

TYPE 1.5 SUPERCONDUCTIVITY IN MAGNESIUM DIBORIDE SUPERCONDUCTOR: A GINZBURG-LANDAU APPROACH TO CRITICAL CURRENT DENSITY

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Abstract

In this study, we present a theoretical investigation into the temperature-dependent critical current density of MgB₂ superconductors. Using the Ginzburg-Landau theory, we explore the behavior of this superconductor within the framework of a free energy model, which has been adapted from the type-1.5 superconductor model. The theoretical calculations derived from this model are then compared with experimental data obtained for MgB₂ superconductors. The results demonstrate a notable correlation between the theoretical predictions and the experimental findings, offering new insights into the temperature-dependent properties of MgB₂ and potentially advancing the understanding of its superconducting behavior.

Keywords: Critical Current Density; MgB₂ Superconductors; Type-1.5 Superconductor; Ginzburg-Landau Theory

Introduction

Magnesium diboride (MgB₂) was discovered to be a superconductor in 2001 (Nagamatsu *et al.*, 2001). This discovery sparked significant interest due to its relatively high critical temperature (T_c) of 39 K, which is the highest among simple binary compound superconductors. MgB₂ is characterized by its simple hexagonal structure and the presence of two superconducting gaps, an unusual feature for

conventional superconductors (Buzea and Yamashita, 2001). The superconductivity in MgB₂ is primarily driven by electron-phonon interactions, as confirmed by various experimental techniques such as photoemission spectroscopy and neutron scattering (Kortus *et al.*, 2001). MgB₂ has a simple binary crystal structure, which simplifies its synthesis and fabrication compared to high-

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temperature superconductors (HTS) like $\text{YBa}_2\text{Cu}_3\text{O}_7$ (YBCO) and $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_x$ (BSCCO) (Vinod *et al.*, 2007). Additionally, the raw materials (magnesium and boron) are abundant and inexpensive, making MgB_2 a cost-effective alternative to other superconductors (Muralidhar *et al.*, 2023). Recent research has focused on enhancing its critical current density (Batalu *et al.*, 2023; Liu *et al.*, 2024a; Liu *et al.*, 2024b), flux pinning mechanisms (Zhao *et al.*, 2022; Qi *et al.*, 2024), and scalability for practical use (Xing *et al.*, 2022; Herbirowo *et al.*, 2023).

Type-1.5 superconductivity is a relatively new concept in the field of superconductivity, first proposed by Moshchalkov *et al.* (2009). This state is characterized by the coexistence of both type-I and type-II superconducting properties within the same material, leading to unique vortex interactions. Experimental evidence for type-1.5 superconductivity has been observed in MgB_2 and other multiband superconductors. Bitter decoration experiments and scanning tunneling microscopy have visualized the unique vortex patterns predicted by theoretical models (Moshchalkov *et al.*, 2009; Babaev *et al.*, 2012). These experiments confirm the coexistence of type-I and type-II superconducting properties within the same material.

Critical current density (J_c) is the highest current it can transport without losing its superconducting state. For the practical use of superconductors in a variety of applications, such as magnetic levitation, power transmission, and medical imaging, it is essential to comprehend and optimize J_c (Dew-Hughes, 2001). Several factors influence the critical current density in superconductors, such as material properties (Larbalestier *et al.*, 2001), temperature (Dew-Hughes, 2001), magnetic field (Sun *et al.*, 2015), and flux pinning (Koshelev *et al.*, 2016).

The Ginzburg-Landau (GL) theory, developed by Vitaly Ginzburg and Lev Landau in 1950, provides a phenomenological framework for understanding superconductivity. This theory extends the macroscopic description of superconductors and introduces the concept of the order parameter, which describes the superconducting state (Ginzburg and Landau, 1950). For unconventional superconductors, the Ginzburg-Landau theory provides a comprehensive framework to explain various superconducting phenomena, including the upper critical field; H_{c2} (Askerzade *et al.*, 2002a; Udomsamuthirun *et al.*, 2006), lower critical field; H_{c1} (Askerzade *et al.*, 2002b; Changjan and Udomsamuthirun, 2011), surface critical field; H_{c3} (Askerzade, 2003a; Makuwan *et al.*, 2023a; Changjan *et al.*, 2017), thermodynamic magnetic field; H_c (Askerzade and

Gencer, 2002; Makuwan *et al.*, 2023b), coherence length; ξ (Changjan and Udomsamuthirun, 2019), London penetration depth; λ (Askerzade, 2003b; Changjan and Udomsamuthirun, 2014; Meakniti and Udomsamuthirun, 2023), specific heat jump; ΔC (Askerzade and Gencer, 2002; Makuwan *et al.*, 2023b), and critical current density; J_c (Askerzade, 2004). The GL theory, extended to account for the anisotropic and multiband nature of unconventional superconductors, helps in understanding the complex behavior of these materials. In this paper, we investigated the temperature-dependent critical current density of MgB_2 superconductors using the Ginzburg-Landau theory and a modified free energy model based on the type-1.5 superconductor model. The theoretical calculations derived from this model are then compared with experimental data obtained for MgB_2 superconductors (Kunchur *et al.*, 2003).

Materials and Methods

For a type 1.5 superconductor with coupled superconducting order parameters, the Ginzburg-Landau (GL) free energy density can be expressed as follows (Babaev and Silaev, 2013):

$$f = \frac{1}{2}(\alpha_1|\psi_1|^2 - 1)^2 + \alpha_2|\psi_2|^2 + \frac{1}{2}\beta|\psi_2|^4 + \frac{1}{2m^*} \left| \left(\hbar \vec{\nabla} - i \frac{e^*}{c} \vec{A} \right) \psi_1 \right|^2 + \frac{1}{2m^*} \left| \left(\hbar \vec{\nabla} - i \frac{e^*}{c} \vec{A} \right) \psi_2 \right|^2 + \varepsilon(\psi_1^* \psi_2 + c.c.) + \varepsilon_1 \left[\left(\hbar \vec{\nabla} - i \frac{e^*}{c} \vec{A} \right) \psi_1^* \left(\hbar \vec{\nabla} - i \frac{e^*}{c} \vec{A} \right) \psi_2 + c.c. \right] + \frac{H^2}{8\pi}. \quad (1)$$

In this case, m^* stands for the carriers' effective mass. The order parameter is ψ_i , and the density of carriers ($i = 1, 2$) is proportional to $|\psi_i|^2$. The coefficient α depends on the temperature, but the coefficient β is temperature-independent, with γ_i being proportionality constant. The kinetic energy of the carriers is taken into account by the fourth and fifth terms. The coupling of two order parameters and their gradients are described in sixth and seventh term by the variables ε and ε_1 , respectively. The final term considers the energy stored in local magnetic fields.

The critical current density has been demonstrated to be determined via (Tinkham, 1975; Abrikosov, 1988)

$$J_c(t) = 2en_s(t)v_c(t). \quad (2)$$

Where $t = \frac{T}{T_c}$ and $n_s(t)$ is the superfluid density, $n_s \approx |\psi|^2$. The critical velocity is denoted by $v_c(t)$. Critical velocity in a superconductor refers

to the maximum speed at which Cooper pairs can move without breaking apart and causing the superconductor to lose its superconducting properties. This velocity depends on several factors, including temperature and the presence of magnetic fields. The relationship between superfluid density and London penetration depth (λ) can be expressed as $\frac{n_s(T)}{n_s(0)} = \frac{\lambda^2(0)}{\lambda^2(T)}$ and the critical velocity is related with the coherence length (ξ) and upper critical field (H_{c2}) by $v_c = \frac{2\hbar}{m_s \xi(t)}$, $\xi^2(t) = \frac{\phi_0}{2\pi H_{c2}}$ (Tinkham, 1975; Abrikosov, 1988). Afterward, we can examine critical current density utilizing London penetration depth and coherence length as

$$\frac{J_c(T)}{J_c(0)} = \frac{\lambda^2(0)}{\lambda^2(T)} (h_{c2})^{1/2}, h_{c2} = \frac{H_{c2}(T)}{H_{c2}(0)}. \quad (3)$$

London penetration depth is the distance to which a magnetic field penetrates into a superconductor and becomes equal to e^{-1} times that of the magnetic field at the surface of the superconductor. According to Niyomsilpchai *et al.* (2003), London penetration depth was determined with the free energy density for the type 1.5 superconductor model, as shown in equation (1), and provided by

$$\frac{\lambda^2(0)}{\lambda^2(T)} = \eta \left(\frac{1+Q+4\varepsilon_1 m^* \sqrt{Q}}{1+Q'+4\varepsilon_1 m^* \sqrt{Q'}} \right). \quad (4)$$

Here

$$\eta = \frac{(\gamma_1 \gamma_2 T_c^2 \theta^2 + \varepsilon^2)}{\theta^3 (\gamma_1 \gamma_2 T_c^2 + \varepsilon^2)}, t - 1 = \theta, t = \frac{T}{T_c}, Q = \frac{\gamma_1 T_c^2 \theta^2 (\varepsilon^2 \gamma_2 - \beta \gamma_1)}{(\gamma_1 \gamma_2^3 T_c^4 \theta^4 - \beta \varepsilon^2)}$$

$$\text{and } Q' = \frac{\gamma_1 T_c^2 (\beta \gamma_1 - \varepsilon^2 \gamma_2)}{(\beta \varepsilon^2 - \gamma_1 \gamma_2^3 T_c^4)}.$$

Coherence length is the distance which a cooper pair spreads, or to the necessary distance for a superconducting wave to develop. In the framework of type 1.5 superconductor model, coherence length was calculated by equation (1) and the assumption that

$$\alpha_i = \alpha_{0i} \left((1-t) - \frac{1}{2}(1-t)^2 \right) \approx \alpha_{0i} \ln(2-t)$$

$$\text{and } \varepsilon = \varepsilon_0 \left((1-t) - \frac{1}{2}(1-t)^2 \right) \approx \varepsilon_0 \ln(2-t)$$

(Changjan and Udomsamuthirun, 2013; Changjan and Udomsamuthirun, 2019) as

$$\frac{\xi(T)}{\xi(0)} = \left[1 - B \ln \left(1 - \frac{t}{2} \right) \right]^{-\frac{1}{2}}. \quad (5)$$

Where

$$B = \frac{-\left(-\alpha_{01} \frac{\hbar e}{2mc} + \alpha_{02} \frac{\hbar e}{2mc} + \frac{4\hbar \varepsilon \varepsilon_1 e}{c}\right)^2 + (\alpha_{01} \alpha_{02} - \varepsilon^2) \left(\frac{\hbar^2 e^2}{4m^2 c^2} - \frac{4\hbar^2 \varepsilon_1^2 e^2}{c^2} \right)}{\left(-\alpha_{01} \frac{\hbar e}{2mc} + \alpha_{02} \frac{\hbar e}{2mc} + \frac{4\hbar \varepsilon \varepsilon_1 e}{c}\right)^2 (\ln 2) + (\alpha_{01} \alpha_{02} - \varepsilon^2) \left(\frac{\hbar^2 e^2}{4m^2 c^2} - \frac{4\hbar^2 \varepsilon_1^2 e^2}{c^2} \right) (\ln 2)}.$$

Finally, we get the temperature-dependent critical current density by substitution equation (4) and equation (5) into equation (3).

Results and Discussion

The results for the normalized temperature-dependent critical current density in two-band (TB) Ginzburg-Landau theory in comparison with that given by the single-band (SB) GL theory had been studied by Askerzade (2004). The normalized temperature-dependent critical current density in TB and SB model showed different behaviors. In this paper, we were interested in the effect of the type 1.5 superconductor model to the normalized temperature-dependent critical current density of MgB₂. The effect of type 1.5 superconductor model on the normalized temperature-dependent critical current density of MgB₂ from our calculation (type 1.5 model) comparison with SB model (Tinkham, 1975; Abrikosov, 1988), TB model (Askerzade, 2004) and experimental data of MgB₂ (Kunchur *et al.*, 2003) were shown in Figure 1.

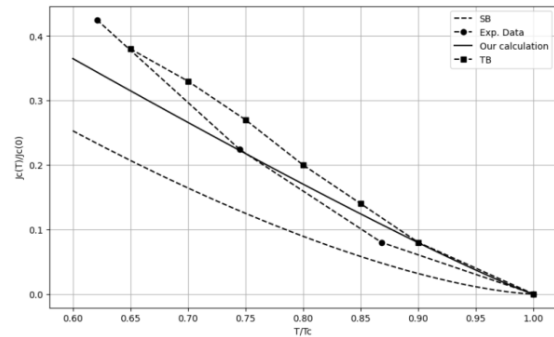


Figure 1. $\frac{J_c(T)}{J_c(0)}$ versus reduce temperature $\left(\frac{T}{T_c}\right)$.

As shown in Figure 1, the calculation from two-band Ginzburg-Landau theory, (TB; solid square) (Askerzade, 2004) and type 1.5 superconductor model (our calculation; solid line) gives a good agreement with experimental data (Exp. Data; solid circle) (Kunchur *et al.*, 2003) at near T_c . The single-band model of superconductivity (SB; dash line), assumes a single isotropic energy gap across the Fermi surface. This model has been successful in explaining the superconducting properties of many conventional superconductors. However, it fails to account for the unique

characteristics of MgB_2 , particularly the presence of two distinct energy gaps (Liu *et al.*, 2001). Experimental evidence from techniques such as point-contact spectroscopy and Andreev reflection has confirmed the existence of these two gaps, with values of approximately 7 meV and 2.8 meV (Szabó *et al.*, 2001). Table 1 displays the raw critical current density data for each scenario.

To determine the statistical significance of each model, it is essential to consistently differentiate between the impacts of systematic approaches and errors arising from variations in modeling techniques. In this study, we employ Analysis of Variance (ANOVA) to assess the statistical significance of each model. The results obtained are compared with the critical current density data of MgB_2 from the SB model, TB model, and our calculations with experimental data.

Table 1. Raw data of critical current density from experimental data, the results of calculation by single-band model, two-band model and type 1.5 superconductor model from Figure 1

T/T_c	$J_c(T)/J_c(0)$			
	Exp. Data	TB	SB	Our calculation
0.65	0.39	0.38	0.21	0.32
0.7	0.31	0.33	0.16	0.266
0.75	0.23	0.27	0.13	0.22
0.8	0.17	0.2	0.09	0.17
0.85	0.11	0.14	0.08	0.12
0.9	0.06	0.08	0.03	0.079
0.95	0.03	0.03	0.01	0.04
1	0	0	0	0
Mean	0.163	0.179	0.089	0.152
Maximum	0.39	0.38	0.21	0.32
Minimum	0	0	0	0
Standard Deviation	0.1391	0.1401	0.0747	0.1122
Root Mean Square Difference (RMSD)	-	14.267	58.985	18.838
Mean Bias Difference (MBD)	-	10.000	-45.384	-6.538

Before the analysis of variance, we used the Kolmogorov-Smirnov test to check the normality of the set of raw data, which comes from each modeling approach in Figure 2.

The test maximum statistic D of (a) Actual (Exp. Data), (b) Two-band model (TB), (c) Single-band model (SB), and (d) Type 1.5 superconductor model (our calculation) equals 0.15, 0.13, 0.16, and 0.12, respectively, as shown in Figure 2. These values have a trend lower than the critical test maximum statistic D , which equals 0.46. It was assumed that the data distribution in those outcomes

was not normal. Stated differently, there was not a substantial enough difference between the raw data sample and the normal distribution. As a result, the ANOVA and the raw data sample disagreed with the parametric statistical model's components (Siegel, 1956). The procedure employs a nonparametric approach to two-way analysis of variance to compare three or more groups on a dependent variable measured at least at the ordinal level. This method is sometimes referred to as the Friedman two-way analysis of variance by ranks test.

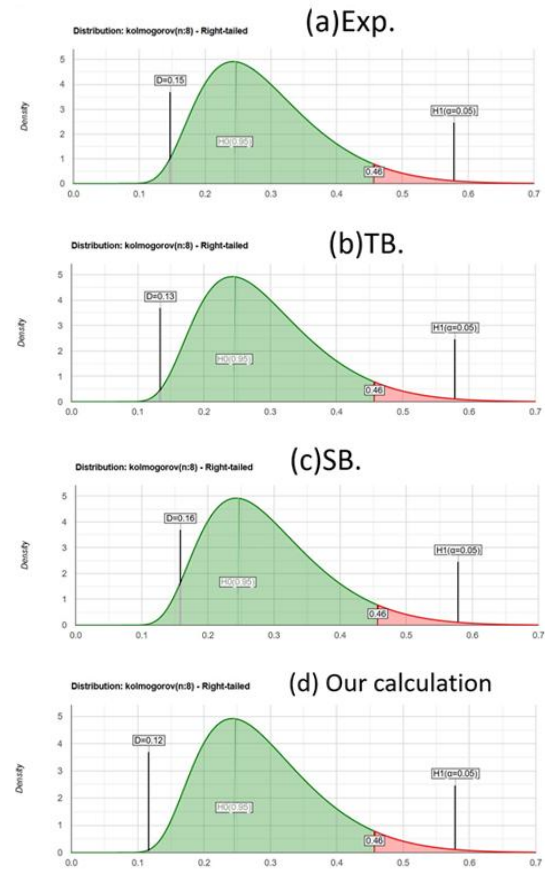


Figure 2. The Kolmogorov-Smirnov Test of Normality of (a) Actual (Exp. Data), (b) two-band model (TB), (c) single-band model (SB) and (d) type 1.5 superconductor model (our calculation)

The test maximum statistic D of (a) Actual (Exp. Data), (b) Two-band model (TB), (c) Single-band model (SB), and (d) Type 1.5 superconductor model (our calculation) equals 0.15, 0.13, 0.16, and 0.12, respectively, as shown in Figure 2. These values have a trend lower than the critical test maximum statistic D , which equals 0.46. It was

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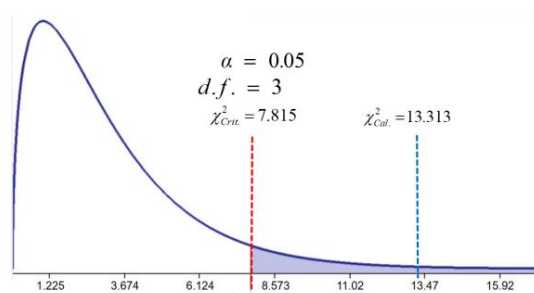


Figure 3. Nonparametric approach to the two-way ANOVA to compare four groups: (a) Actual (Exp. Data), (b) two-band model (TB), (c) single-band model (SB) and (d) type 1.5 superconductor model (our calculation)

Figure 3 shows the nonparametric approach to the two-way ANOVA to compare four groups: (a) Actual (Exp. Data), (b) two-band model (TB), (c) single-band model (SB) and (d) type 1.5 superconductor model (our calculation). The test statistic under chi-square distribution equals 13.313, which is in the 95% region of alternative hypothesis acceptance: $[0, 7.815]$. This result means that the mean ranks of all groups are assumed to be difference. In other words, the difference between the mean ranks of all groups is big enough to be statistically significant. The results from the two-way ANOVA comparing four groups, as shown in Figure 3, indicate that the difference between the mean ranks of all groups was significant. However, we still do not know how the predictions of each model compare to the experimental values. Therefore, we need to perform pair wise comparisons using the measure of central tendencies of the two groups under the nonparametric approach, known as the Wilcoxon signed-rank test. The results of the two-sample Wilcoxon signed-rank test are presented non-statistically significantly different from the

experimental results (Exp. Data) at $\alpha = 0.05$ with two-band model (TB) and type 1.5 superconductor model (our calculation) but statistically significantly different from the experimental results (Exp. Data) at $\alpha = 0.05$ with single-band model (SB). Based on the results of this analysis, the concept of type 1.5 superconductors and two-band model are supported as being suitable for studying MgB_2 superconductors. This is because the calculated results show no significant difference from the experimental results.

Conclusions

The temperature-dependent critical current density in MgB_2 superconductors was investigated by Ginzburg-Landau theory within the framework of a free energy model, which has been adapted from the type-1.5 superconductor model. Our calculation indicate a strong agreement at near T_c between the theoretical predictions and the experimental observations and the statistical analysis results shows non-statistically significantly different from the experimental results (Exp. Data) at $\alpha = 0.05$ with our calculation. Based on the results of this analysis, the concept of type 1.5 superconductors is supported as suitable for studying MgB_2 superconductors. This provides new insights into the temperature-dependent characteristics of MgB_2 and may contribute to a better understanding of its superconducting behavior.

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