

STRENGTH PREDICTION OF LAP SPLICE CONFINED WITH FIBER REINFORCED POLYMER (FRP)

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Abstract

This paper presents a tri-uniform bond stress model for simulating the bond stress distribution along the lap splice length at the critical post-yield force of a reinforcing bar. The bond distribution consists of 3 parts, namely, the splitting part, the post-splitting part, and the yielding part. In each part, the bond stress is assumed to be constant. The model is applicable to simulate the bond strength of the lap splice strengthened by both transverse steel and fiber reinforced polymer (FRP). The lap splice strength in either the elastic range or post-yield range can be predicted by applying the proposed tri-uniform model. The strength of the lap splice calculated by the proposed model agrees well with experimental results of the columns confined by FRP sheets reported by other researchers. In general, it is found that the strength of the lap splice increases with increasing lap splice length, cover to bar diameter ratio and number of the FRP sheets.

Keywords: Post-yield, laps splice strength, bond stress model, nonlinear model, fiber-reinforced polymer

Introduction

The strength of the lap splice is very important for the development of strength and ductility of a reinforced concrete column. The investigations of buildings constructed according to a sub-standard (pre-1971) seismic design method by Chai *et al.* (1991) and Melek *et al.* (2003) have led to the conclusions that lap splices in sub-standard columns were typically designed as compression splices with the lap length of about 20 to 24 times bar diameter, and were poorly confined by a small amount of transverse steels. Consequently, lap splice failure was widely

reported as the primary failure mode that caused low lateral strength and poor ductility in cyclic responses of sub-standard reinforced concrete columns subjected to seismic excitation. Various methods of strengthening the short lap splice zone have been studied, such as external confinement by column or steel jacketing and fiber reinforced polymer sheets.

Fiber-Reinforced Polymer (FRP) sheets are now considered the state-of-the-art technology in rehabilitation and strengthening of reinforced concrete (RC) structures. Due to

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the confinement effect, the bond stress between the reinforcing bar and surrounding concrete is enhanced, thus increasing the lap splice strength. Some RC columns confined with FRP were tested by Xiao and Ma (1997); Ma *et al.* (2000); Bousias *et al.* (2006); Harajli and Dagher (2008). The tests showed that FRP confinement could prevent the lap splice failure before yielding, in other words, the stress of the short lap splice bar can be developed into the post-yield range.

To estimate the lap splice strength, some bond slip models were proposed. One of the most widely used bond slip models was proposed by Eligehausen *et al.* (1983) based on an experimental program at the University of California, Berkeley, but the model is only applicable for bars in the elastic range. Based on Eligehausen's works, Harajli (2006) proposed a bond slip model based on the test results of beams with a lap splice length of 5 times bar diameter at the middle of the beam. This model can be used for estimating the strength of lap splices confined by both transverse steel and FRP sheets, but in pre-yield range only.

The bond characteristics in the post-yield range of steel bars must be known to estimate the strength of the lap splice. Hassan and Hawkins (1977) developed the model for predicting the pull-out strength of an anchored steel bar in the post-yield range by assuming the distribution of the bond stress along the lap

splice length. An investigation of the bond-slip and stress-strain relationships in the post-yield range of reinforcing bars embedded in massive concrete was conducted by Murayama *et al.* (1986), but there was no formula drawn. Another long embedment pull-out test series was conducted by Shima *et al.* (1987) to investigate the bond characteristics in the post-yield range of deformed bars. The result showed the bond stress distribution in the yielding zone of the bars. However, these studies have not focused on the effect of FRP confinement on bond stress characteristics in the post-yield range. In the past studies, there was very scarce information on a bond slip model or bond stress distribution of the lap splice strengthened by FRP in the post yield range of reinforcing bars. This paper attempts to propose the model to predict the lap splice strength of bars in the post-yield range.

Post-yield Strength of Lap Splice Confined by FRP

Uniaxial Stress-strain Model for Confined Concrete

The uniaxial stress-strain relationship of confined concrete is an important factor that affects the lap splice strength. A uniaxial stress-strain model for concrete confined with either FRP or transverse reinforcement as proposed by Harajli *et al.* (2006) is adopted to calculate the compressive strength of the

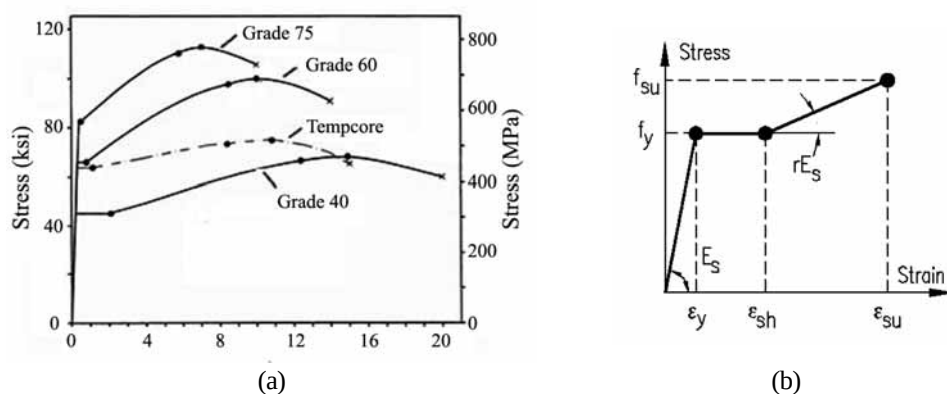


Figure 1. Tri-linear stress-strain model for steel bars (Park and Paulay, 1975)

confined concrete.

$$f'_{cc} = f'_c + k_1 \left(f_{lf} + f_{ls} \frac{A_{cc}}{A_g} \right) \quad (1)$$

$$\epsilon_{cc} = \epsilon_0 \left(1 + k_2 \left(\frac{f'_{cc}}{f'_c} - 1 \right) \right) \quad (2)$$

where A_{cc} is the area of the concrete core, f_{lf} ; f_{ls} are the lateral passive confining pressure exerted by FRP and the ordinary transverse steel respectively, and k_1 ; k_2 are the confinement effectiveness coefficients. The model is applicable to various conditions of cross section (rectangular and circular), wrapping method (partial and full), thickness and number of FRP sheets, and types of steel stirrups (spiral and tie).

Uniaxial Stress and Strain Model for Steel Bar

A tri-linear stress-strain model is adopted from Park and Paulay (1975). Figure 1(a) shows the experimental stress-strain curve for steel grades 40, 60, and 75. Figure 1(b) shows the idealized tri-linear stress-strain model. The steel stress-strain relationship is described by equations (3) to (5).

$$f_s = E_s \epsilon_s \quad \text{If } \epsilon_s \leq \epsilon_y = f_y / E_s \quad (3)$$

$$f_s = f_y \quad \text{If } \epsilon_y \leq \epsilon_s \leq \epsilon_{sh} \quad (4)$$

$$f_s = f_y + r E_s (\epsilon_s - \epsilon_{sh}) \quad \text{If } \epsilon_{sh} \leq \epsilon_s \leq \epsilon_{su} \quad (5)$$

where f_s ; ϵ_s are the bar stress and strain, E_s is the modulus of elasticity or Young modulus of steel, f_y ; ϵ_y are the yield stress and yield strain of steel, ϵ_{sh} is the bar strain at the start of the strain hardening, and ϵ_{su} is the strain at the steel fracture.

Bond - slip Model of Spliced Bars Confined with FRP

Since wrapping FRP around the lap splice zone provides additional lateral stress, thereby increasing the bond stress acting on the spliced

bar. In order to simulate the bond-slip behavior of spliced bars strengthened by FRP, a bond-slip model (Figure 2) by Harajli (2006) is adopted. In this model, the peak bond stress u_{sp} and corresponding slip s_{sp} at bond splitting failure are expressed in equations (6) and (7) respectively. In the equations, u_m is the maximum bond stress at pullout mode of the bond failure given as $u_m = 2.57 \sqrt{f'_c} \cdot n_f$ and t_f are the number of FRP layers and the thickness of FRP sheet, respectively. The post-splitting bond strength u_p for FRP confined concrete is calculated using equation (8). The factor α_f representing the method of wrapping is expressed by equation (9), in which N_f is the number of FRP strips with equal width b_f . For full wrapping, $\alpha_f = 1$.

$$u_{sp} = 0.75 \sqrt{f'_c} \left(\frac{c + 56 \frac{E_f \alpha_f n_f t_f}{E_s n_s}}{d_b} \right)^{2/3} \leq u_m \quad (6)$$

$$s_{sp} = s_1 e^{3.3 \ln \left(\frac{u_{sp}}{u_m} \right)} + s_0 \ln \left(\frac{u_m}{u_{sp}} \right) \quad (7)$$

$$u_p = u_{sp} \left(0.5 + 46 \frac{E_f \alpha_f n_f t_f}{E_s c n_s} \right) \leq u_{sp} \quad (8)$$

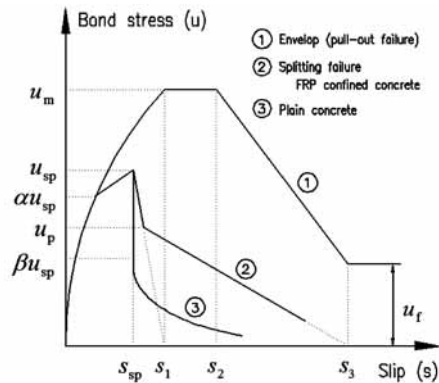


Figure 2. Bond stress-slip model by Harajli (2006)

$$\alpha_f = \frac{N_f b_f}{L_s} \quad (9)$$

Tri-uniform Bond Stress Model

Figure 3 shows a column with longitudinal bars spliced with the starter bars at the base of column. When the load is applied to the column, a crack opens at the interface between the column base and footing (point O). Consider the outermost bar on the tension side; the developed bar stress $f_s \geq f_y$ at the starting point of lap splice zone (point O) must be in equilibrium with the bond stress on the bar surface along the lap splice zone. The bond stress distribution along the lap splice length L_s depends on many factors such as the pull-out force $A_s f_s$, the length of lap splice L_s , the confinement condition, and so on.

An example of bond stress distribution before lap splice failure state is shown in Figure 3-(1), in which u_y and u_e , are bond stresses acting on the yielding part and elastic part, respectively.

In the post-yield critical state of splitting failure, the bond stress distribution along the lap splice length is assumed to follow the tri-uniform stress model as shown in Figure 3-(2a). In the model, the lap splice length is divided into 3 parts; namely, yielding part (OA), post-splitting part (AB) and splitting part (BC),

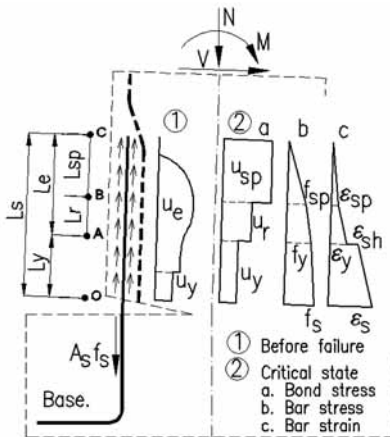


Figure 3. Proposed tri-uniform bond stress model

with bond stresses acting in each part denoted as u_y , u_r , and u_{sp} respectively. The lengths of the yielding part, post-splitting part, and splitting part are L_y , L_r and L_{sp} respectively. The length of these 3 parts must be summed to the lap splice length L_s .

$$L_s = L_y + L_r + L_{sp} \quad (10)$$

It is noted that one of the differences between this model and the bi-uniform stress model proposed by Sezen and Moehle (2003) is that the bond stress was not constant along the elastic part (AC), length L_e . Figure 3-(2b, 2c) shows the tri-linear distribution of the reinforcing bar stress and strain, in which the bar stress and strain are zero at the end of lap splice (C). In the post-splitting part, the bar stress and strain are at yielding (f_y and ϵ_y) at the start point (A) and at the peak splitting point (f_{sp} and ϵ_{sp}) at the end point (B). At point A which separates the bar stress into the elastic and yielding zone, the strain shows a jump from ϵ_y to ϵ_{sh} , which is the strain at the start of strain hardening.

Figure 4 represents the free-body diagrams of the splitting part (Figure 4(a)), the post-splitting part (Figure 4(b)) and the yielding part (Figure 4(c)). In the next section, the equilibrium condition of these parts and condition of the slip along the lap splice to satisfy the bond slip model will be described.

Equilibrium Condition for Splitting Part

A free body diagram and distribution of bar stress and bar strain along this part are shown in Figure 4(a). The bond stress and bar stress have to satisfy equation (11).

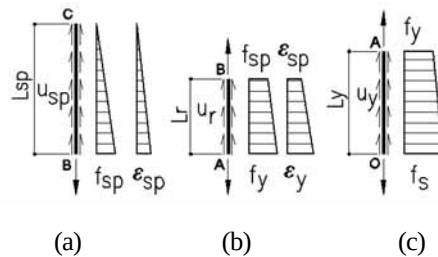


Figure 4. Free body diagrams

$$u_{sp}(\pi d_b)L_{sp} = f_{sp} \frac{\pi d_b^2}{4} \Leftrightarrow f_{sp} = \frac{4u_{sp}L_{sp}}{d_b} \quad (11)$$

To ensure that the splitting bond stress reaches the bond strength, the slip at point B has to attain the value of s_{sp} as shown in equation (12).

$$s_B = \frac{\epsilon_{sp}L_{sp}}{2} = s_{sp} \Leftrightarrow \frac{f_{sp}L_{sp}}{2E_s} = s_{sp} \quad (12)$$

Substituting equation (11) into (12) to obtain the expression in equation (13) for calculating the length of splitting part. The splitting bond strength u_{sp} and corresponding strain s_{sp} are given in equations (6) and (7) respectively.

$$L_{sp} = \sqrt{\frac{E_s d_b}{2} \frac{s_{sp}}{u_{sp}}} \quad (13)$$

Equilibrium Condition for Post Splitting Part

Figure 4(b) shows the free body diagram and distribution of the bar stress and strain along this part. The bond stress and bar stress have to satisfy equation (14).

$$\begin{aligned} u_r(\pi d_b)L_r &= (f_y - f_{sp}) \frac{\pi d_b^2}{4} \Leftrightarrow L_r \\ &= (f_y - f_{sp}) \frac{d_b}{4u_r} \end{aligned} \quad (14)$$

To compute the value of the post-splitting part length (L_r), the uniform bond stress u_r has to be determined. Figure 5 shows how to calculate the bond stress on the post-splitting part based on the bond-slip model of the lap splice strengthened by FRP jacketing. In the Figure, the relationship between the post splitting bond stress (u_r) and corresponding slip is represented by a polyline m-n-q. An example of calculating bond stress u_r for a specific slip s_A is shown in Figure 5.

In Figure 5, C_0 is the distance between the ribs of the reinforcing bar. Depending on the

slip at point A, the bond stress can be computed following the line m-n-q. Equations (15a) and (15b) show the equations for these lines. In these equations, s_p is the slip at bond stress u_p which is calculated by equation (8).

$$u_r = \frac{0.15C_0 - s_A}{0.15C_0 - s_{sp}} u_{sp} \quad \text{If } s_A \leq s_p \quad (15a)$$

$$u_r = \frac{C_0 - s_A}{C_0 - s_p} u_p \quad \text{If } s_A \geq s_p \quad (15b)$$

$$s_p = s_{sp} + \left(1 - \frac{u_p}{u_{sp}}\right)(0.15C_0 - s_{sp}) \quad (16)$$

The bond slip at point A is calculated from the bar strain distribution along this part as follows.

$$s_A = s_{sp} + \frac{\epsilon_{sp} + \epsilon_y}{2} L_r \quad (17)$$

Substituting u_r and s_A from equations (15a, b) and (17) into equation (14), and rearranging the terms, equations (18a) and (18b) for determining the length L_r of the post-splitting part are derived. It is noted that the length of the post-splitting part will be determined from equation (18a) if the length is less than L_n given in equation (19), otherwise the length will be obtained by solving equation (18b).

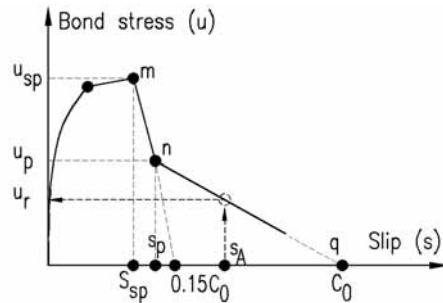


Figure 5. Calculation of bond stress for a specific slip

$$\frac{\varepsilon_{sp} + \varepsilon_y}{2(0.15c_0 - s_{sp})} L_r^2 - L_r + (f_y - f_{sp}) \frac{d_b}{4u_{sp}} \quad (18a)$$

$$\frac{\varepsilon_{sp} + \varepsilon_y}{2(c_0 - s_p)} L_r^2 - \frac{c_0 - s_{sp}}{c_0 - s_p} L_r + (f_y - f_{sp}) \frac{d_b}{4u_p} \quad (18b)$$

$$L_n = \frac{2(s_p - s_{sp})}{\varepsilon_{sp} + \varepsilon_y} \quad (19)$$

Equilibrium Condition for Yielding Part

A free body diagram and distribution of bar stress and bar strain along this part are shown in Figure 4(c). The bond stress and bar stress have to satisfy equation (20)

$$u_y (\pi d_b) L_y = (f_s - f_y) \frac{\pi d_b^2}{4} \Leftrightarrow \quad (20)$$

$$L_y = (f_s - f_y) \frac{d_b}{4u_y}$$

To compute the length of the yielding part, the bond stress on this part has to be determined. However, there is no model of bond and slip relation proposed for the yielding bars. In an effort to calculate the bond slip of an anchorage bar from the basement, Sezen and Moehle (2003) used a bi-uniform bond stress model to simulate the behavior of anchorage in the yield range.

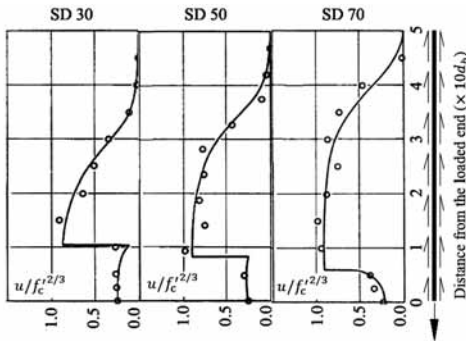


Figure 6. Experimental result by Shima *et al.* (1987)

They assumed bond stress of $0.5 \sqrt{f'_c}$ acting on the yielding part. An experimental program on a pull-out test was conducted to make an assessment on the bond characteristic in the post-yield range of a deformed bar by Shima *et al.* (1987), the test also was found in Maekawa *et al.* (2003). In the test, the embedment length was set to $50 d_b$ ($d_b = 90$ mm) which was so sufficiently long that no free end slip occurred even if the bar reached yielding during pull-out. Figure 6 shows the experimental result of 3 types of steel with different nominal yield strengths 300, 500, and 700 MPa, referred to as SD30, SD50, and SD70 respectively. It can be seen that the yield zone of SD30 is penetrating to a larger distance than that for SD50 because the SD30 bar has lower yield strength but larger yield plateau as compared with SD50. As observed from the Figure 6, it is found that the bond stress acting along the yield part tends to be uniformly distributed and equal to $u_y = 0.25 f_c'^{2/3}$ (or $u_y = 0.41 \sqrt{f'_c}$ since $f'_c = 19.6$ MPa) with 3 grades of steel SD30, SD50, and SD70.

From the investigation above, the bond stress in the yielding part is assumed to be $u_y = 0.25 f_c'^{2/3}$, following the Shima *et al.* (1987) experimental data. Since wrapping FRP around the column lap splice increases the compressive strength of concrete (equation 1), the confined compressive strength is used for calculating bond stress in the yielding zone.

$$u_y = 0.25 f_{cc}'^{2/3} \quad (21)$$

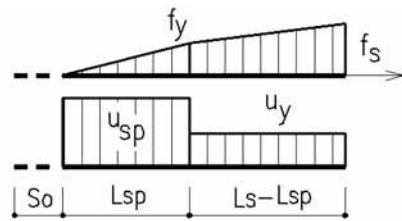


Figure 7. Free slip at the end of lap splice length (Alsiwat and Saatcioglu, 1992)

Substituting equation (21) into equation (20), we obtain equation (22) for calculating the length of the yielding zone.

$$L_y = (f_s - f_y) \frac{d_b}{f_{cc}^{2/3}} \quad (22)$$

Prediction of Lap Splice Strength

For a reinforced concrete column with the lap splice zone confined by FRP, and with known parameters including the length of lap splice, the concrete cover depth of the reinforcing bar, the properties of concrete and steel materials, the characteristics of FRP sheets, and so on, it is possible to determine the length of the splitting part, post-splitting part, and yielding part as functions of the number and thickness of FRP sheets $n_f t_f$ by using equations (13), (18a, b), and (22) respectively. By increasing the number and/or thickness of FRP sheets, it may be possible to develop the post-yield strength of the lap splice. For a given FRP parameter $n_f t_f$, the post-yield strength of the lap splice $[f_s]$ can be calculated according to equation (23) obtained by satisfying the condition of equation (10) that

the total length of 3 parts must be equal to the length of the lap splice. In equation (23), the variable L_r can be obtained by solving the formula (18a) or (18b).

$$[f_s] = f_y + \left(L_s - L_r - \sqrt{\frac{E_s d_b}{2} \frac{s_{sp}}{u_{sp}}} \right) \frac{f_{cc}^{2/3}}{d_b} \quad (23)$$

In the case that the bar stress at point B reaches the yield stress ($f_{sp} = f_y$), or the lap splice length is not sufficient for the formation of the splitting part, the bond stress distribution along the splice length will consist of 2 parts, that is, the yielding part and splitting part. In this case, a free slip s_0 must exist at the end point (C) of the lap splice length to compensate the slip at point B in order to develop bond splitting u_{sp} strength at a critical state (Alsiwat and Saatcioglu, 1992). Figure 7 shows the bond and bar stress distribution along the lap splice length. The lap splice strength can be estimated by equation (24) and finally by equation (25), in which L_{sp} is the length of the splitting part which can be computed from equation (11) by replacing the terms f_{sp} by yield stress f_y .

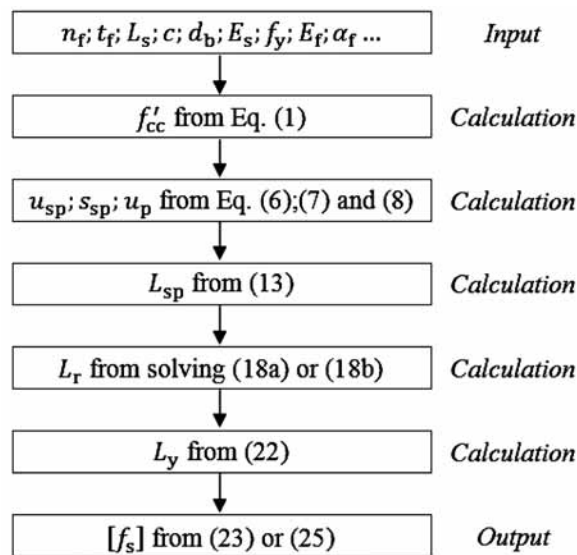


Figure 8. Computational flow chart

$$[f_s] = \frac{4}{d_b} (u_y L_s + (u_{sp} - u_y) L_{sp}) \quad (24)$$

$$[f_s] = \frac{f_{cc}^{2/3}}{d_b} \left(L_s + \frac{f_y d_b}{f_{cc}^{2/3}} - 0.25 \frac{f_y d_b}{u_{sp}} \right) \quad (25)$$

Figure 8 shows the flow chart of the computational procedure to predict the post-yield strength of the lap splice confined by FRP.

It is noted that the discussion above concerns the post-yield strength of the lap splice. However, the lap splice strength in the elastic range of the reinforcing bar can also be computed by applying 1 or 2 parts of the tri-uniform bond stress distribution model proposed in section 2.4.

There are 2 circumstances regarding the elastic strength of the lap splice. The first circumstance occurs when the lap splice length is sufficient so that the bond stress can be developed to the splitting strength ($L_s \geq L_{sp}$). In this case, the bond stress distribution along the lap splice length will consist of 2 parts, that are, the splitting part and post-splitting part. The elastic strength of the lap splice is given by equation (18c) if $L_s - L_{sp} \leq L_n$, or equation (18d) if $L_s - L_{sp} \geq L_n$. Equations (18c) and (18d) are derived from the equations (18a) and (18b) by replacing f_y by $[f_s]$. The second circumstance occurs when the lap splice length is insufficient for the bond stress to reach the splitting strength ($L_s \leq L_{sp}$). In this case, there is a free slip at the end of the lap splice (point C in Figure 3). There will be only the splitting part in the bond stress model and the elastic strength of the lap splice is given by equation (26).

$$[f_s] = f_{sp} + \frac{4(L_s - L_{sp})u_{sp}}{d_b} \left(1 - \frac{(\epsilon_{sp} + \epsilon_y)(L_s - L_{sp})}{2(0.15c_0 - s_{sp})} \right) \quad (18c)$$

$$[f_s] = f_{sp} + \frac{4(L_s - L_{sp})u_p}{d_b} \left(\frac{c_0 - s_{sp}}{c_0 - s_p} - \frac{(\epsilon_{sp} + \epsilon_y)(L_s - L_{sp})}{2(c_0 - s_p)} \right) \quad (18d)$$

$$[f_s] = \frac{4u_{sp}L_s}{d_b} \quad (26)$$

Prediction of Required FRP Thickness for Desired Stress

Section 1.5 shows the procedure to calculate the lap splice strength in either the elastic or post-yield range for a given amount and thickness of FRP sheets. Inversely, it is possible to estimate a required FRP thickness for a desired lap splice strength (or bar stress). As observed in equation (23), the right-hand side of the equation contains a complicated expression for $n_f t_f$, therefore it is very difficult to convert equation (23) into the form that calculates the thickness of FRP as a function of lap splice length. However, the required thickness and number of FRP sheets can be determined by applying the numerical produce as shown in Figure 9, in which the FRP thickness $n_f t_f$ was adjusted in order to satisfy equation (10).

Comparison with Experiment

Six RC columns strengthened by FRP were tested by Harajli and Dagher (2008); the peaks of the reinforcing bars were also reported. This section applies the proposed prediction steps to estimate the lap splice strength of these columns. Table 1 shows the comparison between the experimental peak bar stresses and the analytical lap splice strength estimated by applying the tri-linear bond stress model described above. The verification shows a close agreement between the experimental results and analytical ones. The discrepancy is within the range of 10.7%. It is noted that there is a strange result in the experimental peak bar stress of column C14FP2 confined with 2 FRP layers

which is less than that of C14FP1 confined with 1 FRP layer, even though the 2 columns were set up in the same configuration. The analysis estimation shows the correct trend of increasing lap splice strength with an increasing number of FRP sheets.

Parametric Investigation

Based on the proposed tri-uniform bond stress model, the calculation procedure for the strength of the lap splice confined by FRP has been proposed. In this section, the parametric investigation is presented. Here, the ratio of splice length to bar diameter L_s / d_b and the ratio of concrete cover to bar diameter c / d_b are the 2 parameters that affect the lap splice performance. The column section is 200 mm wide and 400 mm high as shown in Figure 10. The section is reinforced longitudinally by 8 ϕ 14 (deformed bar with diameter 14 mm) and

transversely by ϕ 8 spaced at 200 mm. The yield strength f_y of the bars is 550 MPa and elastic modulus $E_s = 1.96 \times 10^5$ MPa. The concrete compressive strength is $f'_c = 39$ MPa. The FRP sheet is 0.13 mm thick, with the elastic modulus $E_s = 2.30 \times 10^5$ MPa. Four values of L_s / d_b are considered, which are 20, 25, 30 and 35. These selected values of L_s / d_b (20, 25, 30, and 35) are supposed to cover the common ranges of lap splice length specified in design codes and in practices.

The ratio c / d_b of concrete cover and bar diameter varies from 1.0 to 2.4. For each value of ratio, the number of FRP sheets is changed in order to assess the effect of FRP thickness on the strength of the lap splice. Figures 11 to Figure 14 show the relationship between the lap splice strength and the number of FRP sheets for the 4 values of investigated.

As observed in Figure 11, for the columns

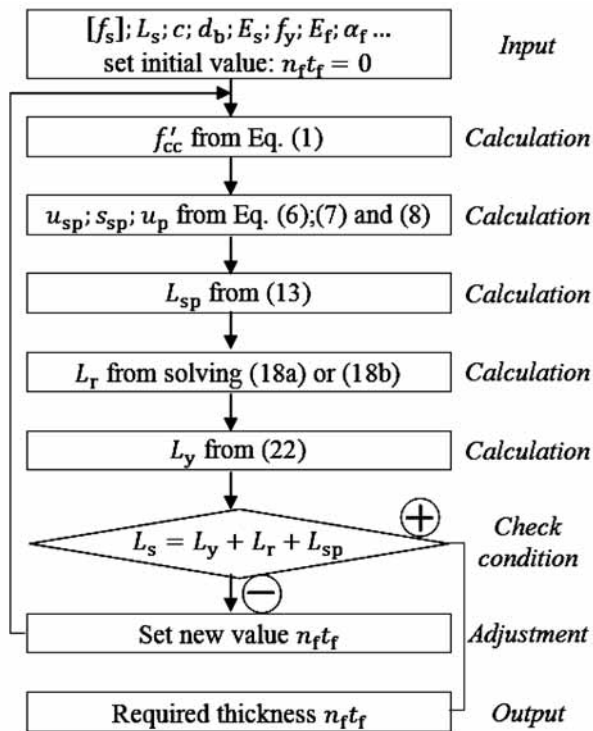


Figure 9. Computational flow chart

confined by the same amount of FRP, the lap splice strength will be higher in the columns that have a larger concrete cover. It is also found that with the same configurations, the lap splice strength proportionally increases with the number of FRP sheets and the increasing rate of post-yield strength is less than that in the elastic range.

Figures 15(a) and 15(b) show an example of bar stress (dotted line) and bond stress (solid line) distributions along the lap splice length with 1 and 2 FRP layers respectively, for $L_s / d_b = 20$. The concrete cover is set to $2.0 d_b$. In elastic behavior (Figure 15(a)), bond stress is $1.337 \sqrt{f'_c}$ and $1.168 \sqrt{f'_c}$ in the splitting part and post-splitting part, respectively, the lap splice strength is computed to be 0.994. Figure 15(b) shows the tri-uniform bond stress when wrapped by 2 FRP sheets. In this case, the lap splice strength is $1.025 f_y$, and the bond stresses are $0.546 \sqrt{f'_c}$, $1.299 \sqrt{f'_c}$ and $1.417 \sqrt{f'_c}$ in the yielding part, post-splitting part and splitting part respectively.

Figures 16(a) and 16(b) show an example of bar stress and bond stress distributions for $L_s / d_b = 25$ for the case of $c / d_b = 1.4$. In this case, the required number of FRP sheets is 3 to develop yielding in the reinforcing bars. Figures 17(a) and 17(b) show an example of bar stress and bond stress distributions for

$L_s / d_b = 30$ for the case of $c / d_b = 1.0$. When comparing with the case of $L_s / d_b = 20$, $c / d_b = 2.0$ (Figures 17(b) versus Figure 15(b)), it is seen that the required number of FRP sheets is equal to 2 in both cases. This indicates that the longer splice length can substitute the smaller concrete cover to bar diameter ratio. Figure 18 shows an example of bar stress and bond stress distributions for $L_s / d_b = 35$. When comparing with the case of $L_s / d_b = 30$ for the same c / d_b and the same number of FRP sheets ($n_f = 2$), it is seen that the maximum developed strength is $1.154 f_y$ when $L_s / d_b = 35$ as compared with $1.05 f_y$ when $L_s / d_b = 30$. This again verifies that the longer length of the lap splice is beneficial to the development of bar stress.

Conclusions

The tri-linear uniform model for bond stress distribution at critical lap splice post yield force was proposed to estimate the lap splice strength confined by FRP sheets in the pre- and post-yield range of the reinforcing bar. The model consists of 3 parts namely splitting, post-splitting and yielding part. The bond stresses in the elastic zone including the splitting part and post-splitting part can also be calculated by the proposed bond model. The bond stress in the plastic or yield zone was adopted from the experimental data. The lap splice strength can therefore be estimated in either the pre- or post

Table 1. Comparison of strength between experiment and analysis

Column	L_s / d_b	c / d_b	f_y (MPa)	Exper. $[f_s] / f_y$	Analysis $[f_s] / f_y$	Diff. (%)
C14FP1	30	1.4	550	1.025	1.053	2.64
C14FP2	30	1.4	550	1.009	1.117	10.70
C16FP1	30	2.1	528	1.008	1.034	2.67
C16FP2	30	2.1	528	1.009	1.092	8.20
C20FP1	30	1.0	617	0.746	0.718	-3.75
C20FP2	30	1.0	617	1.000	0.902	-9.79
Average						1.78
S.D						7.57

yield range of the bar. Inversely, using the proposed model, the thickness and number of FRP sheets required to develop the desired lap splice strength can also be calculated.

The comparison between the analytical predictions and experimental results shows a good agreement. The discrepancy is within the range of 11%. Based on the proposed model, it is found that the lap splice strength increases with the lap splice length, c/d_b and number and thickness of FRP sheets. The above model can be used for determining the thickness and number of FRP sheets to develop the desired lap splice strength.

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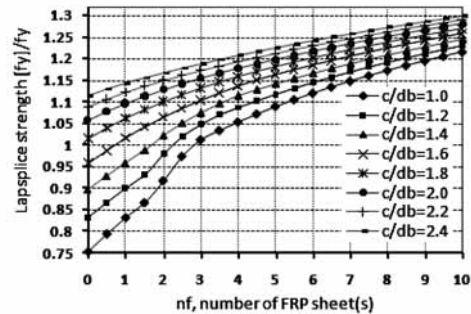


Figure 12. Strength of lap splice ($L_s / d_b = 25$)

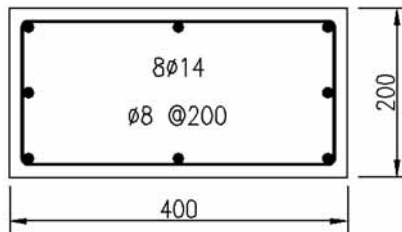


Figure 10. Column section used in parametrical investigation

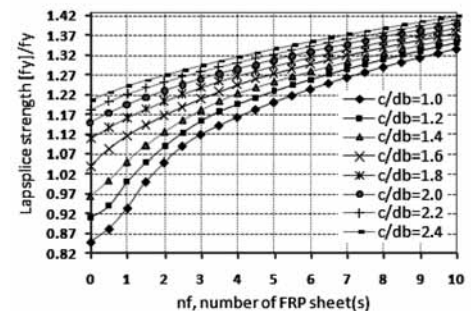


Figure 13. Strength of lap splice ($L_s / d_b = 30$)

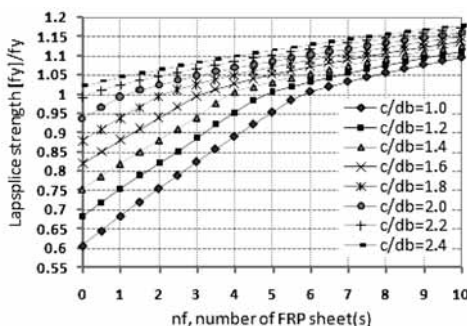


Figure 11. Strength of lap splice ($L_s / d_b = 20$)

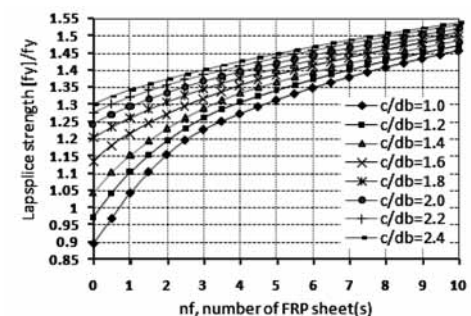


Figure 14. Strength of lap splice ($L_s / d_b = 35$)

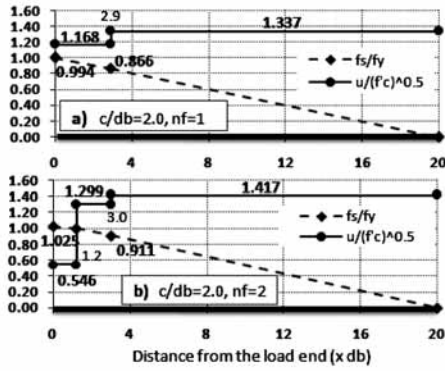


Figure 15. Example of bar stress and bond stress distribution ($L_s / d_b = 20$)

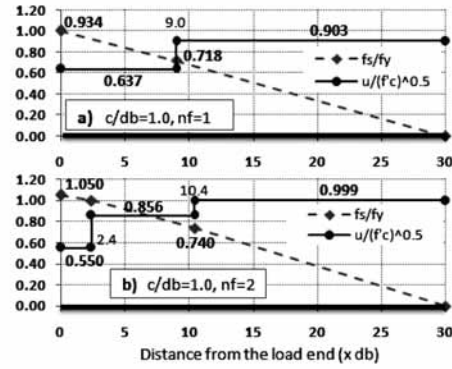


Figure 17. Example of bar stress and bond stress distribution ($L_s / d_b = 30$)

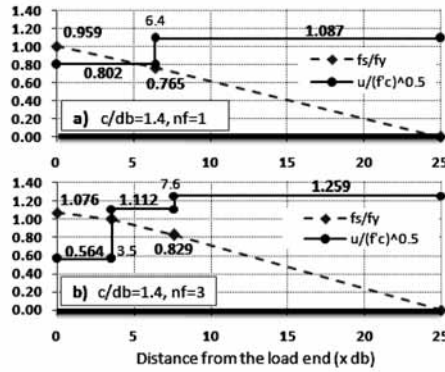


Figure 16. Example of bar stress and bond stress distribution ($L_s / d_b = 25$)

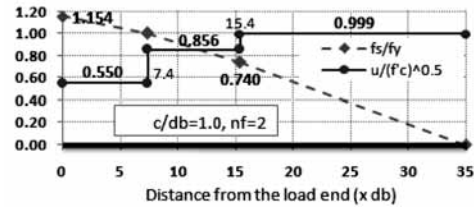


Figure 18. Example of bar stress and bond stress distribution ($L_s / d_b = 35$)

Nomenclature

A_s Area of a reinforcing bar
 b_f Width of FRP strip
 c Concrete cover depth
 c_0 Distance between the ribs of the reinforcing bar
 d_b Diameter of reinforcing bar
 E_f Modulus of elasticity of transverse FRP
 E_s Modulus of elasticity of reinforcement
 $[f_s]$ Lap splice strength
 f_s Stress of longitudinal reinforcement
 f_{sp} Bar stress at the start of splitting part
 f_y Yield stress of longitudinal reinforcement
 f'_c Compressive strength of unconfined concrete

f'_{cc} Compressive strength of confined concrete
 L_s Length of lap spliced zone
 L_y Length of yielding part
 L_r Length of post splitting part
 L_{sp} Length of splitting part
 n_f Number of transverse FRP layers
 N_f Number of partial FRP strips
 n_s Number of lap splices in tension
 r Ratio of plasticity modulus to elasticity modulus
 S_0 Local slip factor
 S_1 Local slip at peak pullout mode failure
 S_p Local slip at corresponding post-yield stress

t_f	Thickness of 1 FRP layer
u_e	Bond stress in elastic part
u_m	Maximum bond stress at pullout mode
u_p	Post splitting bond stress
u_r	Bond stress in post splitting part
u_{sp}	Bond stress in splitting part
u_y	Bond stress in yielding part
α_f	Factor of partial wrapping
ε_f	Strain of reinforcement
ε_{sh}	Strain of reinforcement at start of strain hardening
ε_{sp}	Strain of reinforcement at start of splitting part
ε_y	Strain of reinforcement at yield state

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