

DISCRETE-TIME FEEDBACK ERROR LEARNING WITH TIME-DELAY

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Abstract

In this study, a theoretical foundation on the Discrete-Time Feedback Error Learning (DTFEL) method is extended. This is analogous to the original continuous-time version Feedback Error Learning (FEL) method which is proposed as a control model of cerebellum in the field of computational neuroscience. The DTFEL method has been developed because a digital controller is now replacing an analog one and the FEL method can not be applied to this type of controller directly. However, the validity of the stability of DTFEL has been proved based on the strictly positive realness. From the viewpoint of adaptive control, this is very restricted because some required assumptions can not be satisfied by real systems. Additionally, it is stated that the stability of the system is guaranteed by selecting a sufficiently large positive constant feedback gain. However, from the viewpoint of the control system design, it is not clear how large the feedback gain should be, since such largeness depends on unknown parameters. This study proposes another scheme for the DTFEL method which does not require any positive real conditions. This can be done by utilizing the error signal more effectively. Removing the positive realness leads to a proof of the stability of the DTFEL with the time-delay system.

Keywords: Discrete-time system, time-delay system, feedback error learning, strictly positive realness, learning control, adaptive control

Introduction

One of the great challenges of science today is to understand the human brain, and the biological basis of perceiving, learning, action and memory. The problem is, of course, the brain's incredible complexity of structure and subtlety of design. A great deal is known about the lowest levels: the biochemical mechanisms underlying the operation of individual neuron cells. Similarly the operation and interactions of

various simple neural circuits in the brain is well understood. But at higher levels our knowledge becomes less detailed. Little is known about how the brain works at a "systems level", in other words how its various parts interact to produce coordinated, adaptive and intelligent behavior.

Doya and his group (Doya *et al.*, 2001) introduced some examples of converging efforts from both aspects towards understanding and

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building adaptive autonomous systems, and he aims to promote future collaboration between the neuroscience and the systems theory communities.

The biological motor system is acceptably considered as an ideal realization of control. It consists of actuators, sensors and controllers, like usual control systems do. Unlike artificial control systems, however, it exhibits much higher performance with great flexibility and versatility in spite of the nonlinearity, uncertainties and large degree of freedom of animal bodies.

Kawato and his group (Kawato *et al.*, 1987) proposed a novel architecture of the brain motor control called Feedback Error Learning (FEL) method which uses two degrees of freedom control and system consists of an adaptive feedforward controller and a fixed feedback controller. For continuous-time linear time-invariant systems, the stability of this scheme was theoretically discussed by many researchers (Kawato *et al.*, 1987; Miyamura and Kimura, 2000; Doya *et al.*, 2001; Wongsura *et al.*, 2004).

However, all the presented theoretical results are for continuous-time systems. They must be modified before they can be really implemented to real systems. This is due to the fact that today a controller is computer-based. The transformation from a continuous-time system to a discrete-time system may create some problems in the stability of the system or lead to a poor-tracking performance. The purpose of this study is to propose a new theoretical control knowledge system for discrete systems that is able to be directly applied to real applications.

Recently, Wongsura and his group (Wongsura *et al.*, 2005a, 2005b, 2006a, 2006b; Charenovisal *et al.*, 2006) proposed a discrete-time version of DTFEL called "Discrete-Time Feedback Error Learning (DTFEL)". They considered stability of the DTFEL system and established a control theoretical validity. The formulation of the problem is in the framework of adaptive control theory. The stability of the DTFEL algorithm was proved based on the notion of strictly positive realness.

The strictly positive realness is imposed

on a certain transfer function which contains the feedback controller and unknown parameters of the inverse model. The condition can be satisfied if the feedback controller is chosen as a sufficiently large positive constant. However, from the viewpoint of control system design, it is not clear how large the feedback gain should be in order to guarantee the positive realness, since such largeness depends on unknown parameters.

The DTFEL without strictly positive realness is proposed by Wongsura and Kongprawechnon (2006a). This approach utilizes the error signal between the reference and the output signals, as well as the feedback input, for learning the inverse model, more effectively. The stability analysis is then done without using the positive realness theory.

This study aims to extend the DTFEL controller to the time-delay system. In the first part of this study, the mathematical knowledge which is useful for analyzing the DTFEL system, is briefly discussed. Then, the theory of the original DTFEL and the DTFEL without strictly positive realness is summarized. Then, the extension of the DTFEL to the time-delay systems is shown. Next, the numerical simulated examples to illustrate the theoretical stability properties of the controlled systems are presented. Finally, the conclusions of this study are drawn.

With a slight abuse of notation, $y(k) = G(z)u(k)$ is written to express the input-output relation by the transfer function $G(z)$. Also, the term "(z)" representing the z-domain variable can be skipped without notation throughout this study. That is, for example, P and $P(z)$ can be written interchangeably.

DTFEL Theory without Recourse To Positive Realness

This section is devoted to summarizing the DTFEL without strictly positive realness proposed by Wongsura and Kongprawechnon (2006a). In Figure 1, the plant P is controlled by the feedback controller K_f and the feedforward controller K_f which is adaptable. In the DTFEL

scheme, the objective is to tune K_2 to be P^{-1} which is the inverse model of P . If this is done, the output signal will track the reference input. In this scheme, the plant P is assumed to be linear time-invariant and satisfy the following assumptions.

Assumptions

- (A1) The plant P is stable and has stable inverse P^{-1} .
- (A2) The upper bound of the order of is known.
- (A3) Input signal is bounded and satisfies the Persistently exciting (PE) condition.

The parameterization of the unknown P^{-1} is expressed as

$$\eta_1(k+1) = F\eta_1(k) + gr(k), \quad (1)$$

$$\eta_2(k+1) = F\eta_2(k) + gu_0(k), \quad (2)$$

$$u_0(k) = c_0^T \eta_1(k) + d_0^T \eta_2(k) + l_0 r(k), \quad (3)$$

$$= \theta_0^T \eta(k).$$

$$\theta_0 := [c_0^T \ d_0^T \ l_0]^T, \quad (4)$$

$$\eta(k) := [\eta_1^T(k) \ \eta_2^T(k) \ r(k)]^T, \quad (5)$$

where F is any stable matrix and g is any vector with $\{F, g\}$ being controllable and $u_0(k)$ is the desired output of this system. With this parameterization, appropriate selection of parameters $c(k) = c_0$, $d(k) = d_0$ and $l(k) = l_0$ can yield

an arbitrary transfer function from $r(k)$ to $u_0(k)$. The advantage of the parameterization (1) - (5) is that the unknown parameters enter linearly in the system description. The continuous version of this parameterization was firstly used in adaptive observer (Narendra and Lin, 1980).

The adaptive feedforward controller $K_2(z)$ in Figure 1 is parameterized by

$$\xi_1(k+1) = F\xi_1(k) + gr(k) \quad (6)$$

$$\xi_2(k+1) = F\xi_2(k) + gu(k) \quad (7)$$

$$u_{ff}(k) = c^T(k)\xi_1(k) + d^T(k)\xi_2(k) + l(k)r(k) \quad (8)$$

$$u(k) = u_{ff}(k) + K_1 e(k), \quad (9)$$

where F is stable and $\{F, g\}$ is controllable and $e(k)$ is the error signal defined as $e(k) = r(k) - u(k)$. In Eqns. (8) and (9), the variables $c(k)$, $d(k)$ and $l(k)$ are adaptable parameters. In order to obtain the inverse model adaptively, the feedback error signal $e(k)$, as well as the feedback input $u_{fb}(k)$ are used in the adaptation algorithm. The use of $e(k)$ is based on the idea that $e(k) \rightarrow 0$ is achieved when $K_2(z) \rightarrow P^{-1}(z)$. The adaptive law of this scheme is then given as

$$\theta(k+1) - \theta(k) = \alpha \tilde{\xi}(k) \{u_{fb}(k) + c^T(k)\xi_e(k) + l(k)e(k)\}. \quad (10)$$

Then, the stability of this DTFEL scheme is proved and summarized as following.

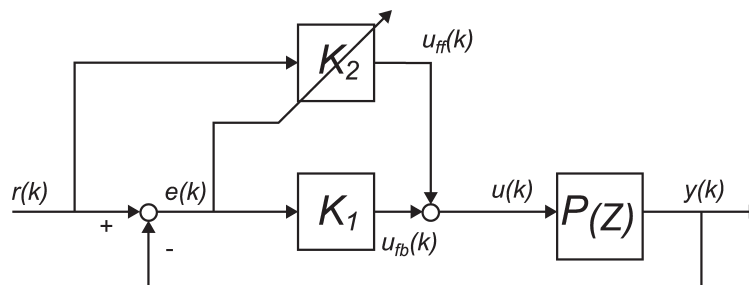


Figure 1. Original DTFEL scheme

Theorem 1: Under the assumptions (A1) - (A3), the adaptive law

$$\theta(k+1) - \theta(k) = \alpha \tilde{\xi}(k) \{u_{fb}(k) + c^T(k) \tilde{\xi}_e(k) + l(k)e(k)\} \quad (11)$$

yields $e(k) \rightarrow 0$. If $\tilde{\xi}(k)$ satisfies the PE condition, the feedforward controller $K_2(z)$ described by Eqns. (6) - (9) tends to $P^{-1}(z)$. The law of (11) can be interpreted as a feedback error learning rule, since $\theta(k)$ is tuned by the feedback input $u_{fb}(k)$, the feedback error $e(k)$, an $\tilde{\xi}_e(k)$ which is generated by $e(k)$.

DTFEL Control of Time Delay System

In the previous section, DTFEL without strictly positive realness is discussed. In this scheme, the error signal $e(k)$ is required for parameter adaptation of K_2 . However, from a practical point of view, at the sample time of k , $e(k) = r(k) - y(k)$, is not available, $y(k)$ cannot be obtained due to the time delay.

In this section, we consider DTFEL control of time delay systems where the plant output is fed back with time delay L which is assumed to be known. Corresponding to the delay of the output signal, delay of the reference signal is introduced as shown in Figure 2. The concept of DTFEL without positive realness algorithm is then applied to time delay systems.

At the sample time of k , $y(k)$ cannot be obtained due to the time delay, $y(k-L)$ can be used. Then, we consider utilizing

$$e(k-L) = r(k-L) - y(k-L) \quad (12)$$

Then, the parameterization of K_2 is described by Eqns. (6) - (8) and

$$u(k) = u_{ff}(k) + K_1(z)e(k-L) \quad (13)$$

To derive the parameter tuning rule, we consider expressing the control input $u(k)$ as a linear parameterization by θ_0 . By using the relation of

$$\begin{aligned} u_0(k-L) - u(k) &= P^{-1}r(k-L) - \\ P^{-1}y(k-L) &= P^{-1}e(k-L) \end{aligned} \quad (14)$$

together with Eqn. (3), $u(k)$ can then be described as

$$\begin{aligned} u(k) &= u_0(k-L) - P^{-1}e(k) \\ &= \theta_0^T \eta(k-L) - P^{-1}e(k-L) \end{aligned} \quad (15)$$

By comparing Eqn. (1) with Eqn. (6), it can be seen that

$$\eta_1(k) \rightarrow \xi_1(k) \quad (16)$$

since F is stable. From Eqns. (2) and (7), together with Eqn. (14), it is obviously seen that

$$\eta_2(k) \rightarrow \xi_2(k) + (zI - F)^{-1} g P^{-1} e(k) \quad (17)$$

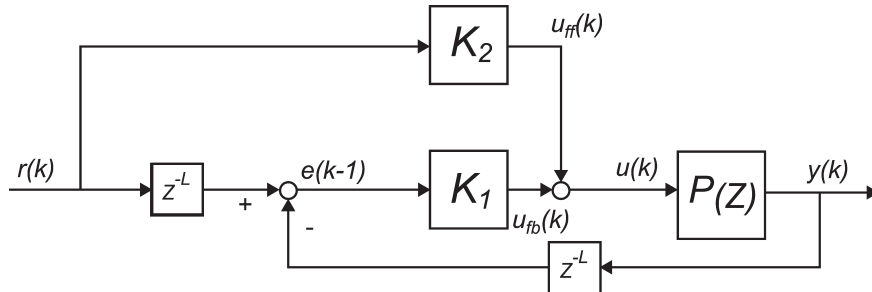


Figure 2. DTFEL scheme with time-delay

Substituting Eqns. (16) and (17) into Eqn. (15) yields

$$\begin{aligned} u(k) &= \theta_0^T \xi(k-L) + \{-1 + d_0^T(zI - F)^{-1} \\ &\quad g\} P^{-1}(z) e(k-L) \\ &= \theta_0^T \xi(k-L) + \{-l_0 - c_0(zI - F)^{-1} \\ &\quad g\} e(k) \\ &= \theta_0^T \xi(k-L) - c_0^T \xi_e(k) - l_0 e(k), \end{aligned} \quad (18)$$

where the $\xi_{eL}(k)$ is generated from $e(k-L)$ as defined by

$$\xi_{eL}(k+1) = F \xi_{eL}(k) + g e(k-L) \quad (19)$$

By defining

$$\tilde{\xi}_L(k) \triangleq \begin{bmatrix} \xi_1(k-L) - \xi_{eL}(k) \\ \xi_2(k-L) \\ r(k-L) - e(k-L) \end{bmatrix}, \quad (20)$$

Eqn. (18) can then be written as

$$u(k) = \theta_0^T \tilde{\xi}_L(k), \quad (21)$$

which is linearly parameterized by θ_0 . Corresponding to Eqn. (20), we define

$$\hat{u}_L(k) \triangleq \theta^T(k) \tilde{\xi}_L(k) \quad (22)$$

and

$$\begin{aligned} \varepsilon_L(k) &\triangleq u(k-L) - \hat{u}_L(k) \\ &= \{\theta_0 - \theta(k)\}^T \tilde{\xi}_L(k) \\ &= -\psi^T(k) \tilde{\xi}_L(k), \end{aligned} \quad (23)$$

where

$$\psi(k) \triangleq \theta(k) - \theta_0. \quad (24)$$

Based on the adaptive law of

$$\theta(k+1) = \theta(k) + \alpha \tilde{\xi}_L(k) \varepsilon_L(k), \quad (25)$$

we can obtain the following theorem.

Theorem 2: Under the assumption (A1) - (A3), the adaptive law

$$\begin{aligned} \theta(k+1) - \theta(k) + \alpha \tilde{\xi}_L(k) \{\theta(k) \\ - \theta(k-L)\}^T \xi(k-L) \\ = \alpha \tilde{\xi}_L(k) \{u_{fb}(k-L) + c^T \\ (k) \xi_{eL}(k) + l(k) e(k-L)\} \end{aligned} \quad (26)$$

yields $e(k) \rightarrow 0$. If $\tilde{\xi}_L(k)$ satisfies the PE condition, the feedforward controller K_2 described by Eqns. (6) - (8) and (18) tends to P^{-1} .

(Proof of Theorem 2)

Substitute Eqns. (23) - (24) into the adaptive law of (25) yields

$$\begin{aligned} \psi(k+1) - \psi(k) &= -\alpha \tilde{\xi}_L(k) \psi^T(k) \tilde{\xi}_L(k) \\ &= -\alpha \tilde{\xi}_L(k) \tilde{\xi}_L^T(k) \psi(k) \end{aligned} \quad (27)$$

The stability of this difference equation is guaranteed by the following lemma.

Lemma 1: (Wongsura and Kongprawechon, 2005a) Let $\xi(k)$ be an arbitrary time-varying vector. Then, the solution of the differential equation

$$z(k+1) = (I - \xi(k)L(z)\xi^T(k))\zeta(k) \quad (28)$$

tends to a constant vector z_0 such that $\xi^T(k)z_0 \rightarrow 0$. If $\xi(k)$ satisfies the PE condition, the previous z_0 is equal to 0.

Set $L(z) = 1$ in Eqn. (28) of Lemma 1 and compare with Eqn. (27), $\psi(k)$ tends to a constant vector ψ_0 such that $\tilde{\xi}_L^T(k)\psi_0 \rightarrow 0$. Thus Eqn. (24)

$$\theta(k) \rightarrow \text{Constant} \quad (29)$$

and

$$\varepsilon_L(k) = -\tilde{\xi}_L^T(k)\psi(k) \rightarrow 0 \quad (30)$$

If $\tilde{\xi}_L^T(k)$ satisfies the PE condition, then $\theta(k) \rightarrow \theta_0$. The signal $u(k-L)$ can be written as

$$\begin{aligned} u(k-L) &= u_{fb}(k-L) + u_{fb}(k-L) \\ &= \theta(k-L)\xi(k-L) + u_{fb}(k-L) \end{aligned}$$

By using this expression and Eqns. (20) - (22), the signal $\varepsilon_L(k)$ is transformed to

$$\varepsilon_L(k) = u_{ff}(k-L) + c^T(k)\xi_{eL}(k) + l(k)e(k-L) + \delta_L(k) \quad (31)$$

where

$$\delta_L(k) \triangleq \{\theta(k-L) - \theta(k)\}^T \xi(k-L) \quad (32)$$

Substituting Eqn. (31) into Eqn. (25) yields Eqn. (26). From Eqn. (29), $c(k)$ and $l(k)$ tend to constant $\bar{c}$ and $\bar{l}$ as $k \rightarrow \infty$. By this fact and Eqn. (19), the Eqn. (31) can be asymptotically described as

$$\varepsilon_L(k) = [K_1(z) + \bar{c}(zI - F)^{-1}g + \bar{l}]e(k-L) \quad (33)$$

From this and Eqn. (30), we obtain $e(k) \rightarrow 0$.

Remark: If the time delay L is 0, the law of (26) is identical to (11).

Results and Discussion

In this section, the simulation and implementation results are illustrated to demonstrate the effectiveness of the theoretical results obtained in this study. In the first part, the simulation of the DTFEL system with $L=1$ is demonstrated.

The pulse-transfer function of the plant is

$$P(z) = \frac{z+0.3}{z+0.5}$$

A feedback controller is selected as

$$K_1(z) = 10$$

In Figure 3, the tracking performance between the input signal and the output signal is shown. It shows the convergence of the signal and the comparison of the tracking performance of the system when the adaptation began, from 0 sec, and after adaptation was successfully done, after 6.5 sec.

Additionally, the simulation when the time delay in the feedback path is not known exactly are also studied. In these cases, the delay of the output signal does not match the delay of the reference signal. Figure 4 shows the tracking performance of the DTFEL systems with unit delay of the input signal and eight delay of the output signal. The results show good tracking performance of DTFEL control systems. However, it should be noted that the performance slightly reduces when the difference between the delay of the input signal and delay of the output signal increases.

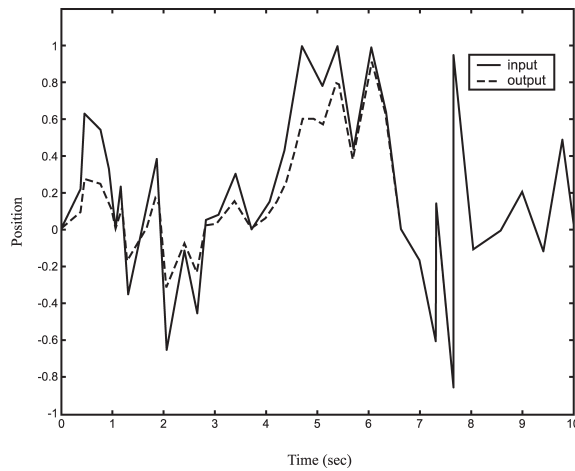


Figure 3. The simulation result of the DTFEL with time delay system

Finally, the implementation of DTFEL system is studied. The hardware configuration of the motor control in this experiment can be implied as the block diagram shown in Figure 5. This is done by using the Simulink Realtime Workshop in conjunction with the LabVIEW RT Simulation Interface Toolkit to generate a Virtual Instrument (VI) used in a LabVIEW program, for execution on a real-time embedded controller to

control a DC servo motor.

Figure 6 shows the successful control performance of such a system. It is interesting to note that the system is stable. Furthermore, the tracking performance is quite good, that is, the output position tracks the reference input with acceptable time-delay. Thus, it can be said that the DTFEL system was able to control the tracking performance of the motor effectively.

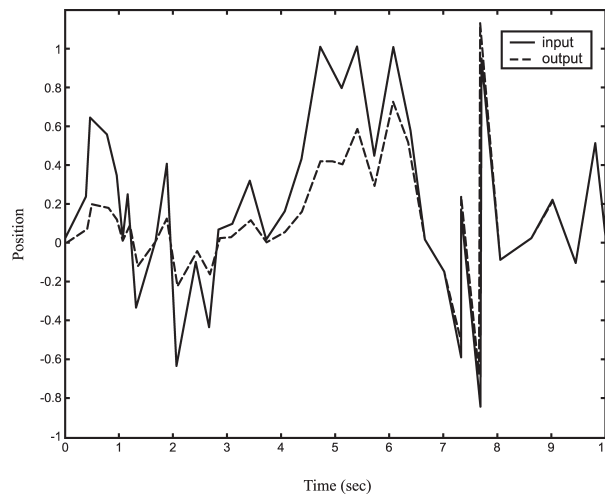


Figure 4. The simulation result of the DTFEL with unmatched delay

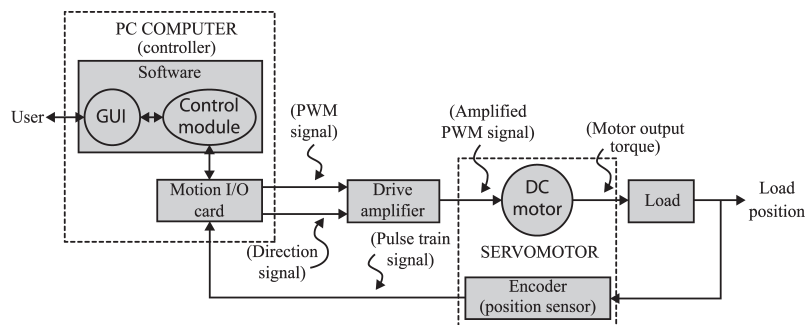


Figure 5. A block diagram of servomotor position control

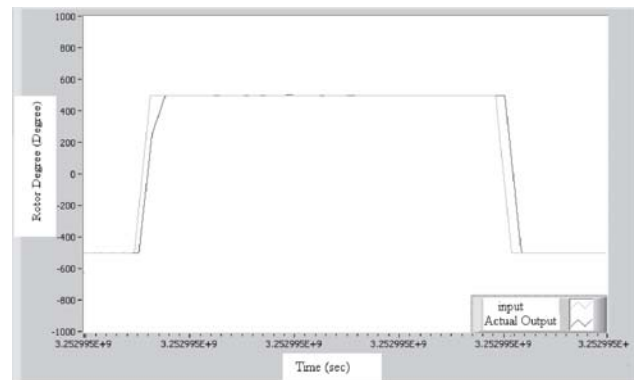


Figure 6. The implementation result of the DTFEL system

Conclusion

This study has presented a new adaptive control scheme based on DTFEL. DTFEL without recourse to strictly positive realness (Wongsura and Kongprawechonon, 2006a) removes many restrict required conditions of the original DTFEL (Wongsura *et al.*, 2005a, 2005b, 2006a, 2006b; Charenovisal *et al.*, 2006). However, the adaptive law of this scheme requires the output signal at the feedback path which is practically not available at instantaneous sample time.

This study solves such problems by introducing delay at the reference signal. The adaptation is then obtained in the same manner as that of the DTFEL without recourse to a strictly positive realness scheme. Also, the stability is guaranteed in the same way.

The numerical simulation study is done in two cases, when the reference signal delay matches and does not match the feedback delay. The simulation results show a good tracking performance of this control scheme in both cases. Finally, DTFEL is practically tested in motor control application. The result shows that the DTFEL system was able to control the tracking performance of the DC servo motor effectively.

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