set of X such that $\emptyset \in \mathcal{g}_X$ and arbitrary union of elements of $\mathcal{g}_X$ belongs to $\mathcal{g}_X$. It's clear that each topology is a generalized topology, but not conversely. Several mathematician have studied this concept in

many direction. Boonpok (2010) introduced the notation of bigeneralized topological spaces which are provided with two arbitrary generalized topologies. He not only studied some of the fundamental properties of close and open set in such space but also introduced the concept of weakly open functions on this space. In this paper, the concept of separated, disconnected and compact sets in bigeneralized topological spaces is presented.

¹ Department of Mathematics and Statistics, Faculty of Science and Technology, Sakon Nakhon Rajabhat University, Sakon Nakhon, 47000, Thailand. E-mail: s_sompong@snru.ac.th

² Division of Mathematics, School of Science, University of Phayao, Phayao, 56000, Thailand.

³ School of Mathematics, Institute of Science, Suranaree University of Technology, Nakhon Ratchasima, 30000, Thailand.

* Corresponding author

The paper is organized in the following manner. In section 2, we set up notation and review some of the standard fact on generalized and bigeneralized topological spaces. Section 3 is devoted to the study of separated and disconnected in bigeneralized topological spaces. Next, we present the results of compact in such space which include some useful observations.

Preliminaries

In this section we will list some concepts and some previous results which are needed for this paper.

Let X be a nonempty set and g_X be a collection of subsets of X . Then g_X is called a *generalized topology* (briefly *GT*) on X if $\emptyset \in g_X$ and if $G_i \in g_X$ for $i \in I \neq \emptyset$ then $G = \bigcup_{i \in I} G_i \in g_X$.

By (X, g_X) denote a nonempty set X with a generalized topology g_X on X and it is called a *generalized topological space* (briefly *GTS*) on X . The elements of g_X are called g_X -open sets and the complements are called g_X -closed sets. Set $g_X O(X) = \{U \subseteq X \mid U \in g_X\}$ and $g_X O(x) = \{U \in g_X \mid x \in U\}$. If $A \subseteq X$ then the union of all g_X -open sets contained in A is denoted by $g_X Int(A)$, the interior of A . Similarly, the intersection of all g_X -closed sets containing A is denoted $g_X Cl(A)$, the closure of A (Császár, 2005). In a space (X, g_X) , if g_X is closed under the finite intersection, then (X, g_X) is called a quasi-topological space (Boonpok, 2010).

Theorem (Min, 2009a) Let (X, g_X) be a GTS. Then

1. $g_X Cl(A) = X \setminus g_X Int(X \setminus A)$;
2. $g_X Int(A) = X \setminus g_X Cl(X \setminus A)$.

Proposition (Min, 2009b) Let (X, g_X) be a GTS and $A \subseteq X$. Then

1. $x \in g_X Int(A)$ if and only if there exists $V \in g_X O(x)$ such that $V \subseteq A$.
2. $x \in g_X Cl(A)$ if and only if $V \cap A \neq \emptyset$ for every $V \in g_X O(x)$.

Proposition (Dungthaisong *et al.*, 2011) Let (X, g_X) be a GTS. For any subsets A and B of X , the following properties hold;

1. $g_X Cl(X \setminus A) = X \setminus g_X Int(A)$ and $g_X Int(X \setminus A) = X \setminus g_X Cl(A)$;
2. If $X \setminus A \in g_X$ then $g_X Cl(A) = A$ and if $A \in g_X$ then $g_X Int(A) = A$;
3. If $A \subseteq B$, then $g_X Cl(A) \subseteq g_X Cl(B)$ and $g_X Int(A) \subseteq g_X Int(B)$;
4. $A \subseteq g_X Cl(A)$ and $g_X Int(A) \subseteq A$;
5. $g_X Cl(g_X Cl(A)) = g_X Cl(A)$ and $g_X Int(g_X Int(A)) = g_X Int(A)$.

Lemma (Jamunarani and Jeyanthi, 2012) Let (X, g_X) be a GTS. If (X, g_X) is a quasi-topological spaces, then the following properties hold;

1. If A and B are g_X -open sets, then $A \cap B$ is g_X -open set.
2. $g_X Int(A \cap B) = g_X Int(A) \cap g_X Int(B)$ for every subset A and B of X .
3. $g_X Cl(A \cup B) = g_X Cl(A) \cup g_X Cl(B)$ for every subset A and B of X .

Next, we recall some notions and notations of bigeneralized topological spaces.

Definition (Boonpok, 2010) Let X be a nonempty set and g_X^1, g_X^2 be generalized topologies on X . A triple (X, g_X^1, g_X^2) is called a *bigeneralized topological space* (briefly *BGTS*).

Let (X, g_X^1, g_X^2) be a BGTS and A a subset of X . The closure and the interior of A with respect to g_X^i are denoted by $g_X^i Cl(A)$ and $g_X^i Int(A)$, respectively, for $i = 1, 2$.

Definition (Boonpok, 2010) A subset A of a BGTS (X, g_X^1, g_X^2) is called $g_X^i g_X^j$ -closed if $A = g_X^i Cl(g_X^j Cl(A))$ where $i, j = 1$ or 2 and $i \neq j$. The complement of $g_X^i g_X^j$ -closed set is called $g_X^i g_X^j$ -open.

Proposition (Boonpok, 2010) Let (X, g_X^1, g_X^2) be a BGTS and $A, B \subseteq X$. Then

1. A is $g_X^i g_X^j$ -closed if and only if A is both g_X -closed in (X, g_X^i) and (X, g_X^j) .

2. If A and B are $g_X^i g_X^j$ -closed, then $A \cap B$ is $g_X^i g_X^j$ -closed.

3. A is $g_X^i g_X^j$ -open subset of (X, g_X^1, g_X^2) if and only if $A = g_X^i \text{Int}(g_X^j \text{Int}(A))$.

4. If A and B are $g_X^i g_X^j$ -open, then $A \cup B$ is $g_X^i g_X^j$ -open.

Definition (Sompong and Rodjanadid, 2016) Let (X, g_X^1, g_X^2) be a BGTS and Y be a subset of X . Define g_Y^1 and g_Y^2 as follows: $g_Y^1 = \{A \cap Y : A \in g_X^1\}$ and $g_Y^2 = \{B \cap Y : B \in g_X^2\}$. A triple (Y, g_Y^1, g_Y^2) is called a *bigeneralized topological subspace* of (X, g_X^1, g_X^2) .

Let (Y, g_Y^1, g_Y^2) be a bigeneralized topological subspace of (X, g_X^1, g_X^2) , and $A \subseteq Y$. The g_Y -closure and g_Y -interior of A with respect to g_Y^i are denoted by $g_Y^i \text{Cl}(A)$ and $g_Y^i \text{Int}(A)$, respectively for $i = 1, 2$. Then $g_Y^1 \text{Cl}(A) = Y \cap g_X^1 \text{Cl}(A)$ and $g_Y^2 \text{Cl}(A) = Y \cap g_X^2 \text{Cl}(A)$.

Proposition (Sompong and Rodjanadid, 2016) Let (Y, g_Y^1, g_Y^2) be a bigeneralized topological subspace of (X, g_X^1, g_X^2) and F a subset of Y . If F is $g_X^1 g_X^2$ -closed, then F is $g_Y^1 g_Y^2$ -closed.

Results

Some Results of g_X -Separated and g_X -Connected Set

In this section, we will restrict our attention to the study of g_X -separated and g_X -connected set, respectively. We then start with the following notation.

Definition Let (X, g_X^1, g_X^2) be a BGTS and let A and B be two non-empty subsets of X . A and B are called *g_X -separated sets* if and only if $g_X^i \text{Cl}(A) \cap B = \emptyset$ and $A \cap g_X^i \text{Cl}(B) = \emptyset$, for all $i = 1, 2$.

Example

1. Let $X = \{1, 2, 3\}$. Define g -structures g_X^1 and g_X^2 on X as follows:

$g_X^1 = \{\emptyset, \{1\}, \{1, 2\}, \{2, 3\}, X\}$ and $g_X^2 = \{\emptyset, \{1\}, \{3\}, \{1, 2\}, \{1, 3\}, X\}$. Then we have $\{1\}$

and $\{3\}$ are g_X -separated set. But $\{1\}$ and $\{2, 3\}$ are not g_X -separated set.

2. Let $X = [-10, 10]$, define g_X^1 and g_X^2 as follows:

$g_X^1 = \{[-10, 7), (b, 10), [-10, 7) \cup (b, 10) \mid b \in X, b \geq 7\}$ and $g_X^2 = \{(-10, a), (7, 10], (-10, a) \cup (7, 10] \mid a \in X, a \leq 7\}$.

Then we have $\{-10\}$ and $\{10\}$ are g_X -separated set, but $\{-10, 8\}$ and $\{10, 6\}$ are not g_X -separated set.

3. Let $X = [-1, 1]$, define $g_X^1 = \{\emptyset, [-1, 0], (0, 1], [0, 1], X\}$ and $g_X^2 = \{\emptyset, [-1, 0), [-1, 0], [0, 1], X\}$. Then $[-1, 0)$ and $(0, 1]$ are g_X -separated set.

Lemma Let (X, g_X) be a GTS and $A, G \subseteq X$. If G is g_X -open, then $G \cap A = \emptyset$ if and only if $G \cap g_X \text{Cl}(A) = \emptyset$.

Proof. Let G be g_X -open and $G \cap A = \emptyset$. Assume that $G \cap g_X \text{Cl}(A) \neq \emptyset$. Thus there exists $x \in G$ and $x \in g_X \text{Cl}(A)$. By Proposition 2.2, we have $U \cap A \neq \emptyset$ for every $U \in g_X$ containing x . Since G is g_X -open and $x \in G$, $G \cap A \neq \emptyset$ which contradicts our assumption. Thus, $G \cap g_X \text{Cl}(A) = \emptyset$.

Conversely, it is clearly seen that $G \cap A = \emptyset$.

Theorem Let (X, g_X^1, g_X^2) be a BGTS, A and B are g_X^i -open subsets of X , for all $i = 1, 2$. Then A and B are g_X -separated sets if and only if $A \cap B = \emptyset$.

Proof. It follows from Definition 3.1 and Lemma 3.3.

Theorem Let (Y, g_Y^1, g_Y^2) be a bigeneralized topological subspace of (X, g_X^1, g_X^2) and $A, B \subseteq Y$. Then A and B are g_X -separated sets if and only if A and B are g_Y -separated sets.

Proof. Suppose that $A, B \subseteq Y$, A and B are g_Y -separated sets. Thus $g_Y^i \text{Cl}(A) \cap B = \emptyset$ and $A \cap g_Y^i \text{Cl}(B) = \emptyset$. It follows that $g_X^i \text{Cl}(A) \cap B = g_X^i \text{Cl}(A) \cap Y \cap B =$

$g_Y^i Cl(A) \cap B = \emptyset$ and $A \cap g_X^i Cl(B) = A \cap g_X^i Cl(B) \cap Y = A \cap g_Y^i Cl(B) = \emptyset$, for all $i = 1, 2$. Therefore A and B are g_X -separated sets.

Conversely, suppose that A and B are g_X -separated sets. Thus $g_Y^i Cl(A) \cap B = g_X^i Cl(A) \cap Y \cap B = g_X^i Cl(A) \cap B = \emptyset$ and $A \cap g_Y^i Cl(B) = A \cap g_X^i Cl(B) \cap Y = A \cap g_X^i Cl(B) = \emptyset$, for all $i = 1, 2$. Hence A and B are g_Y -separated sets.

Theorem Let (X, g_X^1, g_X^2) be a BGTS, $\emptyset \neq A, B \subseteq X$ and $A \cap B = \emptyset$. Then A and B are g_X -separated sets if and only if A and B are $g_{A \cup B}^i$ -open and $g_{A \cup B}^i$ -closed in subspace $A \cup B$ of X , for all $i = 1, 2$.

Proof. Let $\emptyset \neq A, B \subseteq X$ and $A \cap B = \emptyset$. Assume that A and B are g_X -separated sets. Thus $g_X^i Cl(A) \cap B = \emptyset$ and $A \cap g_X^i Cl(B) = \emptyset$, for all $i = 1, 2$. For any $i \in \{1, 2\}$, we also have $g_Y^i Cl(A) = g_X^i Cl(A) \cap Y = A \cup (g_X^i Cl(A) \cap B) = A$, where $Y = A \cup B$. Thus A is g_Y^i -closed. Similarly, we also have B is g_Y^i -closed. Since $A \cap B = \emptyset, A = Y \setminus B$ and $B = Y \setminus A$. Hence A and B are g_Y^i -open.

Conversely, we assume that A and B are g_Y^i -open and g_Y^i -closed, where $Y = A \cup B$. From Lemma 3.3, we have $g_Y^i Cl(A) \cap B = \emptyset$ and $A \cap g_Y^i Cl(B) = \emptyset$, for all $i = 1, 2$. It follows that $g_X^i Cl(A) \cap B = g_Y^i Cl(A) \cap B = \emptyset$. Similarly, we obtain that $A \cap g_X^i Cl(B) = \emptyset$. Therefore A and B are g_X -separated sets.

Definition Let (X, g_X^1, g_X^2) be a BGTS and $A \subseteq X$. We called A is a g_X -disconnected set if and only if $A = C \cup D$, where C and D are g_X -separated sets and we called A is a g_X -connected set if and only if A is not g_X -disconnected set.

A BGTS (X, g_X^1, g_X^2) is called a g_X -disconnected space if X is a g_X -disconnected set and (X, g_X^1, g_X^2) is called a g_X -connected space if X is a g_X -connected set.

Example

1. Let $X = \{1, 2, 3\}$. Define g -structures g_X^1 and g_X^2 on X as follows:

$g_X^1 = \{\emptyset, \{1\}, \{1, 2\}, \{2, 3\}, X\}$ and $g_X^2 = \{\emptyset, \{1\}, \{3\}, \{1, 2\}, \{1, 3\}, X\}$. Then we have $\{1, 3\}$ is a g_X -disconnected set, because $\{1\}$ and $\{3\}$ are g_X -separated set.

2. Let $X = [-10, 10]$, define g_X^1 and g_X^2 as follows:

$g_X^1 = \{[-10, 7), (b, 10), [-10, 7) \cup (b, 10) \mid b \in X, b \geq 7\}$ and $g_X^2 = \{(-10, a), (7, 10], (-10, a) \cup (7, 10] \mid a \in X, a \leq 7\}$.

Then we have $\{-10, 10\}$ is g_X -disconnected set.

3. Let $X = [-1, 1]$, define $g_X^1 = \{\emptyset, [-1, 0], (0, 1], [0, 1], X\}$ and

$g_X^2 = \{\emptyset, [-1, 0], [-1, 0], [0, 1], X\}$.

Then $[-1, 0] \cup (0, 1]$ is a g_X -disconnected set.

Theorem Let (X, g_X^1, g_X^2) be a BGTS. Thus X is g_X -disconnected set if and only if there exists a non-empty proper subset A of X such that A is g_X^i -open and g_X^i -closed set, for all $i = 1, 2$.

Proof. First, we assume that X is g_X -disconnected set. Thus there exist a non-empty sets C and D such that $X = C \cup D$, where C and D are g_X -separated sets. Since $g_X^i Cl(C) \cap D = \emptyset$, we have that $C \cap D = \emptyset$. It follows that $C = X \setminus D$ and $D = X \setminus C$. From $X = C \cup D \subseteq C \cup g_X^i Cl(D)$, it follows that $C \cup D = C \cup g_X^i Cl(D)$. Since $C \cap g_X^i Cl(D) = \emptyset$ and $D \subseteq g_X^i Cl(D)$, we have $D = g_X^i Cl(D)$. Thus D is g_X^i -closed set and we also have C is g_X^i -open set. Similarly, we have C is g_X^i -closed set and D is g_X^i -open set, for all $i = 1, 2$.

Conversely, suppose that there exists a non-empty proper subset A of X such that A is g_X^i -open set and g_X^i -closed set, for all $i = 1, 2$. Let $B = X \setminus A$. Thus B is a non-empty proper g_X^i -closed and g_X^i -open subset of X , for all $i = 1, 2$. Therefore $X = A \cup B$ and $A \cap B = \emptyset$. Since A and B are g_X^i -open set and Lemma 3.3, $A \cap g_X^i Cl(B) = \emptyset$ and $g_X^i Cl(A) \cap B = \emptyset$, for all $i = 1, 2$. Hence X is g_X -disconnected set.

Theorem Let (X, g_X^1, g_X^2) be a BGTS. Thus X is g_X -disconnected set if and only if there exists a non-empty proper g_X^i -open subset U and V of X such that $X = U \cup V$ and $U \cap V = \emptyset$, for all $i = 1, 2$.

Proof. Assume that X is g_X -disconnected set. According to Theorem 3.9, there exists a non-empty proper subset A of X such that A is g_X^i -open set and g_X^i -closed set, for all $i = 1, 2$. Then $X \setminus A$ is a non-empty proper g_X^i -closed and g_X^i -open subset of X , which is obtained our results.

Conversely, suppose that there exist a non-empty g_X^i -open subset U and V of X such that $X = U \cup V$ and $U \cap V = \emptyset$, for all $i = 1, 2$. Since U and V are g_X^i -open sets and $U \cap V = \emptyset$, by Lemma 3.3, thus we have $U \cap g_X^i Cl(V) = \emptyset$ and $g_X^i Cl(U) \cap V = \emptyset$, for all $i = 1, 2$. Therefore X is a g_X -disconnected set.

Theorem Let (X, g_X^1, g_X^2) be a BGTS and E is a g_X -connected subset of X . If $E \subseteq A \cup B$, A and B are g_X -separated sets. Then either $E \subseteq A$ or $E \subseteq B$.

Proof. According to our assumption, we have that $E = (E \cap A) \cup (E \cap B)$. Suppose that $E \cap A \neq \emptyset$ and $E \cap B \neq \emptyset$. Since A and B are g_X -separated sets, $(E \cap A) \cap g_X^i Cl(E \cap B) \subseteq (E \cap A) \cap g_X^i Cl(B) = \emptyset$. Similarly, we also have $g_X^i Cl(E \cap A) \cap (E \cap B) = \emptyset$, for all $i = 1, 2$. Then $E \cap A$ and $E \cap B$ are g_X -separated sets. Therefore E is a g_X -disconnected set which is a contradiction. Thus $E \cap A = \emptyset$ or $E \cap B = \emptyset$. It follows that $E = E \cap B$ or $E = E \cap A$. Hence either $E \subseteq A$ or $E \subseteq B$.

Theorem Let (X, g_X^1, g_X^2) be a BGTS and E is a g_X -connected subset of X . If $E \subseteq A \subseteq g_X^i Cl(E)$, for all $i = 1, 2$, then A is a g_X -connected set.

Proof. Suppose that A is a g_X -disconnected set. Thus there exist two non-empty sets C and D such that $A = C \cup D$, C and D are g_X -separated sets. Since E is a g_X -connected set,

$E \subseteq C \cup D$, and Theorem 3.11, we have either $E \subseteq C$ or $E \subseteq D$. If $E \subseteq C$, then $D = A \cap D \subseteq g_X^i Cl(E) \cap D \subseteq g_X^i Cl(C) \cap D = \emptyset$. Contradiction with $D \neq \emptyset$. Similarly, if $E \subseteq D$, then $C = \emptyset$. Therefore A is a g_X -connected set.

Theorem Let (X, g_X^1, g_X^2) be a BGTS and $E \subseteq X$. If for any $x, y \in E$ and $x \neq y$, there exists g_X -connected set $F \subseteq E$ such that $x, y \in F$. Then E is a g_X -connected set.

Proof. Suppose that E is g_X -disconnected set. Thus there exist non-empty g_X -separated sets A and B such that $E = A \cup B$. Then we also have $A \cap B = \emptyset$ and it follows that there exist $x \in A, y \in B$ and $x \neq y$. Thus $x, y \in E$ and by assumption there exists a g_X -connected set $F \subseteq E$ such that $x, y \in F$. From Theorem 3.11 and since $F \subseteq E = A \cup B$, we have either $F \subseteq A$ or $F \subseteq B$. Then $A \cap B \neq \emptyset$. Hence E is g_X -connected set.

Theorem Let (Y, g_Y^1, g_Y^2) be a bigeneralized topological subspace of (X, g_X^1, g_X^2) , $A \subseteq Y$. Then A is a g_X -disconnected set if and only if A is a g_Y -disconnected set.

Proof. According to Theorem 3.5 and for any $C, D \subseteq Y$, we point out that C and D are g_X -separated sets if and only if C and D are g_Y -separated sets. Thus $A = C \cup D$ is g_X -disconnected sets if and only if $A = C \cup D$ is g_Y -disconnected set, which completes the proof.

Corollary Let (Y, g_Y^1, g_Y^2) be a bigeneralized topological subspace of (X, g_X^1, g_X^2) and $A \subseteq Y$. Then A is g_X -connected set if and only if A is g_Y -connected set.

Some Results of $g_X^1 g_X^2$ -Compact Set

Now, we present the concept of $g_X^1 g_X^2$ -compact set and study some of their properties. Let us now begin our study by introduce the following Definition.

Definition Let (X, g_X^1, g_X^2) be a BGTS and $F = \{F_i \mid F_i \subseteq X, i \in J\}$ be a class of subset of

X . Then F is called a *cover* of $A \subseteq X$ if and only if $A \subseteq \bigcup_{i \in J} F_i$ and if F_i is a $g_X^1 g_X^2$ -open in X for all $i \in J$, then F is called a $g_X^1 g_X^2$ -open cover of A . If J is finite set, then F is called a *finite cover* of A .

Example Let R be a set of real number, g_R^1, g_R^2 be usual topologies. Then (R, g_R^1, g_R^2) is a BGTS. Let $F = \{(x - r, x + r) \mid x, r \in R, r > 0\}$. Then $R = \bigcup_{x \in R} (x - r, x + r)$ and $(x - r, x + r)$ is $g_R^1 g_R^2$ -open in R . Hence F is $g_R^1 g_R^2$ -open cover of R .

Definition Let (X, g_X^1, g_X^2) be a BGTS and $A \subseteq X$. If F is cover of A and H is subclass of F , then H is called a *subcover* of F if H is cover of A . If H is a finite cover of A , then H is called a *finite subcover* of F .

Example Let R be a set of real number, Q be a set of rational number and g_R^1, g_R^2 be usual topologies. Then (R, g_R^1, g_R^2) is a BGTS.

From $F = \{(x - r, x + r) \mid x, r \in R, r > 0\}$ is $g_R^1 g_R^2$ -open cover of R and $H = \{(x - r, x + r) \mid x, r \in Q, r > 0\}$ is $g_R^1 g_R^2$ -open cover of R . Hence H is a subcover of F .

Definition Let (X, g_X^1, g_X^2) be a BGTS and $A \subseteq X$. Then A is said to be $g_X^1 g_X^2$ -compact, if any $g_X^1 g_X^2$ -open cover of A has a finite subcover.

Definition A BGTS (X, g_X^1, g_X^2) is said to be $g_X^1 g_X^2$ -compact, if any $g_X^1 g_X^2$ -open cover of X has a finite subcover.

Example

1. A finite bigeneralized topological space (X, g_X^1, g_X^2) is $g_X^1 g_X^2$ -compact.

2. Let R be a set of real number, g_R^1 and g_R^2 be usual topologies.

Then (R, g_R^1, g_R^2) is a BGTS. Hence we have $(0, 1)$ is not $g_R^1 g_R^2$ -compact in R .

Theorem Let a BGTS (X, g_X^1, g_X^2) be $g_X^1 g_X^2$ -compact and $F \subseteq X$. If F is $g_X^1 g_X^2$ -closed, then F is $g_X^1 g_X^2$ -compact.

Proof. Let $F \subseteq X$ be a $g_X^1 g_X^2$ -closed set and $\{G_i \mid i \in J\}$ be $g_X^1 g_X^2$ -open cover of F . Thus $\{G_i \mid i \in J\} \cup \{X \setminus F\}$ is $g_X^1 g_X^2$ -open cover of X . Since X is $g_X^1 g_X^2$ -compact, there is $i_1, i_2, \dots, i_n \in J$ such that $X \subseteq (\bigcup_{k=1}^n G_{i_k}) \cup (X \setminus F)$. It follows that $F \subseteq (\bigcup_{k=1}^n G_{i_k}) \cup (X \setminus F)$ but $F \not\subseteq (X \setminus F)$, thus $F \subseteq \bigcup_{k=1}^n G_{i_k}$. Therefore F is $g_X^1 g_X^2$ -compact.

Theorem Let (X, g_X^1, g_X^2) be a BGTS. Then X is $g_X^1 g_X^2$ -compact if and only if for any class of $g_X^1 g_X^2$ -closed sets $\{F_i \mid F_i \subseteq X, i \in J\}$ such that $\bigcap_{i \in J} F_i = \emptyset$, there is a finite subset $\{i_1, i_2, \dots, i_n\}$ of J and $\bigcap_{k=1}^n F_{i_k} = \emptyset$.

Proof. Suppose that X is $g_X^1 g_X^2$ -compact, $\{F_i \mid F_i \subseteq X, i \in J\}$ is class of $g_X^1 g_X^2$ -closed sets and $\bigcap_{i \in J} F_i = \emptyset$. Thus $X = X \setminus (\bigcap_{i \in J} F_i) = \bigcup_{i \in J} (X \setminus F_i)$. Since F_i is $g_X^1 g_X^2$ -closed set for all $i \in J$, $X \setminus F_i$ is $g_X^1 g_X^2$ -open set for all $i \in J$. It follows that $\{X \setminus F_i \mid i \in J\}$ is $g_X^1 g_X^2$ -open cover of X . By assumption, X is $g_X^1 g_X^2$ -compact, there is a finite subcover $\{X \setminus F_{i_k} \mid k = 1, 2, \dots, n\}$ such that $X = \bigcup_{k=1}^n (X \setminus F_{i_k}) = X \setminus (\bigcap_{k=1}^n F_{i_k})$. Therefore $\bigcap_{k=1}^n F_{i_k} = \emptyset$.

Conversely, Let $\{G_i \mid G_i \subseteq X, i \in J\}$ be $g_X^1 g_X^2$ -open cover of X . Thus $\{X \setminus G_i \mid i \in J\}$ is a class of $g_X^1 g_X^2$ -closed set in X . It follows that $\bigcap_{i \in J} (X \setminus G_i) = X \setminus (\bigcup_{i \in J} G_i) = \emptyset$. By assumption, we will have $\{i_1, i_2, \dots, i_n\} \subseteq J$ such that $\emptyset = \bigcap_{k=1}^n (X \setminus G_{i_k}) = X \setminus (\bigcup_{k=1}^n G_{i_k})$. Thus $X = \bigcup_{k=1}^n G_{i_k}$ is $g_X^1 g_X^2$ -compact.

Theorem Let (X, g_X^1, g_X^2) be a BGTS and $A, B \subseteq X$. If A and B are $g_X^1 g_X^2$ -compact then also $A \cup B$.

Proof. Let $\{U_i \mid i \in J\}$ be $g_X^1 g_X^2$ -open cover of $A \cup B$. Thus $\{U_i \mid i \in J\}$ is $g_X^1 g_X^2$ -open cover of A and B . Since A and B are $g_X^1 g_X^2$ -compact, there is $U_{i_1}, \dots, U_{i_n}, U_{i_{n+1}}, U_{i_{n+2}}, \dots, U_{i_{n+m}} \in \{U_i \mid i \in J\}$ such that $A \subseteq \bigcup_{k=1}^n U_{i_k}$ and $B \subseteq \bigcup_{k=n+1}^{n+m} U_{i_k}$. Therefore $A \cup B \subseteq \bigcup_{k=1}^{n+m} U_{i_k}$, which the proof is complete.

Corollary Let (X, g_X^1, g_X^2) be a BGTS and $A_i \subseteq X$ for all $i = 1, 2, \dots, n$. If A_i is $g_X^1 g_X^2$ -compact for all $i = 1, 2, \dots, n$, then $\bigcup_{i=1}^n A_i$ is $g_X^1 g_X^2$ -compact.

Conclusions

In this paper we defined g_X -separated and g_X -connected set and provided a appropriate conditions on set which can be separated and connected set in bigeneralized topological space. Moreover, we not only studied the behavior of disconnected set but also introduced the concept of compact set in this space.

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