

DERIVATION OF A MATHEMATICAL MODEL OF THE DC HEAVY RAILWAY SYSTEM USING THE DQ METHOD

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Abstract

The electric railway system is an efficient mode of transportation capable of carrying a large number of passengers per trip, thereby reducing travel time and operational costs. As a result, its use has been increasing, following the technological advances. To accurately analyze the behavior of this system, an accurate mathematical model is required. Herein, the DQ method is employed to obtain a time-invariant model via the transformation of time-varying AC signals into DC quantities within a synchronous rotating reference frame. The derived model can be applied to various railway system development applications, including stability analysis, instability mitigation, and the study of dynamic behavior. To verify the accuracy of the proposed model, simulation is employed to validate its dynamic responses by comparing them with those obtained from the SimPowerSystem block set in MATLAB. The results confirmed that the derived mathematical model can accurately capture both the transient and steady-state responses. Furthermore, compared to the exact topological model implemented in MATLAB, the proposed model significantly reduces the computational time by 97%.

Keywords: DC heavy railway system; DQ method; Mathematical model derivation; Time-invariant model

Introduction

Thailand's national transportation strategy plan promotes the railway system as the main mode of transportation. Additionally, it supports sustainable development by encouraging the use of renewable energy sources; as a result, the usage of electric trains in public transportation has increased. In long-distance transportation, AC systems are mostly used because they can efficiently transfer electrical power over long distances (UI Hassan *et al.*, 2024);

additionally, their cost is lower than that of DC systems (Cunningham, 2022). In contrast, in urban and suburban transportation, DC power systems are often used (Khodaparastan *et al.*, 2019) because they can accurately control the electrical power, quickly respond to electrical load variations, and rapidly handle the acceleration and deceleration required in urban and suburban transportation, which is characterized by frequent stops.

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Furthermore, DC systems exhibit lower energy losses over short periods than AC systems and achieve high voltage regulation performance (Shin *et al.*, 2023).

Electric railway systems can be classified into three main groups according to the number of passengers per hour per direction (also known as passengers per hour per direction, PPHPD) that the system can accommodate: 1) Light capacity (the maximum capacity is 30,000 PPHPD, 2) medium capacity (30,000-50,000 PPHPD, and 3) heavy capacity (>50,000 PPHPD) (Sharma *et al.*, 2013). The use of electrical railway systems in urban transportation has been continuously increasing to meet the demand for fast, clean, and convenient transportation. Therefore, studying the behavior of electric railway systems is crucial; for this purpose, mathematical models can be used for controller design or stability analysis to improve reliability.

In the literature, it has been reported that the DC heavy railway system consists of three main parts (Pakdeeto *et al.*, 2023): AC power supplied from the Metropolitan Electricity Authority, substations, and trains. The power obtained by the substations is reduced and converted from AC to DC by conventional rectifiers to supply the trains. The power converter employed in such systems produces time-varying functions; thus, the model describing such systems is also time-varying (Iwaszkiewicz *et al.*, 2025). Although this model provides accurate and realistic simulation results of system behavior, the stability analysis or controller design for such a time-varying system requires advanced control theories and complex mathematics (Lee *et al.*, 2026). Thus, this paper presents a way how to derive an averaged model of a DC heavy railway system; the model is time-invariant and sufficiently accurate to describe the dynamic behavior of the system. Based on the proposed averaged model, the fundamental control theory can be readily applied to analyze system stability, eliminating the complexities associated with time-varying signals.

The derivation of the proposed averaged model is based on the DQ transformation method. This method is widely used to transform three-phase electrical quantities into a rotating reference frame called “direct-quadrature (dq-axis);” specifically, the method transforms time-varying signals into steady-state signals.

The accuracy of the proposed model is validated by comparing its responses with the dynamic responses obtained from simulations using the SimPowerSystem block set in MATLAB/Simulink; in this study, this is referred to as an exact topological model. In addition, the proposed model is used to analyze the error spectrum via the fast

Fourier transform (FFT). This analysis, which does not consider the switching frequency of the averaged model, verifies the accuracy of the model and examines its accuracy regarding steady-state and transient responses. The main advantages of the proposed model are the following:

1. System behavior study: the proposed model effectively captures the dynamic behavior of the system; this helps to understand system performance under various operating conditions.

2. Computational time reduction: compared with conventional simulations using the MATLAB/Simulink power system block set, the proposed model significantly reduces the computational time. This advantage enables faster simulation of large-scale systems.

3. Error spectrum analysis: the proposed model reveals details of the significant errors occurring in the high-frequency range. However, these errors do not affect stability; thus, the proposed model can be used for stability analysis.

This paper is organized as follows. The introduction is presented in Section I. In Section II, the proposed system is explained. The model derivation is presented in Section III. In Section IV, the accuracy of the proposed model is validated. Finally, Section V concludes the paper.

Proposed system

The architecture of an existing DC heavy railway system is presented in Figure 1. Such a system is used in Thailand and consists of three main parts: 1) the AC power supplied from the Metropolitan Electricity Authority (MEA), 2) the traction substation, and 3) the electric trains. The proposed system is shown in Figure 2. A 115-kV_{rms} AC voltage from MEA is supplied to the transformer at the substation. The transformers reduce the voltage to ~324 V_{rms}. Then, the reduced voltage is converted from an AC to an ~750-V DC voltage by a diode rectifier (Sumpavakup *et al.*, 2018) to supply the electric trains.

Conventional DC heavy railway systems employ 12-pulse and 24-pulse diode rectifiers (Wang *et al.*, 2022) because of their ability to reduce harmonics and improve the power conversion efficiency (Yang *et al.*, 2017). However, such rectifiers are complex, and the waveforms of voltage and current they produce are difficult to analyze. Conventional rectifiers only provide the best DC response with a low ripple; however, they cannot affect transient and steady-state responses. In this study, a 6-pulse diode rectifier is selected to derive the proposed time-invariant model. The DC voltage output from the diode rectifier is ~750 V and is fed

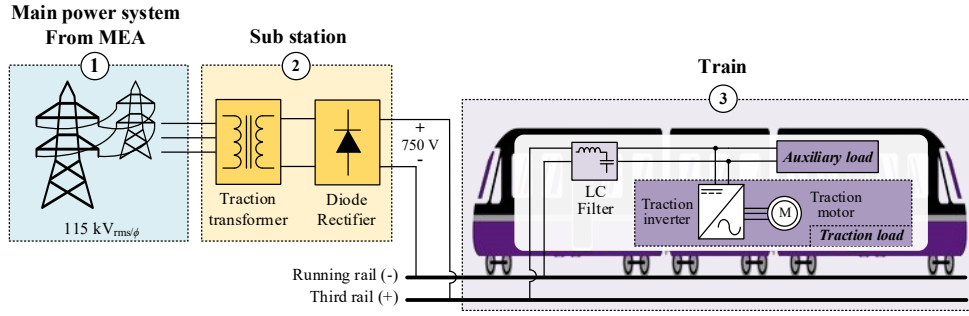


Figure 1. Existing DC heavy railway system

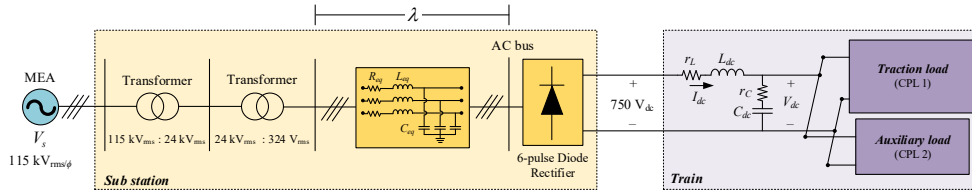


Figure 2. Proposed system

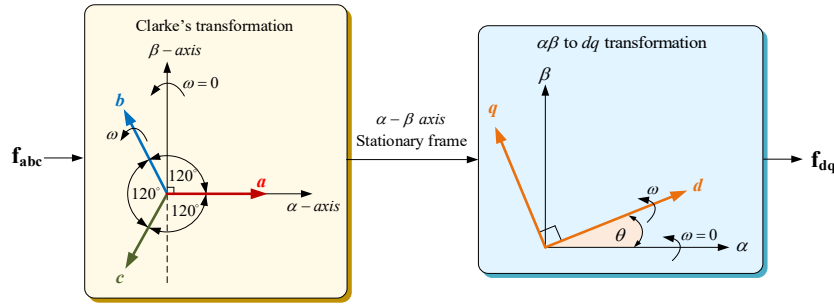


Figure 3. DQ transformation (Park's Transformation)

to the electric train via the third rail (Abd Rahman *et al.*, 2020). In addition, the proposed system employs a DC filter to reduce the voltage and current ripple before supplying the voltage to the train loads. These loads can be classified into two main types (Del Toro *et al.*, 2021): 1) traction load (CPL 1), which is used for motoring the train, and 2) auxiliary load (CPL 2), which supports subsystems such as lighting, air conditioning, and other passenger service components. Both types of loads are controlled converter and behave as constant power loads (CPLs) (Singh *et al.*, 2017); therefore, they are regarded as ideal CPLs for simplicity.

The objective of this study is to derive an averaged model of a DC heavy railway system using the DQ method. This method can be used to eliminate the effect of frequency switching (Leng *et al.*, 2021) in an averaged mathematical model. As a result, the proposed mathematical model is accurate and can be used to effectively analyze

system behavior. Details of the model derivation are presented in the next section.

Mathematical Model Derivation

This section presents the fundamental theory of the DQ transformation, which can be used to derive a mathematical model of three-phase power systems in a time-invariant form.

A) DQ method

The DQ transformation, also known as Park's transformation, is a method used to convert the three-phase electrical components into a DC signal on a rotating reference frame (dq -axis). Figure 3 shows a vector diagram, where a three-phase signal (f_{abc}) is transformed into a DC value (f_{dq}) on both the d and q axes, based on the DQ transformation.

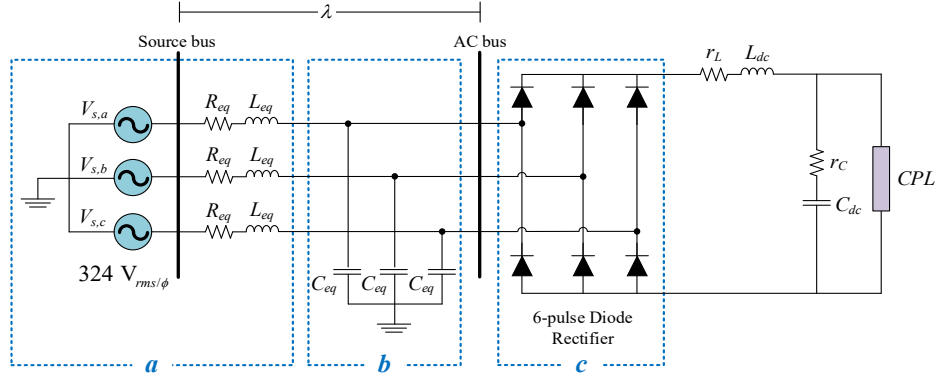


Figure 4. Proposed circuit for the DC heavy railway system

The DQ transformation process involves two main stages: 1) Clarke's transformation, which converts a three-phase signal ($\mathbf{f}_{abc}$) into a stationary reference frame; this is represented by the yellow area shown in Figure 3 and corresponds to a frame with zero rotational speed (0 rad/s). 2) Transformation of the $\alpha\beta$ frame into the dq axes; this is represented by the blue area shown in Figure 3. In this stage, the stationary reference frame is rotated over the reference frame by an angle θ (known as the rotational angle), which synchronizes the reference frame to the same angular frequency as the three-phase system. As a result, the system variables are mapped onto a rotating frame defined by the d and q axes, where the d -axis aligns with the rotating field and the q -axis is orthogonal to the d -axis. This transformation effectively eliminates the time-varying variable of sinusoidal waveforms in three-phase power systems, resulting in a time-invariant model that greatly simplifies system analysis and controller design. The mathematical formulation of this transformation, which is commonly referred to as Park's transformation, is expressed in Equation (1).

$$[\mathbf{f}_{dq0}] = k [\mathbf{T}_{dq0}] [\mathbf{f}_{abc}] \quad (1)$$

where $\mathbf{f}$ is the electrical component matrix, $k = \sqrt{\frac{2}{3}}$

is used for power conserving convention, θ is the rotation angle of the dq -axis, and $\mathbf{T}_{dq0}$ is the matrix transformation

$$[\mathbf{T}_{dq0}] = \begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ -\sin(\theta) & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

B) Derivation of the DQ model

In this study, the source bus voltage was estimated by multiplying the MEA voltage with the transformer's turn ratio. Then the source bus (V_s) will be 324 V_{rms}. The proposed system, which represents the existing heavy rail system, is shown in Figure 4, where the power supply voltage of the heavy rail system can be regarded as an ideal voltage source of 324 V_{rms/phase}. In Figure 4, the equivalent circuit parameters of the power transmission line are R_{eq} , L_{eq} , and C_{eq} . The phase angle between the source and the AC bus is equal to λ . The parameters in the DC filter circuit are r_L , L_{dc} , r_C , and C_{dc} . The train loads are considered ideal CPLs.

In the derivation of the mathematical model of the proposed system (Figure 4), three parts are considered as follows:

Part a: the resistor and the inductor of the transmission line equivalent circuit.

Part b: the capacitor of the transmission line equivalent circuit.

Part c: the diode rectifier circuit.

Then, these parts are converted into the dq -axis to eliminate the time-varying behavior of the system. Details are presented below.

Part a: The resistors connected in series with the inductors (Figure 5(a)) can be analyzed using Equation (2) to determine the voltage drops across both components. Subsequently, these voltage drops are transformed into the dq -axis using Equation (1), which yields Equation (3). The equivalent circuit of the series-connected resistors and inductors on the dq -axis is illustrated in Figure 5(b).

$$\mathbf{V}_{s,abc} - \mathbf{V}_{r,abc} = R_{eq} \mathbf{I}_{abc} + L_{eq} \frac{d}{dt} \mathbf{I}_{abc} \quad (2)$$

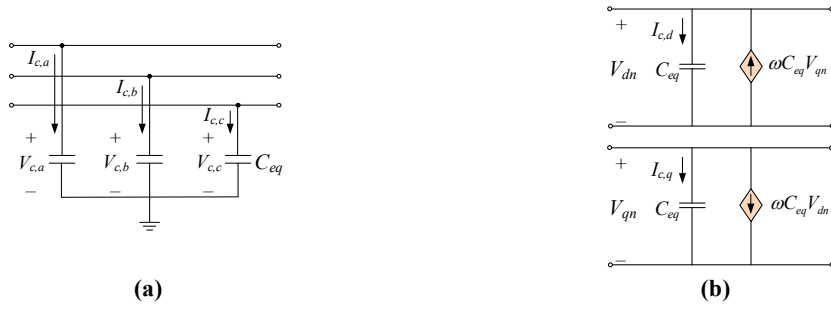
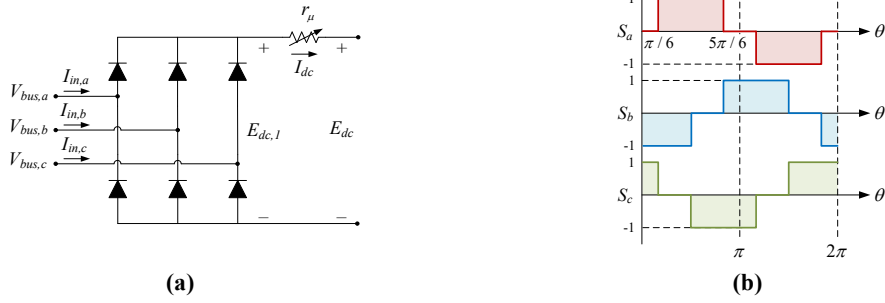
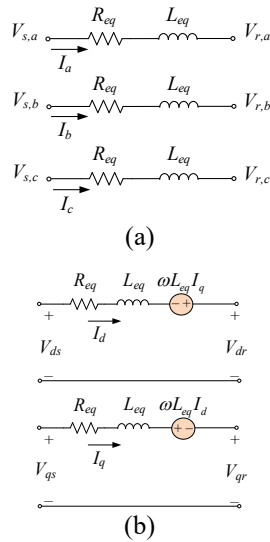

 Figure 6. Equivalent circuits of the capacitors in the (a) *abc* frame and (b) *dq* frame


Figure 7. (a) 6-pulse diode rectifier and (b) switching signal of the rectifier

$$\begin{cases} \Delta V_d = R_{eq} I_d - \omega L_{eq} I_q + L_{eq} \frac{d}{dt} I_d \\ \Delta V_q = R_{eq} I_q + \omega L_{eq} I_d + L_{eq} \frac{d}{dt} I_q \end{cases} \quad (3)$$


 Figure 5. Equivalent circuits of the series-connected resistors and inductors in the (a) *abc* frame and (b) *dq* frame

Part b: The capacitors (Figure 6(a)) can be analyzed using Equation (4) to determine the currents flowing through them. Subsequently, these currents are transformed into the *dq*-axis using

Equation (1), which yields Equation (5). The equivalent circuit of the capacitors on the *dq*-axis is illustrated in Figure 6(b).

$$\mathbf{I}_{c,abc} = C_{eq} \frac{d}{dt} \mathbf{V}_{c,abc} \quad (4)$$

$$\begin{cases} I_{c,d} = -\omega C_{eq} V_q + C_{eq} \frac{d}{dt} V_d \\ I_{c,q} = \omega C_{eq} V_d + C_{eq} \frac{d}{dt} V_q \end{cases} \quad (5)$$

Part c: The commutation effect of L_{eq} in the power transmission lines causes an overlap angle and a DC output voltage drop in the output circuit of the three-phase rectifier circuit. This effect can be represented by a variable resistor r_μ (Figure 7(a)), which can be determined using Equation (6) (Mohan *et al.*, 2003).

$$r_\mu = \frac{3\omega L_{eq}}{\pi} \quad (6)$$

$$\mathbf{S}_{abc} = \frac{2\sqrt{3}}{\pi} \begin{bmatrix} \sin(\omega t + \phi) & \sin(\omega t - \frac{2\pi}{3} + \phi) & \sin(\omega t - \frac{4\pi}{3} + \phi) \end{bmatrix} \quad (7)$$

$$\mathbf{S}_{dq} = \begin{bmatrix} S_d \\ S_q \end{bmatrix} = \sqrt{\frac{3}{2}} \cdot \frac{2\sqrt{3}}{\pi} \begin{bmatrix} \sin(\omega t + \phi - \theta) \\ -\cos(\omega t + \phi - \theta) \end{bmatrix} \quad (8)$$

Figure 7(b) illustrates the switching signal of the diodes shown in the circuit of Figure 7(a). This signal is described in Equation (7) (Areerak *et al.*, 2008) and can be transferred to the dq -axis using Equation (1), which yields Equation (8); ϕ is the phase angle at the AC bus.

In the diode rectifier circuit, $\theta = \omega t - \frac{\pi}{2} + \phi_1$, where ϕ_1 is the phase angle used to rotate the dq -axis. Then, the switching signal on the dq -axis is described in equation (9).

$$S_{dq} = \frac{3\sqrt{2}}{\pi} \begin{bmatrix} \cos(\phi - \phi_1) \\ \sin(\phi - \phi_1) \end{bmatrix} \quad (9)$$

The relationship between voltage and current between the input and output sides of the three-phase bridge rectifier circuit is described in Equations (10) and (11). These equations are subsequently transformed into the dq -axis using Equation (1), which yields Equations (12) and (13), respectively. Based on Equations (12) and (13), the equivalent circuit of a three-phase rectifier on a dq -axis is shown in Figure 8.

$$\mathbf{I}_{in,abc} = \mathbf{S}_{abc} \mathbf{I}_{dc} \quad (10)$$

$$E_{dc,1} = \mathbf{S}_{abc}^T \mathbf{V}_{bus,abc} \quad (11)$$

$$\begin{cases} I_{in,d} = S_d I_{dc} \\ I_{in,q} = S_q I_{dc} \end{cases} \quad (12)$$

$$E_{dc,1} = S_d V_{bus,d} + S_q V_{bus,q} \quad (13)$$

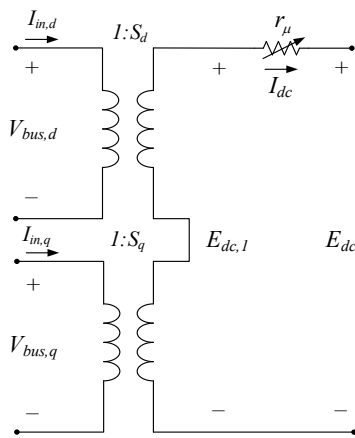


Figure 8. Equivalent circuit of a three-phase rectifier on a dq frame

The balanced three-phase AC power supplied to the system is described in Equation (14). This equation is transformed into the dq -axis using Equation (1). Upon applying the DQ transformation, the voltage sources in the dq -axis can be calculated using Equation (15), where V_m is the peak voltage of the AC source voltage.

$$\begin{bmatrix} V_{s,d} \\ V_{s,b} \\ V_{s,c} \end{bmatrix} = \begin{bmatrix} V_m \sin(\omega t + \lambda + \phi) \\ V_m \sin(\omega t - \frac{2\pi}{3} + \lambda + \phi) \\ V_m \sin(\omega t - \frac{4\pi}{3} + \lambda + \phi) \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} V_{sd} \\ V_{sq} \end{bmatrix} = \sqrt{\frac{3}{2}} \begin{bmatrix} V_m \cos(\lambda + \phi - \phi_1) \\ V_m \sin(\lambda + \phi - \phi_1) \end{bmatrix} \quad (15)$$

The switching signals of the diode and the three-phase AC supply on the dq -axis are described in Equations (9) and (15), respectively; their corresponding vector diagrams are illustrated in Figure 9.

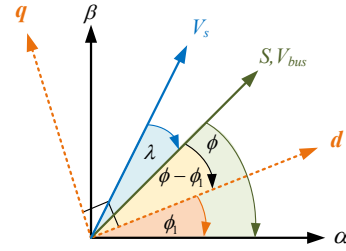


Figure 9. Vector diagram of the DQ transformation

Figure 10 shows the equivalent circuit of the proposed power system after applying the DQ transformation. This circuit can be analyzed using fundamental circuit theory by setting $\phi_1 = \phi$, meaning that the ideal power transformer ratio in the q -axis will be equal to 0. In addition, the dq frame rotates synchronously with the frequency of the AC three-phase system. The transformer ratio and the DC voltage equations are described in Equations (16) and (17), respectively.

$$\begin{bmatrix} S_d \\ S_q \end{bmatrix} = \frac{3\sqrt{2}}{\pi} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (16)$$

$$\begin{bmatrix} V_{sd} \\ V_{sq} \end{bmatrix} = \sqrt{\frac{3}{2}} \begin{bmatrix} V_m \cos(\lambda) \\ V_m \sin(\lambda) \end{bmatrix} \quad (17)$$

Equations (16) and (17) indicate that the transformer ratio on the q -axis (S_q) is zero. Consequently, the equivalent circuit of the power system on the dq -axis (Figure 11) is a simplified version of the circuit shown in Figure 10. The constant power load current is described in Equation (18), where V_{dc} is the voltage across the constant power load. Finally, the mathematical

model of the power system derived by analyzing the simple equivalent circuit using Kirchhoff's voltage law (KVL) and Kirchhoff's current law (KCL), is described in Equation (19).

$$I_{CPL} = \frac{P_{CPL}}{V_{dc}} \quad (18)$$

$$\begin{cases} \dot{I}_{sd} = -\frac{R_{eq}}{L_{eq}} I_{sd} + \omega I_{sq} - \frac{1}{L_{eq}} V_{bus,d} + \frac{1}{L_{eq}} \sqrt{\frac{3}{2}} V_m \cos(\lambda) \\ \dot{I}_{sq} = -\omega I_{sd} - \frac{R_{eq}}{L_{eq}} I_{sq} - \frac{1}{L_{eq}} V_{bus,q} + \frac{1}{L_{eq}} \sqrt{\frac{3}{2}} V_m \sin(\lambda) \\ \dot{V}_{bus,d} = \frac{1}{C_{eq}} I_{sd} + \omega V_{bus,q} - \sqrt{\frac{3}{2}} \frac{2\sqrt{3}}{\pi C_{eq}} I_{dc} \\ \dot{V}_{bus,q} = \frac{1}{C_{eq}} I_{sq} - \omega V_{bus,d} \\ \dot{I}_{dc} = \sqrt{\frac{3}{2}} \frac{2\sqrt{3}}{\pi L_{dc}} V_{bus,d} - \frac{(r_\mu + r_L + r_C)}{L_{dc}} I_{dc} - \frac{1}{L_{dc}} V_{dc} + \frac{r_C}{L_{dc}} \frac{P_{CPL}}{V_{dc}} \\ \dot{V}_{dc} = \frac{1}{C_{dc}} I_{dc} - \frac{1}{C_{dc}} \frac{P_{CPL}}{V_{dc}} \end{cases} \quad (19)$$

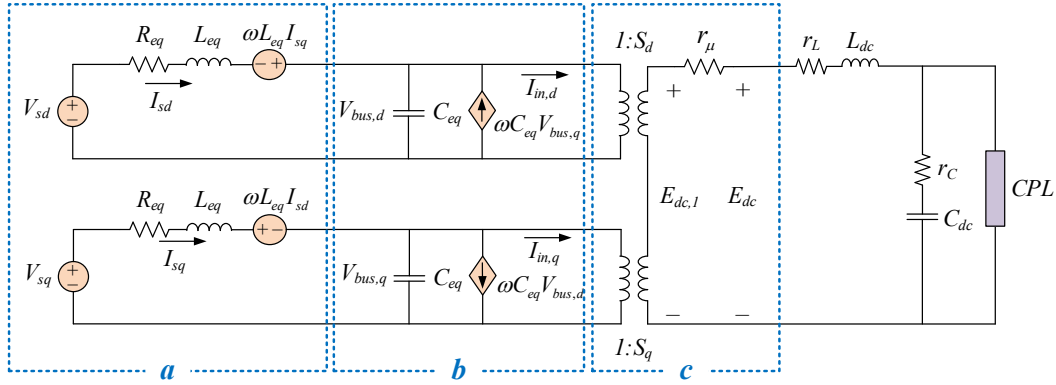


Figure 10. Equivalent circuit of the proposed system on the dq frame

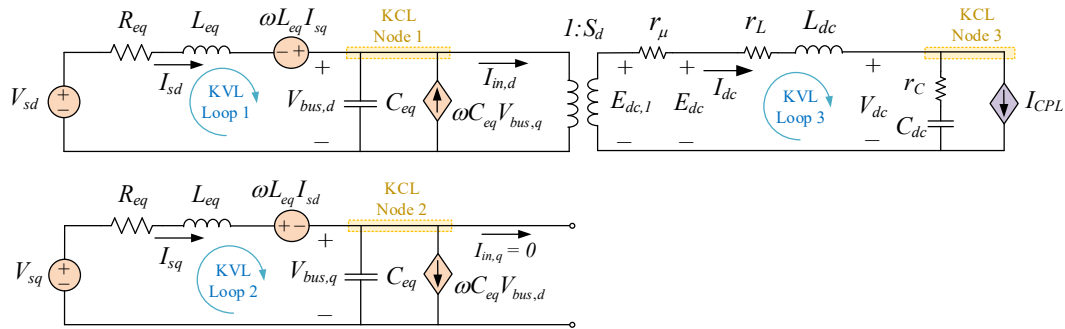


Figure 11. Simplified equivalent circuit of the proposed system

Model validation

In this section, the mathematical model derived using the DQ method is validated by simulating the responses of the proposed system. The selection of the parameter values used in this study was based on engineering design considerations to ensure physically reasonable conditions in the simulation of the DC heavy railway system. These parameters and their values are presented in Table 1. The responses obtained from the proposed averaged model are compared with those obtained from the switching model using the SimPowerSystem block set in Simulink; this is referred to as the “exact topological model.” This comparison indicates the accuracy of the model. Furthermore, the error spectrum calculated using the FFT was employed to

examine the accuracy of the model regarding dynamic and switching frequencies.

Figures 12 and 13 show the validation results of the mathematical model considering two scenarios: 1) power of CPLs (P_{cpl}) in the 0.912–0.962 MW range at 1.5 s; 2) P_{cpl} in the 0.962–1.012 MW range at 2.5 s. The simulation results verify that the mathematical model derived using the DQ method is accurate in both transient and steady-state responses. Additionally, the proposed method eliminates the effect of the switching frequency. Therefore, the system responses obtained from the proposed model represent average values of the actual system, capturing transient dynamics that closely approximate those of the exact topological model.

Table 1. The system parameter

Parameters	Values	Descriptions
V_s	324 V _{rms}	Voltage source at the output of the transformers obtained from MEA
ω	$2\pi \times 50$ rad/s	System angular frequency
R_{eq}	0.005 Ω	Resistance of the transmission line equivalent circuit
L_{eq}	40 μ H	Inductance of the transmission line equivalent circuit
C_{eq}	22 nF	Capacitance of the transmission line equivalent circuit
r_L	0.001 Ω	Equivalent series resistance (ESR) of the inductor used in the filter circuit
r_C	0.001 Ω	Equivalent series resistance (ESR) of the capacitor used in the filter circuit
L_{dc}	0.4 mH	Inductance of the filter circuit
C_{dc}	47 mF	Capacitance of the filter circuit

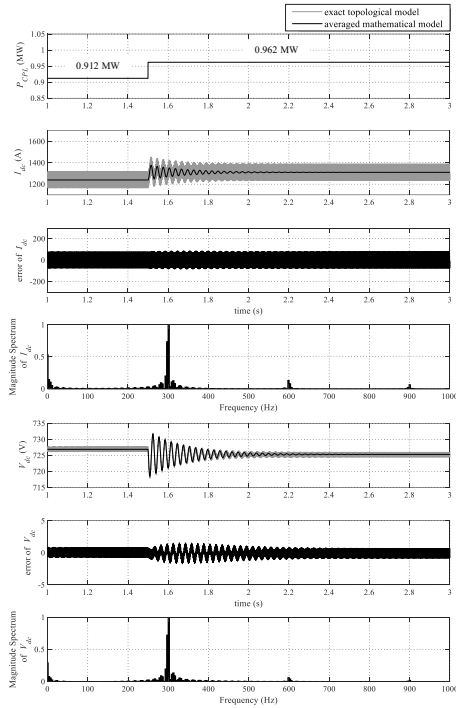


Figure 12. Mathematical model validation for P_{cpl} in the 0.912–0.962 MW range

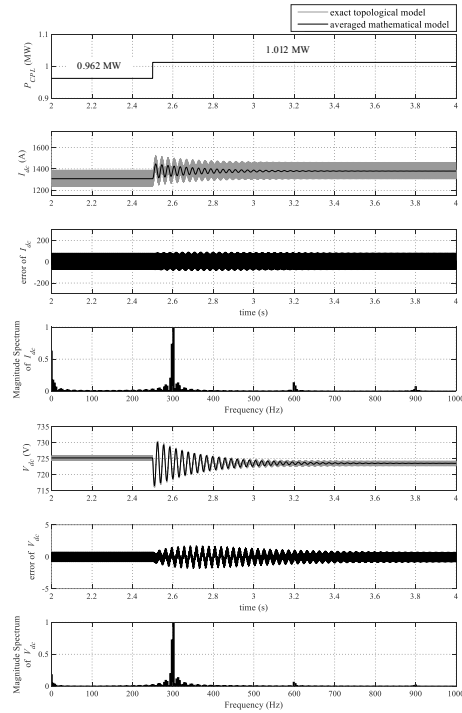


Figure 13. Mathematical model validation for P_{cpl} in the 0.962–1.012 MW range

The discrepancies between the averaged model and the exact simulation results can be calculated to indicate the effect of the switching frequency. When the error spectrum of I_{dc} and V_{dc} is considered, a maximum error of ~ 1 p.u. occurs at 300 Hz, which corresponds to the rectifier switching frequency; the error at lower frequencies remains relatively small. However, at zero frequency, the error of I_{dc} is ~ 0.5 p.u.; this is attributed to the offset DC components between the averaged model and the SimPowerSystems® simulation model. In contrast, V_{dc} remains accurate because the capacitor effectively suppresses the low-frequency errors. Overall, the proposed averaged model accurately represents the low-frequency behavior of the system; therefore, it is valid and reliable in control and system analysis applications because the significant error observed at the switching frequency does not coincide with the system resonance or the dominant operating frequencies calculated using $f_{resonance} = 1/2\pi\sqrt{L_{dc}C_{dc}}$.

The averaged model produces sufficiently accurate results, particularly in the analysis of system behavior in the low-frequency region. In addition to its accuracy, the model is computationally efficient, which is critical in modern power electronics applications, where rapid processing is required (Allmeling *et al.*, 2017; Li *et al.*, 2020); this is because the model solves the governing equations directly in the MATLAB command window. In comparison, the exact topological model is highly reliable but computationally demanding.

The reduction in the computation time can be quantitatively evaluated using Equation (20), where t_{exact} denotes the execution time of the exact topological model simulated in Simulink with a PowerGUI sampling time of 1 μ s, and t_{ave} denotes the execution time of the proposed averaged model solved directly in the MATLAB command window using an ordinary differential equation (ODE) solver with the same sampling time. A comparison between t_{exact} and t_{ave} clearly demonstrates the computational advantage of the proposed approach, verifying its suitability to engineering analysis and design, where both accuracy and efficiency are essential.

$$\%t_{saving} = \frac{t_{exact} - t_{ave}}{t_{exact}} \times 100\% \quad (20)$$

During model validation, all simulations were performed on a personal computer equipped with an Intel® Core™ i7-12700H CPU @2.30 GHz, 32 GB

of RAM, and a 64-bit operating system running Windows 11. t_{ave} and t_{exact} were calculated using the *tic-toc* command in MATLAB to estimate the elapsed time. The results, which are summarized in Table 2, indicate that the averaged model achieves a reduction of $\sim 97\%$ in the simulation time compared with the exact topological model. It should be noted that the actual percentage reduction may vary, depending on the complexity and characteristics of the simulated system.

Table 2. Computational time reduction

Computational time	Load variation	
	scenario#1	scenario#2
t_{exact}	58.1544 s.	30.1028 s.
t_{ave}	1.7054 s.	1.0188 s.
% saving time (% t_{saving})	97.0674%	96.6155 %

Table 2 shows that the averaged model consistently achieves a substantial decrease in the computational time under both load variation scenarios, demonstrating its efficiency. Additionally, it is a practical tool for engineers and researchers, enabling fast controller design and system response evaluation.

Conclusions

This paper presents the derivation of an averaged model of a DC heavy rail system. The model is based on the DQ transformation method, which converts three-phase electrical quantities into a rotating reference frame. This transformation eliminates the time-varying variables, yielding a time-invariant mathematical model. The proposed model was validated by comparing its response with that of the exact topological model using the SimPowerSystems blockset in MATLAB/Simulink. The results verified that the proposed averaged model can accurately capture both the transient and steady-state responses. Additionally, a spectrum error analysis indicated that the discrepancy between the proposed and the exact topological model is related to the switching frequency of the rectifier diode circuit; the system stability and the steady-state response were not affected. Therefore, the proposed averaged model is highly suitable for engineering applications such as control system design and power system stability analysis. Furthermore, the model achieves a substantial reduction in the simulation time, demonstrating its computational efficiency and enabling rapid design and evaluation processes.

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