

STOCHASTIC SUBSPACE SYSTEM IDENTIFICATION OF A MULTI-STORY BUILDING

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Received: May 21, 2008; Revised: Nov 14, 2008; Accepted: Aug 18, 2008

Abstract

System identification is often necessary to monitor large civil engineering structures such as buildings or bridges. A subspace method for identifying the structural dynamic parameters in the state-space form of a building's model is examined in this paper. Random input forces are applied to the structure, and accelerations from all floors are used to identify the structure. Then, the mass, damping, and stiffness matrices are extracted from the identified system by reconstructing from the system's modal parameters. Results from numerical simulations show that the method can identify the structural damage successfully.

Keywords: System identification, subspace identification, multi-story building, structural dynamic parameters, state-space models

Introduction

Vibration monitoring of large civil engineering structures, e.g. bridges, buildings, and dams, is necessary, since service loads, environmental, and accidental actions may cause damage to these structures. In recent years, system identification of civil structures has received considerable interest (Cunha *et al.*, 2006). Through the identification process, we can determine the modal characteristics (frequencies, damping ratios, and mode shapes) of the observed structures. System identification methods are well explained by Juang (1994) and Ljung (1999).

Among many existing system identification algorithms, N4SID (Numerical Subspace State Space System Identification) (Van

Overschee and De Moor, 1994), identifies the system with an easy parametrization and a fast, non-iterative convergence. The N4SID algorithm estimates the structural model in state-space form from which modal properties can be extracted. This algorithm can be applied to systems subjected to known or unknown excitation.

Stochastic Subspace Identification

The multi-story building model as used in this paper is supposed to be a lumped, discrete time, linear, and time-invariant model. Mathematically, the system is described by the following stochastic state-space model:

$$x_{k+1} = Ax_k + Bu_k + w_k \quad (1a)$$

$$y_{k+1} = Cx_k + Du_k + v_k, \quad (1b)$$

where x_k is the state vector, u_k is the vector of input forces, and y_k is the vector of output measurements, respectively, and where A , B , C , and D are the system matrices of appropriate dimensions to be estimated. w_k and v_k are the process noise and the output measurement noise vectors, respectively. Both of them are assumed to be zero mean, white Gaussian, and with a covariance matrix:

$$E\left[\begin{pmatrix} w_p \\ v_p \end{pmatrix} \begin{pmatrix} w_q^T & v_q^T \end{pmatrix}\right] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \delta_{pq} \geq 0, \quad (2)$$

where E is the expected value operator. The Kronecker delta δ_{pq} means $\delta_{pq} = 0$ if $p \neq q$, and $\delta_{pq} = 1$ if $p = q$.

The stochastic identification is performed by determining the order n of the system and the system matrices A , C , Q , S , and R . The output measurements are used to construct a block Hankel matrix that can be divided into a “past” and a “future” part:

$$Y_{0|2i-1} = \begin{pmatrix} y_0 & y_1 & \cdots & y_{j-1} \\ y_1 & y_2 & \cdots & y_j \\ \vdots & \vdots & \ddots & \vdots \\ y_{i-1} & y_i & \cdots & y_{i+j-2} \\ y_i & y_{i+1} & \cdots & y_{i+j-1} \\ y_{i+1} & y_{i+2} & \cdots & y_{i+j} \\ \vdots & \vdots & \ddots & \vdots \\ y_{2i-1} & y_{2i} & \cdots & y_{2i+j-2} \end{pmatrix}$$

$$= \begin{pmatrix} Y_{0|i-1} \\ Y_{i|2i-1} \end{pmatrix} = \begin{pmatrix} Y_p \\ Y_f \end{pmatrix} = \begin{matrix} \text{"past"} \\ \text{"future"} \end{matrix}. \quad (3)$$

The subscripts of $Y_{0|2i-1}$ denote the subscript of the first and last element in the first column of the block Hankel matrix. The value of i must be greater than the maximum

order of the system that must be identified. For statistical reasons, it is assumed that $j \rightarrow \infty$. The Hankel matrix can be divided into 2 parts, Y_p and Y_f , which denote past and future, respectively.

In van Overschee and de Moor (1996), P_i is defined as the projection of the row space of the “future” outputs on the row space of the “past” outputs:

$$P_i \cong Y_f / Y_p. \quad (4)$$

Then the main stochastic subspace identification theorem states that the projection P_i is equal to the product of the extended observability matrix Γ_i and the state sequence X_i :

$$P_i = \Gamma_i X_i = \begin{pmatrix} C & CA & \cdots & CA^{i-1} \end{pmatrix}^T \begin{pmatrix} x_i & x_{i+1} & \cdots & x_{i+j-1} \end{pmatrix}. \quad (5)$$

Γ_i and X_i can be extracted using the singular value decomposition (SVD) of the projection P_i :

$$P_i \cong (U_1 \ U_2) \begin{pmatrix} S_1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix} = U_1 S_1 V_1^T. \quad (6)$$

U and V are unitary matrices, and S is a diagonal matrix; U_1 , V_1 , and S_1 are similar to U , V , and S without their zero singular values.

The extended observability matrix Γ_i can be recovered from the SVD of the projection P_i as:

$$\Gamma_i = U_1 S_1^{1/2}. \quad (7)$$

The state sequence X_i is the other half of the decomposition.

$$X_i = S_1^{1/2} V_1^T. \quad (8)$$

The system matrices A and C can be determined in the following equation system:

$$\begin{pmatrix} X_{i+1} \\ Y_{i|i} \end{pmatrix} = \begin{pmatrix} A \\ C \end{pmatrix} (X_i) + \begin{pmatrix} \rho_w \\ \rho_v \end{pmatrix}, \quad (9)$$

where $Y_{i|i}$ is a block Hankel matrix with only 1 row of outputs. This set of equations can be easily solved for A and C . Intuitively, since the Kalman filter residuals ρ_w, ρ_v are uncorrelated with X_i , it seems natural to solve this set of equations in a least squares sense (since the least squares residuals are orthogonal and thus uncorrelated with the regressors X_i).

After identifying the system matrices, the modal analysis is started by performing an eigenvalue decomposition of the system matrix A to obtain the eigenvalues λ_i and the eigenvectors ψ_i . The modal frequency ω_i and modal damping ξ_i are computed from the following formulas:

$$\lambda_i = -\xi_i \omega_i \pm i \omega_i \sqrt{1 - \xi_i^2} = \alpha_i \pm i \beta_i \quad (10)$$

and

$$\omega_i = \sqrt{\alpha_i^2 + \beta_i^2}, \quad \xi_i = \frac{\alpha_i}{\sqrt{\alpha_i^2 + \beta_i^2}}. \quad (11, 12)$$

The mode shape matrix is found from

$$\Phi = C \Psi. \quad (13)$$

Finally, in order to assure the quality of the estimated modal vectors, the modal assurance criteria (MAC) is used to measure a consistency between the estimated modal vector and the measured modal vector. The MAC of each mode can be computed from

$$MAC_i = \frac{(\hat{\psi}_i^T \psi_i)^2}{(\hat{\psi}_i^T \hat{\psi}_i)(\psi_i^T \psi_i)}, \quad (14)$$

where $\hat{\psi}_i$ is the identified modal vector and ψ_i is the measured modal vector.

Numerical Simulation of a Multi-Story Building

All the computations in this study are performed in MATLAB (The MathWorks Inc., Natick, MA, USA). The stochastic subspace identification algorithm is implemented by using the function N4SID in the MathWork's system identification toolbox (Ljung, 2002).

A 4 stories building model is used in the numerical simulations as shown in Figure 1. The mass properties are 4,000 kg for the first floor, 3,000 kg for the second and third floors, and 2,000 kg for the fourth floor, respectively, and the columns are assumed to be massless. The stiffness properties are 7.0×10^7 N/m for all stories. The modal damping ratios are 2% for all modes. The building is subjected to the white noise input forces u_1, u_2, u_3 , and u_4 on all floors. It is well known that the accuracy of any system identification technique depends on the level of noise to signal ratio. In this paper, noises with a 2% noise to signal ratio have been generated and superimposed on all the measured responses. The modal parameters are identified from structural responses and input forces with the sampling rate of 0.01 sec. The floor displacements and the input forces are given as input for the N4SID algorithm and the state-space system matrices are evaluated. Table 1 shows the comparison of modal frequencies, damping, and the MAC of the identified model.

Conclusions

In this paper, there is demonstrated an effectiveness of the stochastic subspace identification algorithm to estimate modal parameters of a multi-story building. Based on the measured noise acceleration responses subject to ambient vibration, the algorithm is capable of identifying natural frequencies, damping ratios, mode shapes, the stiffness matrix, and damping matrix

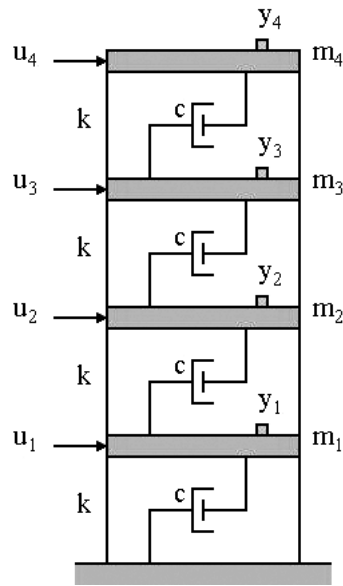


Figure 1. A 4-story building model

Table 1. Natural frequencies, Damping ratios, and MAC

Mode	Frequency (Hz)		Damping ratio		MAC (%)
	Exact	Iden.	Exact	Iden.	
1	9.0075	9.0007	0.0200	0.0192	99.9997
2	24.3113	24.3128	0.0200	0.0199	99.9976
3	36.8758	36.8773	0.0200	0.0201	99.9986
4	45.8833	45.8861	0.0200	0.0199	99.9991

of a structure with reasonable accuracy. The algorithm is quite efficient computationally and can easily be implemented using the N4SID function in MATLAB's system identification toolbox.

References

Cunha, A., Caetano, E., Magalhaes, F., and Moutinho, C. (2006). From input-output to output-only modal identification of civil engineering structures. Structural Assess-

ment Monitoring and Control [SAMCO] Final Report 2006. F11 Selected Papers. Available from: www.samco.org. Accessed date: May 6, 2008.

Juang, J.N. (1994). Applied System Identification. 1st ed. Prentice-Hall, Englewood Cliffs, NJ, USA, 352p.

Ljung, L. (1999). System Identification: Theory for the User. 2nd ed. Prentice-Hall, Englewood Cliffs, NJ, USA, 248p.

Van Overschee, P. and De Moor, B. (1994). N4SID: Subspace algorithms for the

-
- identification of combined deterministic-stochastic systems. *Automatica*, 30(1): 75-93.
- Van Overschee, P. and De Moor, B. (1996). *Subspace Identification for Linear Systems: Theory, Implementation, Applications*. 1st ed. Kluwer Academic Publishers, Boston, Mass., USA, 254p.
- Ljung, L. (2002). *System Identification Toolbox: User's Guide*. 1st ed. Mathwork Inc., Natick, MA, USA, 384p.

