

SIMULATION OF CRACK PATH IN FOAM-CORE CRACKED SANDWICH BEAM SPECIMEN

Paiboon Limpitipanich*

Received: April 01, 2018; Revised: April 30, 2018; Accepted: May 07, 2018

Abstract

A crack path simulation in the core of a mixed mode I and mode II cracked sandwich beam (CSB) specimen, called a non-symmetric CSB, is presented. The loading configuration of the specimen was identical to that used for the mode I condition (crack opening loading). A combination of mode I and mode II loading was achieved by manipulating the apparent flexural stiffness of the upper and lower face/core composite. The simulation of the crack path revealed that the crack grows toward the face with lower flexural stiffness before being propagated to the effective position in the longitudinal strain of the upper and lower parts of the crack plane. The position may be either in the core, in the interface layer, or in the face of the sandwich beam. Simulations of the crack path in pure mode I and pure mode II CSB specimens that were achieved by applying a crack-opening and shear force to the specimen are also demonstrated. The crack path simulation in this study was performed using the results of the finite element analysis in combination with the virtual crack-closure technique and the maximum tangential stress criterion.

Keywords: Crack path simulation, cracked sandwich beam, mixed mode fracture

Introduction

Sandwich structures have been used in many applications because of their generally high strength-to-weight and stiffness-to-weight ratios, good thermal insulation, and good sound absorption characteristics. The strength of the sandwich panel is dramatically reduced when a defect exists in any of its 3 components: the face,

the core, or the adhesive bonding the face and the core. One of the commonly studied failures in a panel is the interfacial fracture between the face and the core. Several specimen configurations have been used for studying the interface failure behavior and fracture toughness of the interface (Carlsson and Prasad, 1993;

Department of Mechanical Engineering, Faculty of Engineering, Burapha University, Chonburi, 20131, Thailand. Tel 0-3810-2222. E-mail: paiboonl@eng.buu.ac.th

** Corresponding author*

Li and Carlsson, 1999; Shivakumar and Smith, 2004; Yokozeki, 2011; Banghai *et al.*, 2015; Odessa *et al.*, 2017). Existing defects in the core are also another serious type of failure, especially for a panel that has a polymeric foam core. The defect in the foam can exist because of an imperfection in the foam material, such as expanded polystyrene (EPS), since the foam sheet is manufactured by expanding polystyrene foam beads. Failure behavior and fracture toughness of the foam core can be obtained using a cracked sandwich beam (CSB) specimen having a pre-crack in the core, as shown in Figure 1.

The problem in testing a foam-core CSB, especially under a mixed mode condition, is that the crack does not grow in a self-similar manner, so it is difficult to observe the onset of the crack's growth. Using a microscope or a high magnification lens helps us to detect the crack tip, but cannot predict the direction of the crack growth. Simulation of the propagation of the crack path allows us to know the growth direction and trajectory of the crack prior to testing.

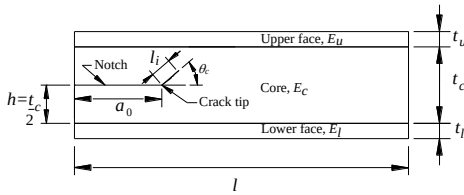


Figure 1. Foam-core CSB specimen having crack in the mid-core layer

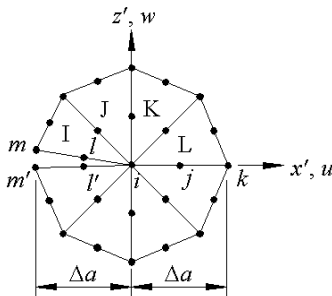


Figure 2. Mesh at the crack tip

The purpose of this paper is to simulate the crack growth path in the core of CSB specimens under mode I, mode II, and mixed mode I and mode II loading. Finite element analysis is used as a tool for simulating the growth of the crack. It is assumed that the crack propagates from the crack tip by an incremental amount l_i in the kinked direction of θ_c , as shown in Figure 1. The specimen used in this study differs from that used for studying the interfacial failure. Commonly, an interfacial failure specimen has identical upper and lower faces, but faces are varied to manipulate the initial mode mixture in this study.

Methods

Computation of Strain Energy Release Rate

Crack path modeling was predicted via the values of the mode I and mode II strain energy release rates, G_I and G_{II} , that were calculated on the basis of Irwin's crack closure integral (CCI) (Irwin, 1957; Tada *et al.*, 2000). Rybicki and Kanninen (1977) applied Irwin's CCI to evaluate G_I and G_{II} using nodal forces and displacements from a 4-node quadrilateral element of the finite element analysis. The method is called the virtual crack-closure technique. Raju (1987) applied the method to evaluate the strain energy release rate for 8-node parabolic and 12-node cubic elements. For a triangular element (used as a crack-tip element in this study, see Figure 2), G_I and G_{II} can be computed from:

$$G_I = \frac{1}{2\Delta a} [F_{z'_i}(w_m - w_{m'}) + F_{z'_j}(w_l - w_{l'})] \quad (1a)$$

$$G_{II} = \frac{1}{2\Delta a} [F_{x'_i}(u_m - u_{m'}) + F_{x'_j}(u_l - u_{l'})] \quad (1b)$$

where $F_{x'}$ and $F_{z'}$ are the shear and crack opening forces, u and w are the nodal displacements in the x' - and z' -directions, and Δa is an element size. Subscripts i, j, m, m', l , and l' refer to the nodal quantities of the crack

closure elements from the finite element results. The triangular element as shown in Figure 2 is obtained by collapsing 1 side of an 8-node element (Raju, 1987). It should be noted that the nodal force at node i should be computed from elements I, J, K, and L and the nodal force at node j should be computed from element L alone (Narayana and Dattajuru, 1996). The summation of Equation (1) defines the total strain energy release rate, assuming that there is no mode III fracture involved:

$$G_{\text{total}} = G_I + G_{II} \quad (2)$$

The strain energy release rate is the amount of strain energy that is released when the crack propagates. It should be noted that the crack will propagate when the following expression is fulfilled (Broek, 1986):

$$\frac{G_I}{G_{Ic}} + \frac{G_{II}}{G_{IIc}} = 1 \quad (3)$$

where G_{Ic} and G_{IIc} are the critical mode I and mode II strain energy release rates, respectively.

The core material of this study was expanded polystyrene (EPS) foam. EPS has a low elastic modulus value and it is a low yield strength material as listed in Table 1. Thus, elasto-plastic fracture mechanics (EPFM) should be used. However, this study emphasizes crack path simulation; the critical loads that cause crack growth are not presented and linear-elastic fracture mechanics (LEFM) is enough to calculate the crack path direction as presented in the next subsection. So, LEFM was used instead of using EPFM.

Perfect interface bonding of an EPS core and face sheets was assumed due to its

geometry. EPS consists of a plastic matrix and gas encased in beads that are thermo-compressively bonded together. Then the bead bonding strength is lower than the interface strength between the face sheets and the EPS foam core.

Computation of Crack Extension Direction

The direction of crack extension for linear elastic fracture mechanics can be computed via 3 well-known methods: the maximum tangential stress criterion, the minimum strain energy density criterion, and the maximum strain energy release rate criterion. The theories state that the crack will start to grow from the crack tip in the radial direction that has the greatest tension, minimum strain energy density, and maximum strain energy.

In the current study, the maximum tangential stress criterion was selected to compute the kinked angle (Figure 1) using the following expression (Erdoğan and Sih, 1963):

$$\theta_c = 2 \tan^{-1} \left(\frac{1}{4} \frac{K_I}{K_{II}} - \frac{1}{4} \sqrt{\left(\frac{K_I}{K_{II}} \right)^2 + 8} \right) \quad (4)$$

where K_I and K_{II} are mode I and mode II stress intensity factors. Irwin (1957) showed that the stress intensity factor is related to the strain energy release rate by

$$K_I^2 = E' G_I \text{ and } K_{II}^2 = E' G_{II} \quad (5)$$

where $E' = E$ and $E/(1-\nu^2)$ for plane stress and plane strain conditions, respectively, E is the modulus of elasticity, and ν is Poisson's ratio. The kinked angle in Equation (4) then can be expressed in the form of G_I and G_{II} as:

Table 1. Material properties

Material	Modulus of elasticity, E (MPa)	Poisson's ratio, ν
Acrylic	3,000	0.40
Aluminum	69,000	0.33
EPS foam	7.2	0.05

$$\theta_c = \begin{cases} 2 \tan^{-1} \left(\frac{1}{4} \sqrt{\frac{G_I}{G_{II}}} - \frac{1}{4} \sqrt{\frac{G_I}{G_{II}} + 8} \right), & \text{if } \tau_i \geq 0 \\ -2 \tan^{-1} \left(\frac{1}{4} \sqrt{\frac{G_I}{G_{II}}} - \frac{1}{4} \sqrt{\frac{G_I}{G_{II}} + 8} \right), & \text{if } \tau_i < 0 \end{cases} \quad (6)$$

The crack-tip shear-stress, τ_i , is used as an indication of the direction of the crack growth: upward or downward. The positive and negative shear stresses orient the crack to be downward and upward, respectively, as corresponding to the principal plane. Term G_I/G_{II} in Equation (6) must not be less than 0, and G_I and G_{II} should have the same sign according to the sign convention of a scalar quantity.

Finite Element Simulation of Crack Path

In the current study, the foam-core CSB was modeled and the nodal forces and displacements determined by the finite element method. The finite element mesh used is shown in Figure 3. A coarse mesh with dominant 8-node quadratic plane strain elements and 6-node quadratic plane strain elements was used throughout the overall specimen. Around the crack tip region, a fine mesh with 8-node elements was used as shown. Eight triangular elements obtained by collapsing 1 side of each element were used around the crack tip node as shown in Figure 2.

The computation procedure for predicting the crack growth path is presented in Figure 4. It was done by first solving the finite element model of the CSB specimen to obtain the nodal forces and displacements at the crack tip elements. The energy release rates G_I and G_{II} and the kinked angle were computed from Equations (1) and (6), respectively. After knowing

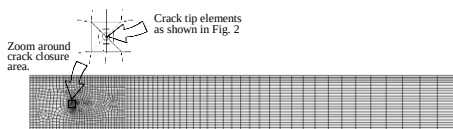


Figure 3. Finite element mesh of foam-core CSB specimen

the crack growth direction, the incremental crack growth was applied to the fracture model by the amount of an incremental length l_i . The procedure was repeated until a satisfactory crack length was obtained. It should be noted that the incremental length l_i should be small enough in order to obtain an actual crack path profile. A length l_i of 1 millimeter was selected in this study. Larger increment lengths of 2.5, 5.0 and 7.5 millimeters may be used for a rough crack profile calculation, as shown in Figure 5.

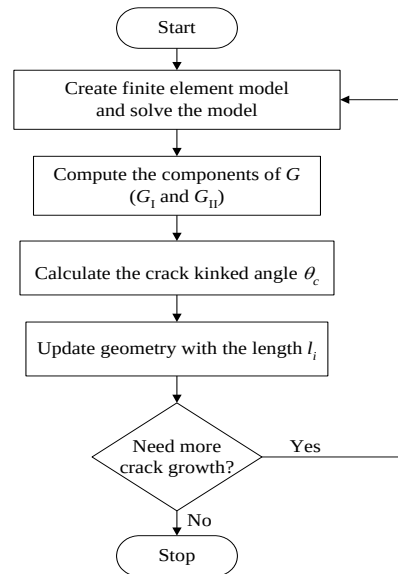


Figure 4. Simulation procedure for predicting the crack growth path

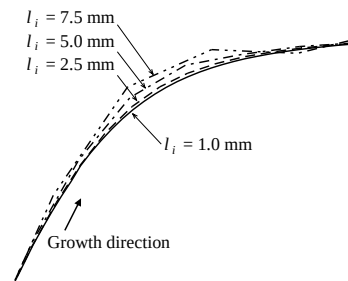


Figure 5. Crack path profile resulting from varying the incremental crack growth length l_i

Results and Discussion

Numerical simulations of the crack path profile in the core of the CSB specimen were described for 3 different cases: mode I, mode II, and mixed mode I and mode II fracture conditions. Acrylic and aluminum were selected as the faces of the specimen. EPS foam having a density of 15.4 kg/m³ was used as the core. The properties of the materials used in the current study are listed in Table 1.

Mode I Fracture Specimen

The foam-core CSB specimen, having identical upper and lower face-sheets, was used for modeling the crack path under mode I fracture conditions. Aluminum with a thickness of 3.175 mm was used for the faces. The specimen was loaded under the mode I crack-opening with a constant loading rate under a quasi-static condition. Even though the strain rate affects the material properties of the EPS foam, since only the crack path was of interest, the strain rate effect was not included. The specimen dimensions and configuration are shown in Figure 6(a). Because the upper and lower beams (over and under the plane of the notch) of the specimen are identical, the kinked angle θ_c for the start of the crack propagation is expected to be 0, so that the crack will propagate in a self-similar manner and be perpendicular

to the direction of the unit applied load P . In other words, the crack should remain in the mid-plane of the specimen.

Finite element modeling of the crack path profile for the start of the crack propagation, and iterations 20, 40, and 60 are shown in Figure 6(b), also represented by a dash line in Figure 6(a). It shows that the crack path trajectory was straight and remained in the mid-layer of the core as expected.

Technically, the mode I CSB specimen need not be identical in the upper and lower face sheets; the apparent flexural stiffness, D_e , of the upper and lower face/core composites is what must be equal. The apparent flexural stiffness is used for a non-symmetric laminate or sandwich beam (Vinson 1999) and can be evaluated from the well-known expression as:

$$D_e = D - \frac{B^2}{A} \quad (7)$$

where A , B , and D are called the extensional stiffness, coupling stiffness, and bending stiffness, respectively. A , B , and D for the face and a half core composite can be computed from the following expressions:

$$A = \frac{E_f}{(1-\nu_f^2)} t_f + \frac{E_c}{(1-\nu_c^2)} \frac{t_c}{2} \quad (8a)$$

$$B = \frac{1}{4} t_f t_c \left(\frac{E_f}{(1-\nu_f^2)} - \frac{E_c}{(1-\nu_c^2)} \right) \quad (8b)$$

$$D = \frac{E_f}{12(1-\nu_f^2)} (t_f^3 + 3t_c^2 t_f) + \frac{E_c}{96(1-\nu_c^2)} (t_c^3 + 12t_f^2 t_c) \quad (8c)$$

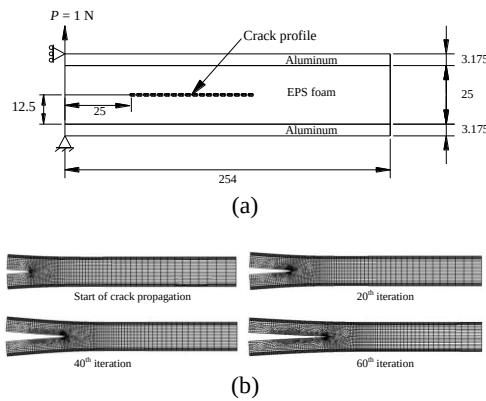


Figure 6. Mode I CSB specimen: (a) specimen configuration (dimensions are in millimeters), and (b) finite element modeling resulting crack profile

In an actual situation of the crack extension of an EPS foam core, the crack profile of the foam core are not straight as simulated in Figure 6. Figure 7 shows the upper and lower profiles of the crack in the EPS foam-core CSB specimen. The specimen was pulled under the crack opening mode as discussed. It can be seen that the crack does not quite propagate in a straight line perpendicular to the applied load. Instead, the crack profile is

jagged and uneven. This is because EPS foam consists of beads bonded together with thermo-compressive bonds. The bead is made of a plastic matrix and gas encased in the beads. However, the overall path of the crack remains perpendicular to the pull direction, as described by Limpitpanich and Rusmee (2004).

For the possibility of the growth of a crack in an EPS foam bead, Stupak *et al.* (1991) and Mills and Kang (1994) described the mechanism of crack growth and reported that even when the crack was starting in the foam bead, there were 2 possibilities for crack growth: growth around and close to the bead boundary as described in Figure 8(a) and 8(b), respectively. However, Limpitpanich and Rusmee (2004) found that the crack could propagate through the beads as shown in

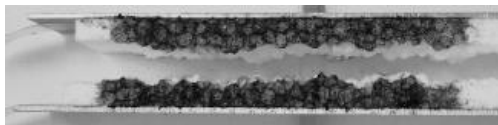


Figure 7. Actual crack growth path profile in the core of EPS foam-core CSB specimen

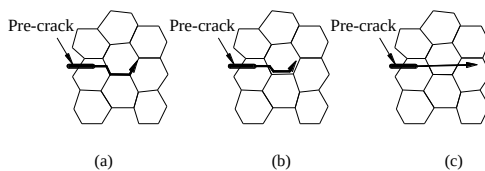


Figure 8. Sketches of crack growth mechanisms: (a) around bead boundary fracture, (b) close to bead boundary fracture, and (c) through bead fracture

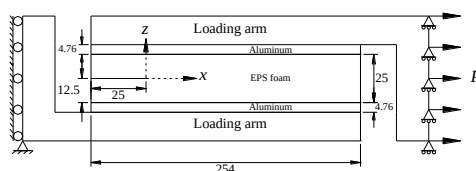


Figure 9. Mode II fracture configuration (dimensions are in millimeters)

Figure 8(c). Good quality foam would produce a crack surface having a mixture of 3 types as the crack front weaves around, finding the weakest point in the foam. On a local scale, the crack front may or may not propagate in the direction as predicted by the finite element analysis simulation. However, the overall crack path being horizontal illustrates that the simulation predicting the crack path on a non-local scale will be applicable.

Mode II Fracture Specimen

For the mode II fracture condition or the sliding mode, the applied load should be in the longitudinal direction resulting in a shear force being applied. A pure shear configuration can be achieved as shown schematically in Figure 9. The function of the loading arm is to apply a pure shear load to the foam-core CSB specimen. They are made from steel that has much higher stiffness than the foam-core specimen. It is expected that the crack should initially grow with a kinked angle of 70.53° since G_I is 0.

Finite element modeling of the crack path profile for the start of the crack propagation, and iterations 5, 10, and 15 are shown in Figure 10(a). The simulation shows that the crack

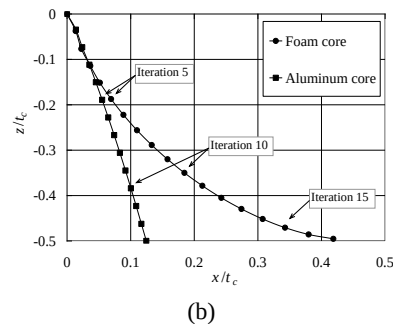
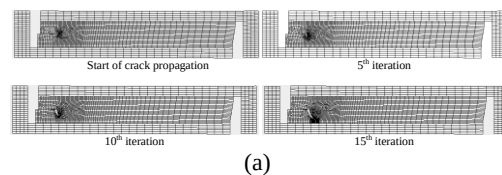


Figure 10. Mode II Crack path profile: (a) finite element modeling, and (b) graphical representation comparing with mode II single material

initially grew downward under the mode II fracture with the kinked angle around 70.04° , showing 0.69 percent deviation from the expected angle. It propagated directly towards the lower face/core interface, but veered toward the longitudinal direction before reaching the interface. It should be noted that the crack may grow toward the upper or lower faces of the specimen depending on the direction of the sliding mode. The crack can grow toward the upper face in the case that an opposite load direction is applied.

The crack profile for a CSB specimen having aluminum as a core was simulated and compared with that of the foam-core specimen. Both results are shown graphically in Figure 10(b). The position of the crack path was normalized to the thickness of the core. For the single material specimen, the crack grew directly to the lower edge within the longitudinal distance of 0.13 of the core thickness as shown. The reason for the difference in the crack path of the single material and the foam-core specimens was that the EPS foam core is very soft compared with the aluminum face-sheet (of the order of 10^4), so the crack profile does not grow directly toward the interface as occurs in a single material specimen.

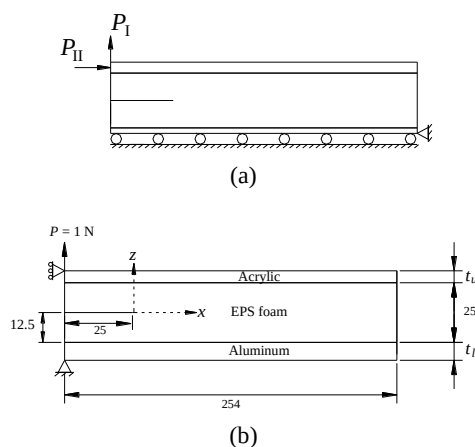


Figure 11. Mixed Mode I and Mode II configurations: (a) changing the load direction, and (b) using non-symmetric foam-core CSB (dimensions are in millimeters)

Mixed Mode I and Mode II Fracture Specimen

Various specimen configurations or loading methods can be used to obtain mixed the mode I and mode II condition at the crack tip. Changing the loading configuration to achieve a mode fracture can be done by either using the configuration as specified in mixed mode delamination of a composite material according to the ASTM D-6671 or by changing the direction of loading (Carlsson and Prasad, 1993) as shown in Figure 11(a). The ASTM D-6671 method is not recommended because it has the potential to compress the soft foam core beyond its yield strength. Changing the load direction requires complex fixturing that does not lend itself to the current study. Changing the configuration of the CSB specimen was the choice chosen in this study. A combination of mode I and mode II can be achieved at the crack tip by using non-symmetric upper and lower beams (over and under the plane of the notch) as described by Bruno and Greco (2000).

The non-symmetric foam-core CSB specimen in this study was obtained by using either different facing materials or face geometry resulting in a difference between the upper and lower apparent flexural stiffnesses of the face/core composite, D_{eu} and D_{el} . Changing the face geometry for this purpose is not limited to the faces having rectangular cross sections, but also other face cross sections, such as an I-beam. In this paper, however, only specimens having a rectangular cross section were used for the crack path simulation. Varying the thickness of the faces and using different facing materials results in a variation of the ratio of the lower and upper apparent flexural stiffnesses, D_{el}/D_{eu} . The specimen and loading configurations are shown in Figure 11(b). The upper and lower faces were acrylic and aluminum, respectively, of varying thickness, while 25-mm thick EPS foam was used for the sandwich core.

As described above, the varying of the upper and lower stiffnesses results in variation of the mode mixture. Figure 12 shows the initial mode mixture, G_{II}/G_I , and the corresponding kinked angle over the range of

lower stiffness, D_{el} , per constant upper stiffness, D_{eu} . The plot of D_u is normalized by flexural stiffness of a half core, $D_c = 0.147 \text{ N}\cdot\text{m}^2$. It can be seen that for low upper stiffness ($D_{eu}/D_c = 5$ and 10), the percentage of mixed mode depends highly on how large the difference is between D_{el} and D_{eu} , but G_{II}/G_I is less variable and not greater than 0.13 , or 11.43 percent of mode II, for specimens having high upper stiffness ($D_{eu}/D_c \geq 100$). This suggests that even though the specimen is loaded under mode I crack-opening, a mixed mode I and mode II fracture can be achieved by manipulating either the facing material or face geometry.

According to Figure 12, a crack under the mixed mode condition starts propagating from the crack tip, so the crack is expected to grow directly toward the face. However, the simulation found that there are 2 possibilities of the crack beginning to grow: the crack may propagate reaching the interface or may turn in the horizontal direction at sub-interface depth d

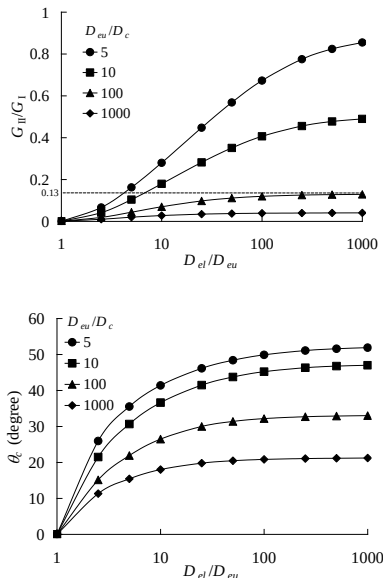


Figure 12. Mode mixtures (left) and corresponding crack kinked angle (right) for the start of crack growth when varying the upper and lower face/core flexural stiffness

away from the interface. This can be demonstrated by finite element modeling of 2 different crack profiles as shown in Figure 13. Figure 13(a) shows that the crack grew directly to the upper face and reached the interface at iteration 15, and then the crack turned into the interface layer. Figure 13(b) shows that the crack grew toward the interface but turned in the horizontal direction at a depth d at iteration 30. It should be noted that the crack will propagate toward the lower stiffness face, e.g. the upper face in Figure 13, and find the effective position of longitudinal strains and curvature of the upper and lower face/core composites. The effective position for the specimen in Figure 13(a) is in the upper face, but the fracture toughness of aluminum is very high compared with that of the adhesive and foam core, so the crack continually propagates in the interface layer.

In order to explain the effect of D_{el}/D_{eu} on the sub-interface depth, the crack profile and sub-interface depth for specimens using 4-mm thick acrylic for the upper face ($D_{eu}/D_c = 22.4$) are presented in Figure 14. It can be seen that the crack profiles show the same behavior as the specimen in Figure 13(b). The sub-interface depth depends on D_{el}/D_{eu} and converges to 1.80 mm for $D_{el}/D_{eu} \geq 100$ as shown. As the upper and the lower flexural stiffnesses of the faces are equal ($D_{el} = D_{eu}$),

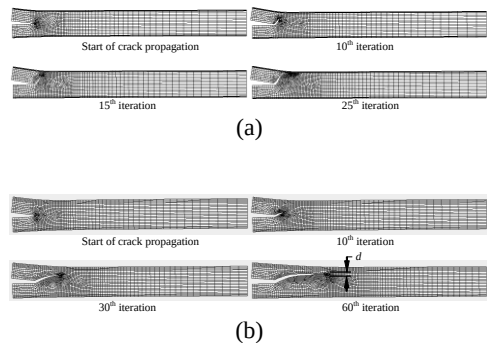


Figure 13. Two types of crack path behavior: (a) crack grows in face/core interface ($D_{el}/D_{eu} = 5, D_{eu}/D_c = 5$), and (b) crack grows sub-interface ($D_{el}/D_{eu} = 5, D_{eu}/D_c = 22.4$)

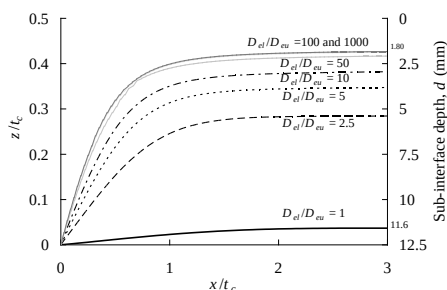


Figure 14. Crack profiles and sub-interface depth for the mixed mode CSB specimens. 4-mm acrylic is used for upper face ($D_{eu}/D_c = 24$)

the crack travels in the mid-plane of the core as in the pure mode I case. However, the crack does not exactly grow in the mid-core layer. It grows 11.6-mm in the sub-interface or deviates from the mid-plane of the core with a distance of 0.9 mm. The reason is that D_e is computed from the face/core composite with a high stiffness ratio of the order of 10^2 and 10^4 for acrylic/EPS foam and aluminum/EPS foam systems, respectively.

Conclusions

Finite element modeling in combination with the virtual crack-closure technique and the maximum tangential stress criterion were used for a simulation of the crack path profile in fracture specimens. The simulation procedure can be applied to other specimen types, such as a compact tension specimen. This is useful for knowing the direction of crack growth prior to testing and can be applied for predicting and controlling the crack to propagate in the desired direction.

The simulation of the crack path in the core of foam-core CSB specimens was presented in 3 different cases: mode I, mode II, and mixed mode I and mode II. The mode I case was achieved by applying the mode I crack-opening to the symmetric CSB specimen. The simulation revealed that the crack grew in the mid-plane of the core and was perpendicular to the direction of the applied load. The concept

of pure shear was applied to a foam-core CSB specimen and it was found that the crack started to grow with a kinked angle of 70.04° (with 0.69 percent deviation from the pure mode II kinked angle) directly to the face/core interface as the single material specimen did, but it veered toward the longitudinal direction before reaching the interface.

The mixed mode I and mode II case was achieved by applying a mode I crack-opening load to the non-symmetric foam-core CSB specimen that has a different facing material or face geometry. The simulation showed that the crack started to grow under the mixed mode condition depending on the upper and lower face/core material and apparent flexural stiffness. The crack grew toward the face/core interface having lower stiffness, to find the effective position which may be in the core, adhesive layer, or the face. In the case that the effective position was in the face and the adhesive layer, the crack grew along the interface layer. On the contrary, the crack turned in the horizontal direction in the core and propagated sub-interface for the specimen having an effective position in the core.

Acknowledgments

This work was supported by the Faculty of Engineering, Burapha University, Chonburi, Thailand, under contract no. 1/2551.

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