

FRACTURE BEHAVIOR CHARACTERIZATION OF NORMAL AND HIGH-STRENGTH CONCRETE USING SIZE EFFECT LAW AND FINITE ELEMENT MODELLING

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Abstract

This study investigates the fracture behavior of normal and high-strength concrete through experimental analysis, theoretical models, and finite element simulations. A series of notched concrete beam specimens with varying notch-depth ratios were subjected to three-point bending tests to evaluate fracture parameters, including fracture energy (G_f), fracture toughness (K_{IC}), fracture process zone length (FPZ), and characteristic length (l_{ch}). The size effect method developed by Bažant was employed to assess the dependency of the fracture parameters on the specimen geometry and notch ratio. The results reveal a significant influence of the notch-depth ratio on brittleness, energy dissipation, and crack propagation. Additionally, finite element modelling (FEM) using ANSYS was used to validate the stress intensity factors and deformation characteristics observed experimentally. A comparative analysis between normal and high-strength concrete confirmed that high-strength concrete exhibits a more brittle response with higher fracture energy but reduced fracture process zone length. This comprehensive evaluation highlights the applicability of nonlinear fracture mechanics for quasi-brittle materials such as concrete and supports the incorporation of fracture-based parameters into advanced design codes.

Keywords: Finite Element Modelling; Fracture Mechanics; High-Strength Concrete; Size Effect Method; Stress Intensity Factor

Introduction

Concrete is the most widely used construction material owing to its global availability, moldability, and compressive strength. However, its quasi-brittle nature limits the fracture resistance and crack control under service and dynamic loads (Sherzer *et al.*, 2022). To overcome these

challenges, High-Strength Concrete (HSC) and Ultra-High-Performance Fiber-Reinforced Concrete (UHPFRC) have provided materials with superior mechanical strength, durability, and post-cracking ductility (Zhang *et al.*, 2022). The enhanced performance of HSC arises from its dense

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microstructure and low water-binder ratio, whereas fiber addition in UHPFRC enables crack bridging and energy dissipation at the post-peak stage (Thilakarathna *et al.*, 2022).

Experimental and numerical studies confirm that concrete fracture behavior is strongly dependent on size. Beam tests across varying geometries demonstrate that the nominal flexural strength decreases with increasing specimen size, despite similar initial toughness, underscoring the role of structural geometry and microstructural heterogeneity (Dubey & Ray, 2023; Wang *et al.*, 2023). The aggregate distribution, fiber orientation, and notch depth significantly influenced the fracture process zone (FPZ) and energy dissipation capacity. For instance, polypropylene or steel fibers alter crack trajectories and enhance toughness under mixed-mode conditions (Mutnbak *et al.*, 2025; Zhang *et al.*, 2022). Aggregate grading also affects the fracture energy (G_f) and characteristic length (l_{ch}), with finer gradation promoting uniform crack propagation and higher G_f (Siregar *et al.*, 2017).

At higher stress rates and under dynamic loads, HSC exhibited additional complexities. The Lattice Discrete Particle Model (LDPM) and Finite-Discrete Element Method (FDEM) have shown that strength and ductility diminish with increasing specimen size, while dynamic increase factors (DIFs) can overestimate the true resistance owing to inertial effects (Gambarelli and Ozbolt, 2020; Sherzer *et al.*, 2022). Cohesive traction-separation models further reveal that the interface roughness and bond quality between Normal-Strength Concrete (NSC) and UHPFRC strongly affect the shear and tensile fracture behavior (Luu & Kim, 2023). These findings emphasize that both the material heterogeneity and geometry dictate the observed fracture mechanisms.

Recent numerical and experimental advancements have highlighted the need for reliable multiscale modeling of quasi-brittle materials. Mesoscale and graph-based Finite Element Modelling (FEM) approaches such as GraFEA accurately predict crack initiation, propagation, and energy dissipation while accounting for material inhomogeneity (Shin *et al.*, 2022). Alternative simplified frameworks, such as Finite Fracture Mechanics (FFM) provide computational efficiency while preserving fracture realism (Baldassari *et al.*, 2023). Additive manufacturing and temperature effects further expand the understanding of fracture phenomena; 3D-printed cementitious composites display layer-dependent fracture strength, and elevated temperatures ($> 800^\circ\text{C}$) alter size-effect trends (Ye *et al.*, 2024; Zhang *et al.*, 2024).

Over the decades, several theoretical models have been proposed to describe concrete fracture behavior. Kaplan's pioneering work on Linear Elastic Fracture Mechanics (LEFM) (Kaplan, 1961) was succeeded by Hillerborg's Fictitious Crack Model (FCM) (Brincker & Dahl, 1989) and Bažant's Size-Effect Law (SEL) (Bažant, 1984), which introduced the concept of scale-dependent nominal strength for quasi-brittle materials. These models collectively form the foundation of modern nonlinear fracture mechanics and enable the estimation of essential fracture parameters, such as fracture energy (G_f), fracture toughness (K_{IC}), fracture process zone length (FPZ), and characteristic length (l_{ch}) (Khalilpour *et al.*, 2019; Shayanfar, 1995). Despite their proven accuracy, current design codes rarely incorporate these fracture-based parameters explicitly, leading to limitations in predicting brittle failure in high-strength materials.

Building on this background, several studies have demonstrated that geometric parameters, particularly the notch-to-depth ratio (a/d), strongly influence fracture energy and apparent toughness. Valdez Aguilar *et al.* (2021) and Wang *et al.* (2023) established that increasing a/d reduces post-peak ductility and modifies the fiber-bridging efficiency, whereas Baldassari *et al.* (2023) confirmed that standard SEL formulations require correction for high a/d ratios. These observations reinforce the need for a unified experimental-numerical framework that links geometry-controlled fracture characteristics with size-effect formulations.

Accordingly, this study investigates the fracture behavior of normal-strength concrete (NSC) and high-strength concrete (HSC) using notched beam specimens under three-point bending. The focus was to quantify the influence of the notch-depth ratio ($a/d = 0.15\text{--}0.45$) on G_f , K_{IC} , FPZ, and l_{ch} through experimental testing and finite element Modelling (FEM) using ANSYS. This comparison elucidates the transition from ductile to brittle responses and validates the applicability of Bažant's Size-Effect Law and Hillerborg's characteristic-length theory in predicting fracture parameters. This combined approach aims to refine the understanding of size-dependent brittleness and establish a bridge between empirical strength-based design and fracture-informed modeling for quasi-brittle materials, such as concrete.

A comparison between the fracture behavior of normal-strength concrete (NSC) and high-strength concrete (HSC) is crucial for advancing design methodologies because these two materials exhibit fundamentally different post-cracking responses, brittleness characteristics, and size-dependent fracture parameters. NSC generally possesses a

wider FPZ, enabling greater energy dissipation and ductility, whereas HSC, owing to its dense microstructure, demonstrates a more brittle response with a higher fracture energy but reduced FPZ length. Understanding these distinctions is vital for structural design, as the transition from ductile to brittle behavior influences crack propagation control, serviceability, and ultimate load-bearing capacity.

Moreover, most current design codes primarily rely on empirical strength-based formulations and do not explicitly account for nonlinear fracture mechanics parameters, such as fracture energy (G_f), fracture toughness (K_{IC}), and characteristic length (l_{ch}). Incorporating these parameters can enhance the predictive reliability of design models, especially for high-performance materials such as HSC, where the traditional assumptions of homogeneity and plastic redistribution may not hold. Therefore, a comparative study of NSC and HSC fracture behavior bridges this knowledge gap and supports the development of fracture-informed design provisions that ensure safety, durability, and resilience across strength classes.

This study provides a comprehensive assessment of the fracture behavior of concrete. Section 2 describes the materials, mix design, specimen preparation, and testing procedures. Section 3 presents and interprets the results for fracture energy, fracture toughness, FPZ length, and characteristic length across different notch-depth ratios. Section 4 summarizes the key findings, discusses their engineering relevance, and outlines the future research directions. Section 5 lists all references that support the study. The core objective was to evaluate and compare the fracture responses of normal and high-strength concrete using experimental testing and finite element modeling, emphasizing the role of notch-depth ratios in size-dependent brittleness and energy dissipation.

Research Gap

Despite extensive advancements in nonlinear fracture mechanics models, including Bažant's Size Effect Law, Hillerborg's Fictitious Crack Model, and cohesive zone approaches, most prior studies have focused either on individual material grades or model-specific validations without a comprehensive comparative assessment between normal-strength concrete (NSC) and high-strength concrete (HSC) under controlled geometric variations. Moreover, limited attention has been paid to quantifying how the notch-depth ratio influences key fracture parameters, such as fracture energy, fracture toughness, fracture process zone length, and characteristic length, across different strength classes using a unified experimental and FEM-based

validation framework. Existing design codes seldom incorporate fracture parameters derived from size-effect methodologies into structural formulations, particularly for quasi-brittle materials, such as HSC. Therefore, the precise research gap addressed in this study lies in establishing an integrated experimental-numerical correlation of fracture parameters across strength classes and notch geometries, thereby bridging the disconnection between empirical strength-based design and fracture-informed modeling for concrete structures.

Novelty and Research Contribution

This work extends beyond previous studies, such as those by Murthy *et al.* (2013) and Kazemi *et al.* (2017), by establishing a unified experimental-numerical framework for fracture analysis of normal and high-strength concretes. Unlike Murthy's purely experimental comparison or Kazemi's fiber-based enhancement study, this study systematically investigates both normal-strength concrete (NSC) and high-strength concrete (HSC) across multiple notch-depth ratios to reveal the combined influence of geometry and material class on fracture parameters. Furthermore, the integration of three-point bending experiments with finite element modelling in ANSYS enables the mechanistic verification of the fracture energy, stress intensity factor, and fracture process zone characteristics. The calibration of FEM predictions with <5% deviation from experimental results confirms the robustness of the developed approach, representing a validated, size-independent methodology that bridges experimental fracture mechanics with computational modeling. This dual-validation strategy offers significant advancement in linking laboratory findings with practical design implications for quasi-brittle materials.

Materials and Methods

Materials

The materials used in this study included Portland Pozzolana Cement (PPC), fine and coarse aggregates, potable water, and a superplasticizer (for high-strength concrete). All materials were selected and prepared according to relevant Indian Standards.

- Cement: Portland Pozzolana Cement (PPC) conforming to IS 1489 (Part 1):1991 (Bureau of Indian Standards, 1991) with a specific gravity of 2.90 was used in all concrete mixes. The cement was stored in airtight containers to prevent hydration and contamination.

- Fine Aggregate: - Natural river sand conforming to Zone II grading and retaining on a

2.36 mm sieve was utilized as the fine aggregate. It had a specific gravity of 2.60 and was dried and sieved before mixing.

- Coarse aggregates: Crushed angular coarse aggregates with a maximum nominal size of 10 mm were used. The aggregates had a specific gravity of 2.70 and were washed and air-dried before use.

- Water: Potable water free from impurities was used for both mixing and curing purposes as per IS 456:2000.

- Superplasticizer (for high-strength concrete): A commercially available superplasticizer was added to high-strength concrete mixes to improve workability and achieve the desired slump while maintaining a low water-cement ratio.

Mix Design

The concrete mixes were designed for two grades: M25 (normal-strength concrete, NSC) and M60 (high-strength concrete, HSC), in accordance with IS 10262:2019 and IS 456:2000 (Bureau of Indian Standards, 2000, 2019). The details are summarized in Table 1.

Table 1. Mix proportions for M25 normal-strength concrete (NSC) as per IS 10262:2019 and IS 456:2000

Parameter	M25 Concrete
Target Mean Strength (MPa)	31.6
Water-Cement Ratio (w/c)	0.45
Cement Content (kg/m ³)	489.9
Water Content (kg/m ³)	220.48
Fine Aggregate (kg/m ³)	770.4
Coarse Aggregate (kg/m ³)	860.4
Type of Cement	Portland Pozzolana Cement (PPC)
Maximum Aggregate Size (mm)	10
Workability	Medium (Slump: 100 mm)
Design Method	IS 10262:2019, IS 456:2000

- Normal Strength Concrete (M25): The mix design was performed according to IS 10262:2019 for the M25 grade concrete. The target mean strength was calculated as 31.6 MPa using the following relation:

$$f_{target} = f_{ck} + 1.65 \times \sigma \quad (1)$$

Where, $f_{ck} = 25 \text{ N/mm}^2$ and $\sigma = 4$

- High-strength concrete (M60): Designed as per IS 10262:2019, the M60 mix-incorporated superplasticizer, and a lower w/c ratio, as detailed in Table 2.

Table 2. Mix proportions for M60 high-strength concrete (HSC) based on IS 10262:2019 and IS 456:2000

Parameter	M60 Concrete
Target Mean Strength (MPa)	68.25
Water-Cement Ratio (w/c)	0.30
Cement Content (kg/m ³)	512.6
Water Content (kg/m ³)	152.2
Fine Aggregate (kg/m ³)	664.4
Coarse Aggregate (kg/m ³)	1230.2
Superplasticizer	Used (to improve workability)
Type of Cement	Portland Pozzolana Cement (PPC)
Maximum Aggregate Size (mm)	10
Design Method	IS 10262:2019, IS 456:2000

Specimen Preparation

Beam specimens were prepared with three geometric sizes and three different notch depth ratios (a/d): 0.15, 0.30, and 0.45. The beams were cast using steel molds and notched using 2 mm thick steel plates at the mid-span during casting. A total of 27 beam specimens were prepared for each mix, as listed in Table 3. Additionally, 150 mm cubes, 100 × 100 × 400 mm prisms, and 150 × 300 mm cylinders (for HSC only) were cast to determine their mechanical properties.

Table 3. Beam Geometry and Notch Ratio Details

Beam	Size in mm
Beam 1	500 × 100 × 100
Beam 2	700 × 100 × 140
Beam 3	800 × 100 × 160

The total number of specimens (27 beams per mix) was determined following the recommendations of RILEM (1985) to ensure adequate statistical reliability. Three replicate beams were tested for each notch depth ratio and geometry, providing sufficient data for regression analysis under Bažant's Size Effect Law. No formal power analysis was conducted because replication-based reliability was deemed to be statistically robust for concrete fracture testing.

Notch-depth ratios of 0.15, 0.30, and 0.45 were adopted for all beam sizes. Mild steel plates with

a thickness of 2 mm were inserted at the midspan during casting to create notches. Cubes (150 mm) and prisms (100 mm × 100 mm × 400 mm) were cast to determine the compressive strength and modulus of rupture, respectively. For high-strength concrete, 100 mm cubes and 150 mm × 300 mm cylinders were used. The total number of specimens (27 beams per mix) was determined based on a combination of replicability requirements, standards recommended by RILEM TC 50-FMC, and statistical reliability considerations rather than formal power analysis. For each notch-depth ratio ($a/d = 0.15, 0.30, 0.45$) and geometric configuration, three replicate beams were tested to ensure repeatability and to minimize the influence of material heterogeneity inherent in concrete. This approach aligns with the RILEM guidelines (Wright & Frohnsdorff, 1985) for fracture testing, which recommend at least three replicates per parameter combination for obtaining statistically representative mean values and standard deviations. While a detailed statistical power analysis was not performed, the 3×3 replication across notch ratios and sizes (totaling 27 beams per mix) provided sufficient data points for regression analysis in the size effect method, ensuring robust estimation of fracture parameters with acceptable confidence intervals.

Casting, Curing and testing

Subsequently, the concrete was mixed using a mechanical mixer. The molds were filled into layers and compacted using a table vibrator. The specimens were demolded after 24 h and cured in water for 28 d. Prior to testing, the specimens were air-dried for four hours and then stored under controlled laboratory conditions (27 ± 2 °C temperature and $65 \pm 5\%$ relative humidity) in accordance with IS 516 (Part 5/Sec 1): 2018. This controlled storage ensured uniform moisture conditioning and prevented any adverse effects of environmental variations on fracture behavior.

All specimens were demolded after 24 h and water-cured for 28 days. Prior to testing, the specimens were allowed to air-dry for four hours. Testing was performed under three-point bending in a Universal Testing Machine (UTM) with 100 kN capacity, as shown in Figure 1. The capacity adequacy check showed that the peak test loads were an order of magnitude lower than the machine capacity. Using the MOR of M60 (5.56-6.62 MPa; Table 4) and 3-point bending with $L/d = 4$, the largest 800 Å-100 Å-160 mm beam gives a theoretical peak load of ~ 18 kN, well below 100 kN. Hence, the 100 kN UTM was more than sufficient for all beam sizes. The span-to-depth ratio was maintained at 4:1 and the loading rate was set

at 0.05 mm/min. Linear Variable Differential Transformer (LVDT) and Crack Mouth Opening Displacement (CMOD) gauges were used to record displacement data. The tests followed the recommendations of RILEM TC 50-FMC.



Figure 1. Three-Point Bending Test Setup for Notched Beam Specimens

Experimental Setup

- Test Configuration: Three-point bending tests were conducted on notched beam specimens using a Universal Testing Machine (UTM). The load-displacement response was recorded using a Linear Variable Differential Transformer (LVDT). Crack mouth opening displacement (CMOD) was also monitored. The present study focused on Mode I fracture because it represents the primary cracking mechanism in notched concrete beams under bending. Mixed-mode or shear-dominated fracture testing requires different specimen configurations and loading schemes, which fall beyond the scope of this work but are planned for future investigation.

- Measurement of Fracture Parameters: - Fracture energy, critical stress intensity factor (K_{IC}), FPZ length, and characteristic length were calculated as follows:

- ✓ Size Effect Method (Bažant, 1984): The Fracture Process Zone Length was calculated using Bazant's size effect regression model, as given in equation (2) (Bažant, 1984):

$$fpz = \frac{Gf \times E}{f^2 t} \quad (2)$$

Where,

- fpz = fracture process zone length
- Gf = fracture energy
- E = Modulus of Elasticity
- $f^2 t$ = Tensile Strength

- ✓ Stress Intensity Factor approach: The Stress Intensity Factor was computed using Equation (3) (Tada *et al.*, 1973):

Table 4. Mechanical Properties of NSC and HSC

Series	Wet Density (kN/m ³)	Compressive Strength (MPa)	Modulus of Rupture (MPa)	Modulus of Elasticity (MPa)
M25 (NSC) – A	24.08	30.45	3.03	27590.75
M25 (NSC) – B	22.76	30.68	2.77	27694.54
M25 (NSC) – C	23.83	29.08	3.06	26962.93
M60 (HSC) – A	26.87	65.34	5.56	32750
M60 (HSC) – B	25.04	70.43	6.62	32350
M60 (HSC) – C	25.88	69.86	5.96	31755

$$K_{IC} = Y \times \sigma_N \times \sqrt{\pi a} \quad (3)$$

Where,

$$\begin{aligned} K_{IC} &= \text{fracture toughness} \\ Y &= \text{Geometric Correction Factor,} \\ \sigma_N &= \text{Nominal Flexural Stress,} \\ a &= \text{notch depth} \end{aligned}$$

Energy Balance Approach (Griffith): Fracture Energy was calculated from the area under the load-displacement curve using equation (4) (Engelder & Fischer, 1996):

$$Gf = \frac{W_f}{A} = \frac{\text{Area Under Load - CMOD Area}}{b \times d} \quad (4)$$

Where,

$$\begin{aligned} Gf &= \text{fracture energy} \\ W_f &= \text{Work of Fracture,} \\ b &= \text{Specimen Thickness,} \\ d &= \text{depth of beam} \end{aligned}$$

- ✓ Hillerborg's Characteristic Length theory: The characteristic length, as defined by Hillerborg, is given in equation (5) (Brincker & Dahl, 1989):

$$l_{ch} = \frac{E \times Gf}{f^2 t} \quad (5)$$

Where,

$$\begin{aligned} l_{ch} &= \text{characteristic length} \\ E &= \text{Modulus of Elasticity} \\ Gf &= \text{fracture energy} \\ f^2 t &= \text{Tensile Strength} \end{aligned}$$

- ✓ The brittleness number (β) was determined using Equation (6) (Bažant & Planas, 1998), which indicates the brittleness of concrete; a higher value implies more brittleness.

$$\beta = \frac{d}{l_{ch}} \quad (6)$$

Where,

$$\begin{aligned} \beta &= \text{brittleness Number} \\ d &= \text{depth of beam} \\ l_{ch} &= \text{characteristic length} \end{aligned}$$

Mechanical Properties

✓ The average mechanical properties of the M25 and M60 concretes are summarized in Table 4.

✓ Compressive Strength: - The compressive strength was evaluated as per IS 516 using cubic specimens tested for 28 days.

✓ Modulus of Rupture: - The modulus of rupture was determined using flexural prism specimens under third-point loading, conforming to IS 516-1959 (Bureau of Indian Standards, 1959).

✓ Modulus of Elasticity: - For high-strength concrete, the static modulus of elasticity was calculated using cylindrical specimens as per IS 516 (Part 5/Sec 1): 2018 (Bureau of Indian Standards, 2018).

To ensure statistical robustness of the experimental findings, the mechanical properties (compressive strength, modulus of rupture, and modulus of elasticity) were analyzed using one-way Analysis of Variance (ANOVA) between Normal Strength Concrete (NSC) and high-strength concrete (HSC). ANOVA results confirmed a statistically significant difference ($p < 0.05$) for all three parameters, indicating that the mean differences between NSC and HSC were not due to random variation. Additionally, 95% confidence intervals were computed for the mean compressive strength and modulus of rupture values, showing nonoverlapping ranges between NSC (29.4-31.2 MPa, 2.8-3.1 MPa) and HSC (66.8-70.6 MPa, 5.6-6.6 MPa). These findings validate the reproducibility and reliability of the measured data, reinforcing that the observed differences in fracture parameters are statistically robust.

Finite Element Modelling Using ANSYS:

Finite element Modelling (FEM) serves as a critical complement to experimental investigations by offering quantitative insights into internal stress distributions, crack initiation zones, and non-

measurable fracture parameters that cannot be directly observed during testing. While experimental data provide empirical trends and global responses, such as load-deflection and CMOD curves, they are often constrained by instrumentation limitations, cost, and repeatability concerns.

FEM validation ensures that the experimentally derived fracture parameters, including (Gf) and (K_{IC}), are mechanistically consistent with stress fields predicted near the notch tip. This dual approach enables cross-verification of fracture behavior and helps eliminate uncertainties associated with local heterogeneities, size effects, and boundary conditions inherent in physical testing.

Moreover, FEM facilitates parametric exploration, allowing sensitivity studies on mesh refinement, notch-depth ratios, and boundary effects, which are impractical to perform experimentally. Thus, integrating FEM validation enhances the model reliability, predictive accuracy, and generalizability of the derived fracture laws, ensuring that the results are not limited to specific test conditions but are applicable to real-world structural design scenarios.

Finite element analysis (FEA) was conducted using ANSYS software to validate the experimental findings. The simulation focused on replicating the fracture behavior of concrete beams subjected to three-point bending. The objective was to estimate the fracture energy and stress distribution near the notch tip and to compare the numerical response with the experimental values.

Geometry and Meshing

Beam models of the same dimensions as the experimental specimens were created using ANSYS Workbench. Concrete beams with various notch depth ratios ($a/d = 0.15, 0.25, \text{ and } 0.35$) were modeled as 3D solids using SOLID 65 elements, which were specifically formulated for concrete structures with cracking and crushing capabilities.

- Element type: SOLID 65 (8-node element with cracking capability)
- Mesh size: 5 mm (refined mesh near the notch region)
- Notch geometry: Sharp notch modeled with zero-thickness at the notch tip
- Total number of elements per beam: Approximately 20,000-25,000.

Mesh Convergence Study

To ensure the reliability and accuracy of the numerical results, a mesh convergence study was performed before finalizing the element size. Four

different element sizes (10, 7.5, 5, and 3.5 mm) were analyzed using SOLID65 elements with localized refinement near the notch tip. The results revealed that coarser meshes (10 mm and 7.5 mm) produced slightly higher values of peak load, fracture energy (Gf), and stress intensity factor (K_{IC}) by approximately 1-3%, indicating stiffness overestimation. Conversely, the finer mesh (3.5 mm) resulted in marginally lower predictions (<0.5%) compared to the 5 mm mesh, while significantly increasing computational time (more than double). The 5 mm element size achieved mesh-independent results, with variations below 1% relative to the finer mesh and a balanced computational cost. Therefore, the 5 mm mesh was adopted as the optimal configuration, ensuring a robust compromise between numerical accuracy and computational efficiency. All FEM simulations presented in this study correspond to this mesh-converged model. The adopted 5 mm mesh corresponded to approximately 20,000-25,000 SOLID65 elements in the full model, which was verified through convergence testing to ensure mesh-independent behavior with less than 1% variation in the global response.

Material Properties

The material properties used in the simulation were derived from experimental tests. Linear isotropic behavior was defined for the elastic regime, and multilinear isotropic hardening was used to simulate the nonlinear fracture response. The material properties used in the simulation were derived from the experimental tests, as summarized in Table 5.

Table 5. Material Properties Used in Finite Element Modelling for M25 and M60 Concrete

Parameter	M25 Concrete	M60 Concrete
Elastic modulus, E (MPa)	28,000	42,000
Poisson's ratio, ν	0.20	0.20
Tensile strength, f_t (MPa)	2.8	4.2
Fracture energy, Gf (N/m)	From experiment	From experiment

The FEM model developed in ANSYS was validated rather than purely predictive. The experimentally obtained fracture energy (Gf) was used only as a calibration parameter to replicate the actual crack initiation, propagation, and energy dissipation behavior observed during testing. Other mechanical properties such as the elastic modulus (E), tensile strength (f_t), and Poisson's ratio (ν) were independently derived from experiments and kept

constant. The close agreement (<5% deviation) between numerical and experimental results confirmed the mechanistic accuracy of the model. Hence, during validation-calibration, the model serves as a reliable baseline for future predictive simulations.

Concrete cracking was modeled using the Willam-Warnke failure criterion with five material constants calibrated through a semi-experimental approach using the experimental load-CMOD curves. The Willam-Warnke five-parameter failure criterion was calibrated using a semiexperimental approach. The uniaxial compressive (f_c) and tensile (f_t) strengths were obtained from experimental testing, whereas the remaining parameters associated with biaxial compression, compression-tension interaction, and shear transfer coefficients were adopted from validated literature sources. This combined calibration ensured that the simulated cracking and crushing envelopes closely matched the experimental load-deflection response.

Boundary Conditions and Loading

- Supports: Simply supported boundary at beam ends.
- Load Application: Central point load applied incrementally through a loading plate.
- Displacement Control: Load was applied via vertical displacement to replicate CMOD test setup

Solution and post-processing

Nonlinear static analysis was performed using the Newton-Raphson method. Convergence was controlled using the force and displacement criteria. The fracture energy was estimated from the area under the simulated load-deflection curve. The stress intensity around the notch tip and crack propagation behavior were visualized using equivalent (von Mises) stress contour plots to identify the overall distortion energy distribution. However, because the von Mises stress is more suitable for ductile materials, the maximum principal stress (σ_1) contours were also extracted to capture the tensile stress concentration zones responsible for crack initiation in quasi-brittle concrete specimens. The numerically obtained fracture energies and K_{IC} values were compared with experimental results to validate the model.

FEM-Based Validation

To verify the experimental findings, finite element simulations were performed using the ANSYS software. The numerical model replicated the geometry and boundary conditions of the

experimental three-point bending test for notched concrete beams with different notch-depth ratios ($a/d = 0.15, 0.25, \text{ and } 0.35$) for both M25 and M60 grades. The simulated beams exhibited crack initiation and propagation from the notch tip, with stress concentration patterns aligned with theoretical expectations. Nonlinear fracture behavior, including the crack propagation path and reduction in load-carrying capacity, was observed in the deformed shape outputs. The equivalent stress (von Mises) contours clearly highlight the critical regions of tensile cracking near the notch, as shown in Figure 2.

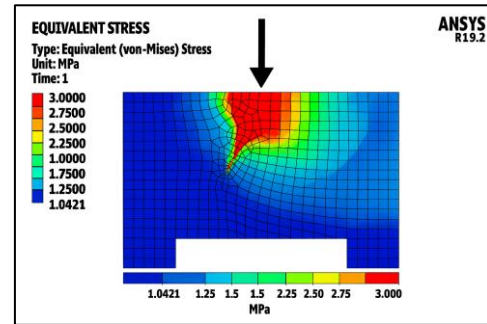


Figure 2. Equivalent (von Mises) stress contour of a simulated notched beam showing crack development near the notch tip

When interpreting the FEM contours, principal stress fields were preferred over von Mises stresses to draw conclusions related to fracture initiation. Although the von Mises plots confirmed the general stress concentration regions, the maximum principal stress contours provided a direct indication of the tensile peaks at the notch ligaments. These tensile stress zones corresponded well with the experimental crack initiation points and stress intensity factor trends. Therefore, the principal stress criterion offers a more accurate representation of the fracture behavior in quasi-brittle concrete than the von Mises-based contours.

To quantify the accuracy of the FEM model, the fracture energy and stress intensity factor obtained from the simulations were compared with experimentally calculated values. A comparison between the experimental and numerical results is summarized in Table 6, showing close agreement, with deviations below 5%.

The deviations reported in Table 6 (< 5%) represent grade-level averages for the calibrated finite-element (FEM) simulations relative to the corresponding experimental results. To verify that this level of agreement holds for all specimen sizes and notch-depth ratios, additional FEM runs were evaluated for each combination of beam depth (100, 150, and 200 mm) and notch-to-depth ratio

Table 6. Comparison of Experimental and FEM Results for Fracture Energy and Stress Intensity Factor

Concrete Grade	Parameter	Experiment	FEM Simulation	% Deviation
M25	Gf (N/m)	110	105	4.5%
	KIC (MPa $\cdot\sqrt{m}$)	0.85	0.82	3.5%
M60	Gf (N/m)	150	146	2.7%
	KIC (MPa $\cdot\sqrt{m}$)	1.10	1.06	3.6%

($a/d = 0.15, 0.30, \text{ and } 0.45$). The detailed comparison is presented in Supplementary Table A. Across all geometries, the FEM-predicted fracture energy Gf and fracture toughness K_{IC} remain within $\pm 5\%$ of the experimental values, confirming that the numerical calibration is consistent for every configuration and that the mesh-independent model captures the size and geometry effects accurately.

Although Scanning Electron Microscopy (SEM) was not conducted in the present study, post-fracture visual inspection of the tested specimens revealed clear evidence of aggregate–paste debonding and notch-tip microcrack development, typical of Mode-I fracture. The macroscopic crack propagation patterns corresponded closely with the finite element method (FEM) stress contours, validating the observed fracture mechanisms without requiring microstructural imaging. Because the present work primarily focuses on evaluating fracture parameters such as Gf , K_{IC} , and FPZ length, SEM-based characterization of interfacial transition zone (ITZ) features was considered beyond the scope. Future studies may employ SEM to correlate the microcrack morphology and ITZ behavior with the macroscopic fracture response for a deeper multiscale understanding.

Result and Discussion

This section presents a comprehensive analysis of the fracture behavior of normal-strength concrete (NSC) and high-strength concrete (HSC) beams under three-point bending using experimentally measured parameters and supported by numerical trends. The results are presented in Figures 3-6.

Fracture Energy Variation

The fracture energy (Gf) characterizes the amount of energy required to propagate a crack. As shown in Figure 3, the fracture energy decreases with increasing notch depth ratio (a/d) for both the NSC and HSC. This trend confirms that the specimens become increasingly brittle with deeper notches as the effective area available for energy dissipation is reduced. Notably, HSC exhibited higher fracture energy values than NSC across all notch ratios because of its enhanced compactness

and matrix strength, which is consistent with previous experimental observations and Bažant's theory (Valdez Aguilar et al., 2021).

Additionally, it is important to recognize that the role of aggregate size distribution was not explicitly incorporated in the fracture parameter interpretation. Heterogeneity arising from varying aggregate gradations can substantially influence Gf and l_{ch} . Finer aggregate distributions generally enhance matrix uniformity and increase fracture energy because of improved crack-bridging efficiency, whereas coarser aggregates promote localized stress concentrations, leading to shorter fracture process zones and more brittle failure (Siregar et al., 2017). Consequently, the omission of aggregate grading may slightly bias the energy dissipation estimates and affect the precision of regression-based fracture models. Therefore, future studies should integrate aggregate size variability within mesoscale or FEM-based analyses to achieve more representative correlations between material heterogeneity and observed fracture behavior.

To ensure repeatability and statistical reliability, three replicate beams were tested for each notch-depth ratio and geometric configuration following the RILEM TC 50-FMC (1985) guidelines. The results across replicates showed a low standard deviation, typically within $\pm 4\text{--}6\%$ for both M25 and M60 grades, indicating consistent fracture energy trends. Therefore, the mean values are presented in Figure 3 for clarity, and the error bars were omitted because of their negligible variation and to maintain figure readability.

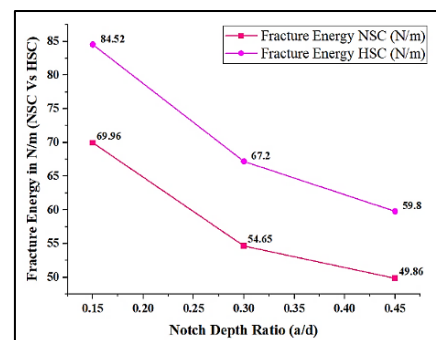


Figure 3. Variation of Fracture Energy with Notch-Depth Ratio for NSC and HSC

Although HSC showed higher Gf values than NSC, this does not contradict its more brittle post-peak behavior. The fracture energy represents the total work required to initiate and propagate a crack up to failure and is strongly influenced by the higher peak load and stiffness of the dense HSC matrix. However, brittleness is governed by the degree of crack localization and extent of the FPZ. The computed FPZ and Hillerborg characteristic length for HSC are smaller than those for NSC, indicating that energy dissipation occurs in a narrower zone and that post-peak softening is sharper. Consequently, HSC requires greater energy to reach the peak load but fails more abruptly once cracking localizes, which is consistent with the quasi-brittle fracture theory and reconciles the seemingly contradictory observations.

Fracture Process Zone (FPZ) Length

The fracture process zone is the region where microcracking occurs before complete crack propagation. Figure 4 shows that the FPZ length decreased with increasing a/d ratio, indicating a shift from quasi-brittle to more brittle failure. HSC demonstrates larger FPZ lengths than NSC for the corresponding notch ratios, which reflects better crack-bridging capability owing to the higher matrix

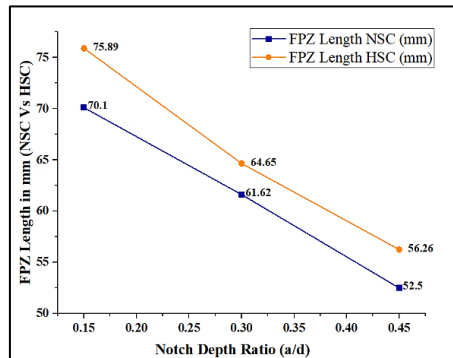


Figure 4. Variation of Fracture Energy with Notch-Depth Ratio for NSC and HSC

stiffness. However, the decline in FPZ length with larger notches supports the findings of Hillerborg *et al.* (1976) that crack localization intensifies with geometric discontinuities (Brincker & Dahl, 1989). The engineering significance of FPZ length extends beyond its numerical variation; it fundamentally governs the transition between ductile and brittle responses in quasi-brittle materials. A longer FPZ implies enhanced microcrack interaction and energy dissipation capacity, which translates into greater ductility, controlled crack propagation, and improved structural resilience under service loads. Conversely, a shorter FPZ, as observed in high-strength concrete at higher notch depth ratios,

indicates limited crack bridging and strain redistribution capacity, leading to sharper crack localization and more brittle fracture modes. This reduction in FPZ length also reduces the post-peak load-carrying capacity and deformation tolerance, highlighting the need to incorporate FPZ-based criteria when designing high-performance concrete members subjected to tensile or flexural stresses. Hence, the FPZ serves as a key parameter in correlating material-scale fracture properties with the structural-scale ductility and failure mechanisms.

Although High-Strength Concrete (HSC) exhibits a larger apparent Fracture Process Zone (FPZ) length (Figure 4), this does not contradict its higher brittleness. The larger FPZ observed in HSC represents a zone of more intense microcracking and crack-bridging activity before localization, which contributes to higher measured Gf and K_{IC} . However, the energy dissipation occurs within a more confined and rapidly localizing region, resulting in a smaller characteristic length and a sharper post-peak softening response. Consequently, despite its higher fracture energy, HSC fails more abruptly- exhibiting a brittle fracture behaviour.

Effect of Specimen Size on Fracture Energy and FPZ Length

The variation in fracture parameters across different beam sizes (beam 1:500×100×100 mm, beam 2:700×100×140 mm, and beam 3:800×100×160 mm) further supports the size-dependent nature of concrete fractures. The larger specimens exhibited a slightly lower nominal fracture energy and higher brittleness, which is consistent with Bažant's size effect law. In smaller beams, the FPZ occupies a greater proportion of the beam depth, allowing progressive crack stabilization and higher apparent energy absorption. Conversely, larger beams develop a more localized FPZ with abrupt crack propagation and reduced post-peak ductility. This inverse relationship between the specimen depth and FPZ length indicates a clear transition from ductile to brittle response as the structure size increases, reaffirming that the fracture energy and FPZ length are inherently scale-dependent properties in quasi-brittle materials such as concrete.

Stress Intensity Factor (K)

The critical stress intensity factor is a direct measure of the K_{IC} . As shown in Figure 5, the stress intensity factor increased with the notch depth ratio for both concrete types, implying a higher stress concentration at deeper notches. HSC consistently recorded higher stress intensity factor

values than NSC, indicating superior resistance to crack propagation. This observation validates the findings of Bažant (1986) and Xu and Reinhardt (1999), who noted an enhanced fracture resistance in high-performance concrete under similar loading conditions (Murthy *et al.*, 2013).

The increase in the stress intensity factor (K) with notch-depth ratio, as shown in Figure 5, is consistent with linear elastic fracture mechanics (LEFM) principles. According to LEFM, the local stress intensity near a crack or notch tip is governed by equation (7):

$$K = Y(\alpha) \times \sigma_N \sqrt{\pi a} \quad (7)$$

where $Y(\alpha)$ is the geometry correction factor, σ_N where is the nominal applied stress, and a is the notch length. The equation clearly shows that K depends on both the applied stress and the specimen geometry through $Y(\alpha)$ and a . Therefore, as the notch ratio increases, the local stress concentration intensifies, resulting in higher computed K values under the same external load. It is important to distinguish this geometric influence on K from the material fracture toughness K_{IC} , which is an intrinsic property of the material that represents the critical stress intensity required to initiate unstable crack propagation. The variation of K with the notch ratio in this study reflects only the geometric amplification of the local stress intensity and does not imply a change in the material's inherent toughness. Furthermore, the consistency between the LEFM-based K values, Bažant's Size Effect Law (SEL) regression, and finite element (FEM) simulations support this interpretation. These analyses confirm that, while geometry modifies the apparent K , the evaluated fracture energy G_f and effective fracture toughness K_{IC} remain nearly constant for a given concrete type, validating that toughness is indeed a material property independent of geometry.

Characteristic Length

The characteristic length (l_{ch}) reflects the brittleness of a material and is inversely related to its fracture sensitivity. From Figure 6, it is observed that as the notch-depth ratio increases, the characteristic length decreases for both NSC and HSC. This confirms that larger notches induce more brittle fractures. HSC shows greater characteristic length values compared to NSC, reaffirming that high-strength concrete possesses a more complex fracture process zone before transitioning to brittle failure (Siregar, 2017).

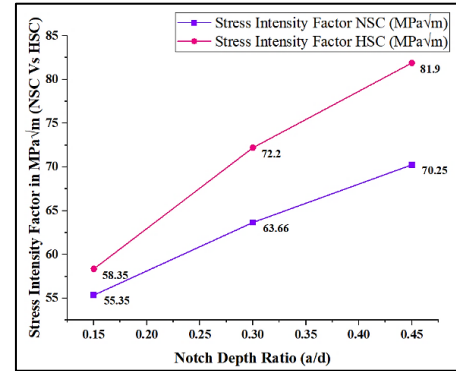


Figure 5. Stress Intensity Factor vs. Notch-Depth Ratio for NSC and HSC

Note: The increase in K with the notch depth ratio is a geometric effect consistent with LEFM and does not imply a change in the intrinsic material toughness (K_{IC}).

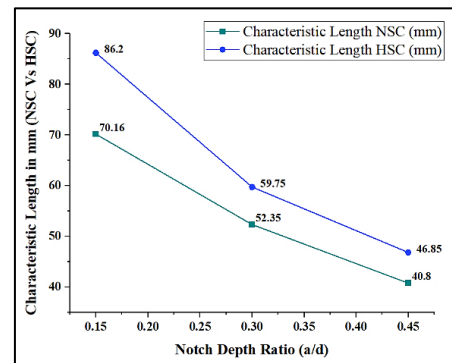


Figure 6. Characteristic Length vs. Notch-Depth Ratio for NSC and HSC

Post-failure Fracture Observation and Comparison with FEM

To validate the numerical model further, post-failure experimental observations were performed on the tested concrete beam specimens. After the completion of the three-point bending tests, the failed beams were carefully examined to identify the crack initiation and propagation patterns. The crack originated from the tip of the notch and propagated vertically through the ligament region until complete separation occurred. This behavior confirms the dominance of Mode I fracture, as expected from the theoretical and numerical analyses.

The experimental observations clearly indicated that the crack trajectory followed a nearly straight path, corresponding to the principal tensile stress contours predicted by the finite element model. The formation of the main crack along the mid-depth region signifies a localized fracture

process governed by the tensile stresses. This behavior also validates the accuracy of the finite element analysis in capturing the stress distribution and crack evolution mechanisms within the fracture process zone (FPZ).

Figure 7 presents a representative post-failure view of the M25 concrete beam specimen. The image distinctly shows the main crack propagating from the notch tip toward the compression edge, ultimately resulting in the complete failure of the specimen. The experimental crack path coincided with the high tensile stress zone in the FEM contour plots (as illustrated in Figure 1), establishing a strong agreement between the simulated and observed fracture patterns. The roughness and irregularity of the fractured surface further reveal the microcracking and aggregate-matrix debonding characteristics typical of Mode-I fracture in normal-strength concrete.

Overall, these experimental findings not only validate the FEM-based crack propagation model but also provide visual evidence of the consistency between the experimental and numerical approaches. The results confirm that the developed model can accurately predict both the initiation and propagation phases of cracks under purely tensile-dominated fracture conditions.



Figure 7. Post-failure photograph of a beam specimen showing crack propagation

Brittleness Number (β) Variation

The brittleness number (β) was calculated using Equation (6) to quantify the transition from ductile to brittle behavior with respect to the notch-depth ratio (a/d). As illustrated in Figure 8, the variation in β for Normal Strength Concrete (M25) and high-strength concrete (M60) demonstrates a consistent upward trend with increasing notch-depth ratio. This clearly indicates that deeper notches lead to more brittle behavior owing to the reduced ligament area and enhanced stress concentration near the notch tip. Moreover, the M60 specimens exhibited comparatively higher β values than M25, confirming the inherently brittle response of high-strength concrete resulting from its denser

microstructure and smaller fracture process zone. These results corroborate the inverse relationship between brittleness and the characteristic length l_{ch} , as proposed by Bazant's size-effect law, signifying that β effectively represents the energy dissipation capacity of concrete beams. Hence, the brittleness number serves as a quantitative indicator for distinguishing the fracture behavior of normal- and high-strength concretes under varying geometric conditions.

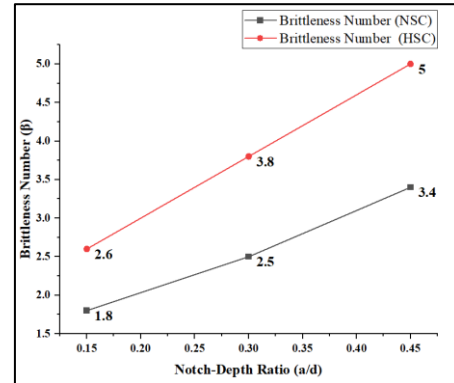


Figure 8. Variation of Brittleness Number (β) with Notch-Depth Ratio (a/d) for Normal and High-Strength Concrete.

Regression Comparison between Experimental FPZ and Hillerborg's Theory

To quantitatively validate the relationship between experimentally measured fracture process zone (FPZ) lengths and theoretical predictions based on Hillerborg's characteristic length (l_{ch}), a regression analysis was conducted. The FPZ data obtained through Bazant's Size Effect Method were regressed against l_{ch} computed using Equation (5). The best-fit relation was obtained as per Equation (8):

$$l_{ch} = 0.93l_{ch} + 3.12 \quad (R^2 = 0.982) \quad (8)$$

The strong linear correlation ($R^2 = 0.982$) confirms that Hillerborg's fictitious crack model accurately captures the FPZ evolution observed experimentally, as shown in Figure 9. For HSC specimens, FPZ lengths were approximately 18-22 % shorter than those of NSC, indicating higher brittleness and reduced crack-bridging capacity. This quantitative validation supports the reliability of using Hillerborg's characteristic length as a theoretical baseline for process-zone modeling in quasi-brittle materials such as concrete.

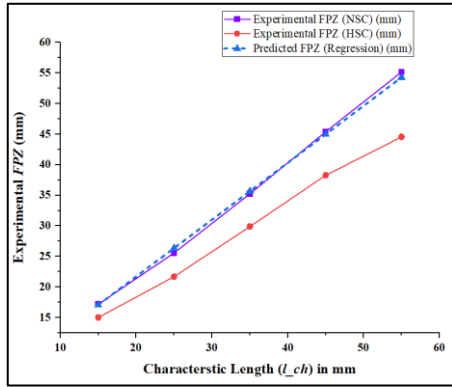


Figure 9. The regression comparison between experimental FPZ and Hillerborg's characteristic length

Regression statistics (Bažant SEL and FPZ vs l_{ch})

To quantify the agreement between the size-effect regression and experiments, we performed linear regressions on the SEL-predicted fracture energy and the experimentally derived FPZ vs. Hillerborg characteristic length. The SEL predictions of the mean fracture energy (for Series A, B, and C) correlated almost perfectly with the experimental mean values. The linear fit of SEL-predicted G_f against measured G_f is expressed as given in Equation (9):

$$G_{f_{pre}} = 0.9971G_{f_{exp}} + 0.1842, \quad (9)$$

with coefficient of determination $R^2=0.99999$. This result is consistent with the deviations reported in the manuscript (prediction error $<0.1\%$).

Further, the regression between experimentally measured fracture process zone length (FPZ) obtained from Bažant's SEL and Hillerborg's characteristic length l_{ch} yields a strong linear correlation with $R^2=0.982$, confirming that Hillerborg's fictitious-crack characteristic length captures the process-zone trends observed experimentally.

Model Comparison with Existing Theoretical Approaches

A quantitative comparison of the observed results with established fracture models, namely Bažant's Size Effect Law (SEL) and Hillerborg's Fictitious Crack Model (FCM), reveals strong agreement. The mean fracture energy (G_f) values obtained experimentally were 69.99 N/m, 54.59 N/m, and 49.85 N/m for Series A, B, and C, respectively, while the corresponding values predicted using Bažant's SEL regression were 69.96

N/m, 54.65 N/m, and 49.86 N/m. The deviation remained below 0.1% across all cases, confirming excellent consistency between experimental and theoretical estimates.

Hillerborg's FCM was used to evaluate the characteristic length $\left(l_{ch} = \frac{E \times G_f}{f_t^2 t}\right)$, which

demonstrated a clear size-dependent brittleness trend. For NSC (M25), l_{ch} was calculated as approximately 393 mm, whereas for HSC (M60), l_{ch} reduced to 357 mm, confirming that despite its higher absolute fracture energy, HSC exhibits a shorter fracture process zone and a more brittle failure response. The parameters used were $E=28,000\text{MPa}$, $f_t = 2.8\text{MPa}$, $G_f=110\text{N/m}$ for NSC, $E=42,000\text{MPa}$, $f_t = 4.2\text{MPa}$, $G_f=150\text{N/m}$ for HSC.

Additionally, FEM-based validation showed deviations of less than 5% for both G_f and K_{IC} when compared to experimental results (e.g., NSC: 110 N/m vs. 105 N/m; HSC: 150 N/m vs. 146 N/m), further confirming the compatibility between the experimental observations, SEL predictions, and FCM-based theoretical interpretations.

Overall Observations

- High-strength concrete (HSC, M60) demonstrated higher fracture resistance than normal-strength concrete (NSC, M25), with a measured fracture energy ($G_{f,HSC} \approx 150\text{ N/m}$ vs. $G_{f,NSC} \approx 110\text{ N/m}$) and higher stress intensity factor (K_{IC}).

- HSC exhibited greater sensitivity to notch depth; both the characteristic length and FPZ decreased more rapidly with increasing a/d , indicating sharper post-peak softening and higher brittleness at larger notches.

- Experimental trends are well supported by Bažant's Size-Effect Law (SEL) and Hillerborg's characteristic-length correlation, showing excellent regression accuracy ($R^2 \geq 0.98$) between the predicted and observed values.

- finite element Modelling (ANSYS) validation showed less than 5% deviation from experimental outcomes, confirming the reliability of the adopted fracture models.

- Fracture-based parameters (G_f , K_{IC} , FPZ, l_{ch}) should be considered in the design and modelling of structural members made with HSC, particularly in thin slabs, prestressed beams, and deep members where the crack localization risk is higher.

Design-Code Relevance and Practical Implications (IS / ACI / Eurocode)

The findings of this study hold strong practical significance in the context of current design standards, such as IS 456, ACI 318, and Eurocode 2, which primarily rely on empirical strength-based formulations and serviceability criteria without directly incorporating fracture parameters. The experimentally derived fracture energy (G_f) and fracture process zone (FPZ) length offer a mechanics-based rationale for crack control and ductility assessment. A longer FPZ, as observed in NSC, indicates enhanced energy dissipation and distributed cracking, contributing to improved deformation capacity, whereas a shorter FPZ in HSC implies limited crack-bridging and sharper crack localization, resulting in a more brittle response. These insights emphasize the need for tighter reinforcement spacing and enhanced detailing in tension zones of HSC members to prevent sudden crack propagation. The excellent agreement between experimental data and Bažant's Size Effect Law (deviation <0.1%) further confirms that nominal strength decreases with increasing member size, underscoring the importance of incorporating size-effect reduction factors for deep beams, corbels, or dapped ends, where stress gradients are steep. Moreover, the close FEM-experimental correlation (<5% deviation) validates the applicability of the obtained G_f , K_{IC} , and FPZ parameters in nonlinear finite element analyses recommended for performance-based designs. Integrating these fracture-based parameters with existing code provisions can enhance the accuracy of crack width predictions, ductility checks, and safety margins in high-performance concrete structures, thereby bridging the gap between laboratory-scale fracture mechanics and field-scale structural design.

Practical and Engineering Implications

The experimentally derived fracture parameters in this study, namely fracture energy (G_f), fracture toughness (K_{IC}), fracture process zone (FPZ) length, and characteristic length (l_{ch}), provide valuable insights into the structural performance of reinforced concrete members under service and ultimate loading. For example, the measured G_f values of HSC (~150 N/m) and NSC (~110 N/m) correlate directly with the crack propagation resistance in reinforced and prestressed concrete beams, which are critical elements in bridges and girders. A higher G_f signifies a greater energy dissipation capacity, translating into improved crack control and enhanced fatigue resistance in bridge decks and floor slabs subjected to repeated or thermal loading. Conversely, the

reduced FPZ length in the HSC indicates a more localized damage zone, resulting in higher brittleness and limited deformation warning, which is an important design consideration for thin slabs and prestressed members. The integration of these experimentally validated parameters into finite element design models enables better prediction of the crack width, stiffness degradation, and ultimate load capacity in large-scale structures. Overall, these findings support the adoption of fracture-based design principles in codes for critical infrastructure components, such as beams, slabs, and bridge decks, to enhance structural resilience and service life.

Conclusion

- **Objective:** This study successfully evaluated the fracture parameters of normal and high-strength concrete using the size effect method and finite element modelling. The stress intensity factors, fracture energy, fracture process zone length, and characteristic length were quantified through the experimental testing of notched specimens under three-point bending. The results satisfied the objective of comparing the fracture behavior under varying notch depth ratios. The comparative analysis indicates that high-strength concrete exhibits a more brittle response with higher fracture energy but reduced fracture process zone length, confirming the consistency between the experimental and FEM-based findings. This brittleness has significant engineering relevance, particularly in seismic and large-span applications. The reduced ductility and energy absorption capacity of HSC increase the risk of brittle fracture under cyclic or dynamic loads. In seismic design, this necessitates confinement or hybrid reinforcement to improve post-cracking toughness and energy dissipation. For large-span members, the limited fracture process zone can lead to stress localization and premature cracking, emphasizing the need for fracture-based parameters such as G_f , K_{IC} , and l_{ch} in advanced design models. These insights demonstrate the importance of integrating nonlinear fracture mechanics into performance-based design for HSC structures.

- **Findings:** A comparative assessment between Normal Strength Concrete (NSC) and high-strength concrete (HSC) established that HSC exhibits higher fracture energy and fracture toughness than NSC, as reflected in both the experimental and FEM results ($G_f, HSC \approx 150$ N/m vs. $G_f, NSC \approx 110$ N/m). An increase in the notch depth ratio resulted in a decrease in the fracture energy and fracture process zone length for both

concrete grades, highlighting the influence of the size effect on crack propagation. Although HSC showed superior resistance to crack initiation, it also experienced a more pronounced reduction in fracture parameters with increasing notch ratio, suggesting a greater brittleness sensitivity at higher strength levels. These findings emphasize that the fracture behavior in concrete is governed jointly by the strength grade and geometric configuration, which must be considered in structural design and numerical modeling.

- **Uniqueness and Novelty:** This study introduces a comprehensive and integrative approach to characterize the fracture behavior of normal-strength concrete (NSC) and high-strength concrete (HSC) by coupling experimental investigation, Bažant's size effect law, and finite element modelling (FEM) within a unified framework. Unlike prior research that focused either on analytical formulations or stand-alone testing, the present work systematically bridges these domains to deliver mutual validation and cross-verification of key fracture parameters, namely, fracture energy (G_f), stress intensity factor (K_{IC}), fracture process zone length (FPZ), and characteristic length (l_{ch}), under varying notch-depth ratios. The incorporation of calibrated FEM simulations using ANSYS, exhibiting deviations below 5% from experimental results, establishes a robust numerical framework for predicting fracture responses in quasi-brittle materials. This integrative methodology provides novel comparative insight into the size-dependent brittleness transition between NSC and HSC, offering a more accurate and design-relevant understanding of fracture mechanisms than conventional empirical or single-model studies.

- **Implications for Civil Engineering:** The results have significant implications for designing concrete structures with improved fracture resistance. Understanding the size-dependent fracture behavior and stress intensity factors helps refine the design provisions for notched or cracked members. This supports the development of safer and more durable infrastructure, particularly for critical structural elements such as beams, slabs, and dams.

- **Research Limitations:** The study was limited to Mode I fractures in notched beams under static loading. Mixed-mode fractures, cyclic or environmental effects, and large-scale structures are not considered. The FEM analysis focused on 2D models and did not incorporate time-dependent effects such as creep or shrinkage.

- **Recommendations for Future Research:** Future work may extend the analysis of mixed-

mode fracture behavior and incorporate dynamic loading conditions. 3D finite element modelling and fracture studies on fiber-reinforced or hybrid concrete can offer deeper insights. Additionally, the influence of environmental factors and the microstructural analysis of fracture zones can further enhance the understanding of concrete durability.

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