

STOCHASTIC THERMAL INSTABILITY ANALYSIS OF ELASTICALLY SUPPORTED LAMINATED COMPOSITE PLATE EMBEDDED WITH SMA FIBERS

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Abstract

This study investigates the second-order statistics of shape memory alloy fibres embedded in elastically supported laminated composite plates subjected to thermal loading. The two-parameter Pasternak foundation model is assumed for the elastically supported model. The parametric thermal instability equation is derived from the Mathieu-Hill equation, employing Bolotin's approach in conjunction with the C^0 finite element method. The stochastic finite element method (SFEM)-based second-order perturbation technique (SOPT) is used to calculate the expected mean and standard deviation (SD) of the input random system parameters. This research employs a mathematical model based on higher-order shear deformation theory (HSDT) to determine the excitation frequency and instability regions. The role of stochastic system parameters is analysed in the excitation frequency and instability regions, as they relate to the volume fraction of SMA fibres, plate length-to-thickness ratio, quantity and placement of SMA fibres, frequency mode, temperature increments, percentage recovery strain, and foundation parameters concerning expected mean and standard deviation.

Keywords: Excitation frequency; Thermal instability; SMA fibres; Stochastic FEM; HSDT; SOPT

Introduction

Studying how thermal instability affects laminated composite plates embedded with shape-memory alloy fibres (LCPESMA) is essential. High-temperature applications widely employ shape memory alloy (SMA) fibres. This is primarily due to their peculiar behaviours, such as pseudo elasticity and shape memory, which are advantageous, especially at elevated temperatures. Using conventional composites embedded with SMA fibres significantly enhances the compressive

thermal buckling resistance subjected to elevated temperatures. Nuclear engineering, civil infrastructure, automobiles, and aerospace industries, among others, are widely adopting these novel types of multi-phase composites with appropriate bonding properties.

These structural elements are subjected to alternating mechanical and thermal out-of-plane and in-plane loads, which can cause dynamic instability and parametric resonance. Thoroughly

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assessing unstable behaviour and regions is crucial for achieving optimal high performance and ensuring the safe design of these components. The LCPESMA comprises two parameters: Pasternak foundation with Winkler non-linearity. It has been proven to be one of the most appropriate models for reliable design, as it controls both transverse and lateral deformations. The structures supported by such elastic foundations are widely used for numerous engineering applications.

Multi-phase structures composed of composite materials supported by elastic foundations exhibit significant variability in materials, geometrical parameters, external mechanical and/or thermal loading, and foundation parameters. These inherent variabilities lead to performance deviations and impact the safety and design. Hence, it is crucial to evaluate the impact of these input random system parameters, both individually and simultaneously, on performance through a stochastic approach to ensure optimal safety and reliability. Researchers have explored LCPESMA's thermal buckling behaviour through various deterministic analytical and numerical techniques. (Park *et al.*, 2004) combined von Karman non-linearity and first-order shear deformation theory (FSDT) to analyse the thermally-induced vibration response of LCPESMA.

Kumar and Singh, (2009) employed the 3D layer-wise displacement model with the C^0 -based finite element method (FEM) to study the thermal buckling behaviour of LCPESMA. (Parhi and Singh, 2015) performed a free vibration analysis of LCPESMA using a nonlinear FEM with the HSDT and von Kármán strain-displacement field.

Chen and Yang, (1990) explored the dynamic stability of anti-symmetric angle-ply rectangular plates under pure pulsating bending and uniform pulsating compressive loads.

Moorthy *et al.*, (1990) adopted the FSDT to study the parametric instability of laminated composite plates (LCP) under harmonically increasing in-plane stresses, emphasising non-negligible transverse shear deformations. Kwon, (1991) used mixed FEM and high-order theory to examine the dynamic instability of LCP. Under several stationary and dynamic in-plane loads, Sahu and Datta, (2000) studied the stability of isotropic, quasi-isotropic, cross-ply, and angle-ply plates.

Employing a plate model derived from the updated HSDT (Chakrabarti & Sheikh, 2006), the influence of in-plane edge pressure on the dynamic instability of laminated sandwich plates was assessed. (Chakrabarti and Sheikh, 2010) introduced a methodology for investigating the dynamic instability of imperfect LCPs. They

applied in-plane partial edge loads using a FEM model based on HSDT. Pradyumna and Bandyopadhyay, (2010) applied HSDT to investigate Bolotin's dynamic instability theory in functionally graded shells. (Ramachandra & Panda, 2012) determined the dynamic instability zones and critical buckling loads for composite and isotropic plates using HSDT. Sahoo and Singh, (2015) combined Bolotin's method with a new inverse hyperbolic zigzag theory to identify primary areas of instability in sandwich and LCP exposed to periodic compressive and in-plane static stresses. Mohanty *et al.*, (2015) examined the parametric instability in delaminated composite plates subjected to periodic in-plane loads via FEM with the help of the FSDT theory. Kumar *et al.*, (2016) employed the HSDT approach to investigate the linear and nonlinear dynamic instability in a plate composed of functionally graded materials.

Sahoo and Singh, (2018) employed an innovative zigzag plate model to assess the dynamic stability of sandwich and LCP using HSDT combined with FEM based on C^0 continuity. Singh *et al.*, (2022) studied the HSDT and von Karman nonlinearity to explore linear and nonlinear dynamic instability in a hybrid composite material with randomly dispersed CNTs. Adhikari and Singh, (2020) estimated laminated composite plate structure dynamic instability under five non-uniform harmonic edge compressive loading based on HSDT with C^0 FEM. Kumar, and Patel, (2021) examined nonlinear vibration and dynamic instability using von Karman nonlinearity and HSDT. They investigated composite plates reinforced with carbon nanotube fibres under non-uniform in-plane load. (Singh *et al.*, 2021) provided a semi-analytical formulation for the nonlinear vibration and instability of CNT-reinforced composite plates using the HSDT and Galerkin approach.

Structural performance evaluations have been conducted using stochastic analysis techniques, encompassing both analytical and sampling methods. The second-order perturbation technique (SOPT) accelerates computations for mean and standard deviation as second-order statistics by including low-order variability in the differential equations. Aside from the other side, various sampling approaches are used, in which MCS is most widely employed due to its simplicity and greater accuracy for higher-order variability. Zhou *et al.*, (2023) proposed a perturbation technique and Particle Swarm Optimisation (PSO) algorithm-based multiscale uncertainty propagation method of dynamic analyses of fibre-reinforced polymer (FRP) structures. Shegokar and Lal, (2016) examined the instability of randomly characterised

functionally graded beams' mean and standard deviation behaviours. Yusof *et al.*, (2020) applied HSDT-based FEM to LCP that included SMA to examine parametric instability. LCPESMA parametric instability literature is scarce compared to hybrid LCP literature.

Lal *et al.*, (2015) studied the uncertain geometrical and material parameters of elastically supported piezoelectric LCP using von Karman non-linearity with HSDT coupled with SOPT. (Verma and Singh, 2009) utilised C^0 FEM and FOPT concurrently to examine the stochastic thermal buckling of LCP. Utilising polynomial chaos (PC), Lal *et al.* (2024) studied the mean and coefficient of variance (CoV) of thermal buckling temperature for SMA fibre-reinforced LCP. Limited literature is available on the thermal instability of elastically supported LCPSMA to find the expected mean and SD of excitation frequency concerning various input random system parameters. A significant gap in the current body of literature is the incorporation of stochastic-based thermally induced instability analysis to quantify various uncertain parameters, thereby enhancing reliability and safety. There is a valuable opportunity to incorporate SOPT and stochastic FEM (SFEM) using HSDT for dynamic thermal instability analysis.

The study aims to determine the instability region in advanced multi-phase materials by analysing the statistics of excitation frequencies, expected mean, and standard deviation. It is essential to understand how input random system parameters, acting simultaneously or individually, affect the temperature-induced instability behaviour of these multi-phase composite systems. In the proposed research, various critical factors, including SMA fibres volume fraction, plate length-to-thickness ratio, positions and number of SMA layers, frequency mode, dynamic factors, temperature increments, and percentage recovery strain with different uncertain system parameters, are expected to significantly influence the thermal buckling temperature, instability regions and excitation frequency of ESLCPESMA.

Mathematical formulation

Geometric configuration and mechanical properties

Figure 1 depicts an ESLCPESMA that typically involves layers of conventional epoxy matrix and SMA fibres arranged in a specific configuration to optimise desired properties. The plate has dimensions: width ' w ,' thickness ' h ,' and length ' l .' It comprises ' n ' orthotropic laminae, each

of which is of thickness ' h_l ,' and making an angle ' θ ' about the X -axis. With uniform temperature variations, there is in-plane compressive thermal loading, ' N^T ,' in each X and Y direction. ' U ,' ' V ,' and ' W ' denote the displacements along the X , Y , and Z axes, respectively. The laminated plate, SMA fibre, and elastic foundations are considered to be perfectly bonded. The LCPESMA is characterised by a Pasternak elastic foundation having linear foundation stiffness, represented by K_1 and K_2 , as shown in Figure 1.

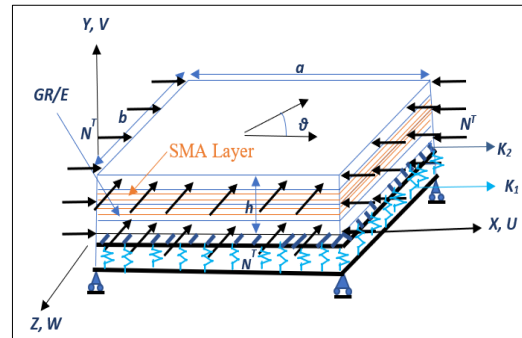


Figure 1. Geometry of ESLCPESMA

The foundation reaction per unit area (p) exerted by supporting two parameters of the Pasternak elastic foundation can be represented as (Lal *et al.*, 2015)

$$p = K_1 w - K_2 (w_{,x^2} + w_{,y^2}) \quad (1)$$

where w , K_1 , and K_2 are the transverse displacement of LCPESMA, Winkler (spring), and Pasternak (shear) stiffness of foundations, respectively. The SMA fibres are embedded within a graphite epoxy matrix to form the LCPESMA. The equivalent thermomechanical properties of LCPESMA are derived using a micromechanical approach (Lal *et al.*, 2015). The equivalent thermomechanical properties of LCPESMA are expressed as

$$E_1 = RE_{1m}V_m + E_sV_s \quad (2)$$

$$E_2 = (1 - \sqrt{V_s})RE_{2m} + \frac{RE_{2m}\sqrt{V_s}}{(1 - \sqrt{V_s}(1 - \frac{RE_{2m}}{E_s}))} \quad (3)$$

$$G_{12} = (1 - \sqrt{V_s})RG_{12m} + \frac{RG_{12m}\sqrt{V_s}}{(1 - \sqrt{V_s}(1 - \frac{RG_{12m}}{G_s}))} \quad (4)$$

$$\nu_{12} = RV_m\nu_m + V_s\nu_s \quad (5)$$

$$\rho = RV_m\rho_m + V_s\rho_s \quad (6)$$

$$\alpha_1 = \frac{RE_{1m}\alpha_{1m}V_m + E_s\alpha_sV_s}{E_1} \quad (7)$$

$$\alpha_2 = R\alpha_{2m}V_m + \alpha_sV_s \quad (8)$$

Subscript 's' is used for SMA fibres and 'm' for matrix and conventional fibres. E_s , ν_s , G_s , ρ_s , V_s , and α_s are the elastic modulus, Poisson's ratio, shear modulus, density, fibre volume fraction, and thermal expansion coefficient of SMA fibres, respectively. Further, E_{1m} , E_{2m} , ν_{12m} , G_{12m} , ρ_m , $V_m (=1-V_s)$, and α_{1m} are the elastic modulus in the longitudinal and transverse direction, Poisson's ratio, shear modulus, density, volume fraction, and thermal expansion coefficient of graphite epoxy composite matrix, respectively. The parameter 'R' is known as the thermal characteristics retention ratio, which is expressed as

$$R = \sqrt{(T_w - T)/(T_d - T_0)} \quad (9)$$

where T denotes the temperature at which the attributes of an ESLCPESMA are to be evaluated, T_d and T_w represent the glass transition temperatures for dry and wet situations, respectively, and T_0 signifies the reference environmental temperature (21°C). In the present case, it is assumed that $T_w = T_d$ and $T_d = 100^\circ\text{C}$.

Displacement field model

The displacement fields of the amended HSDT, considering C^0 continuity with seven degrees of freedom (DOFs) in the X, Y, and Z directions for an LCPESMA, are presented as per Verma and Singh, (2009).

$$\begin{aligned} U(x, y, z) &= u(x, y) + q_1(z)\beta_x(x, y) + q_2(z)\gamma_x(x, y); \\ V(x, y, z) &= v(x, y) + q_1(z)\beta_y(x, y) + q_2(z)\gamma_y(x, y); \\ W(x, y, z) &= w(x, y); \end{aligned} \quad (10)$$

The initial term in each equation represents the axial displacements that occur at the mid-plane. At the same time, it β_x, β_y denotes rotation perpendicular to the middle plane and occurs around the y-axis and the x-axis. The $\gamma_x = w_{,x}$, $\gamma_y = w_{,y}$ represents the plate's slope in the x and y axes, respectively. The functions $q_1(z)$ and $q_2(z)$ are as:

$$q_1(z) = \kappa_1 z - \kappa_2 z^3, q_2(z) = -\kappa_2 z^3, \quad (11)$$

where $\kappa_1 = 1$, $\kappa_2 = \frac{4}{3}h^2$

The modified displacement field vector based on HSDT with seven degrees of freedom can be written as

$$\{q\} = \{u \quad v \quad w \quad \gamma_y \quad \gamma_x \quad \beta_y \quad \beta_x\}^T \quad (12)$$

Stress-strain interaction

The constitutive stress-strain relationship for ESLCPESMA under plane stress conditions is given by Kumar and Singh, (2009)

$$\{\sigma_e\}_k = [Q_e]_k \{\varepsilon\} + \{\sigma_r\}_k V_{sk} - [Q_m]_k \{\alpha_m\}_k V_m \Delta T \quad (13)$$

where, $\{\alpha_m\}$, $\{\sigma_e\}$, and $\{\sigma_r\}$ represent the coefficient of thermal expansion of the matrix, the resultant stress developed in the plate, and the recovery stress vector of SMA fibres of Nitinol induced by a change in temperature (ΔT) due to the change of phase from austenite to martensite. The material properties and temperature-sensitive recovery stress of SMA fibres are provided in Kumar and Singh, (2009). The compact elasticity matrix of the SMA fibres inserted lamina and matrix are denoted by $[Q_e]$ and $[Q_m]$, respectively. ΔT denotes the temperature change ($T - T_0$), with T representing the constant difference and T_0 the surrounding environment temperature (assumed as 21°C). In this analysis, a uniform temperature distribution is assumed.

The plate's strain energy

The strain energy can be stated as

$$U = \frac{1}{2} \int_A \{\sigma\}^T \{\varepsilon\} dA. \quad (14)$$

The strain energy associated with the ESLCPESMA can be defined using the following equation: (13) in equation (14).

$$U_p = 0.5 \int_A \{\bar{\varepsilon}_i\}^T [D] \{\bar{\varepsilon}_i\} dA \quad (15)$$

Here, $[D]$ stands for the elastic stiffness matrices.

The elastic foundation's strain energy

For an elastic foundation with a shear deformable layer, the strain energy is given by Lal *et al.*, (2015)

$$U_f = 0.5 \int_A \{p\} \{w\}^T dA \quad (16)$$

Substituting equation (1) into equation (16), it may be expressed as:

$$U_f = 0.5 \int_A \begin{Bmatrix} w \\ w_{,x} \\ w_{,y} \end{Bmatrix}^T \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_2 \end{bmatrix} \begin{Bmatrix} w \\ w_{,x} \\ w_{,y} \end{Bmatrix} dA \quad (17)$$

Potential energy because of thermal stresses

The potential energy (V) resulting from the external work carried by the thermal load is expressed as

$$V = -\delta W = 0.5 \int_A [N_{Tx} w_{,x}^2 + N_{Ty} w_{,y}^2 + N_{Txy} w_{,x} w_{,y}] dA \quad (18)$$

Here, N_{Tx} , N_{Ty} , and N_{Txy} are in-plane thermal buckling loads per unit length acting on an ESLCPESMA in the x, y, and z directions, respectively.

Vibrating plate kinetic energy

The vibrating plate's kinetic energy is stated as:

$$T = 0.5 \int_A \rho^{(k)} \{\dot{u}_T\}^T \{\dot{u}_T\} dA \quad (19)$$

where ρ and $\{\dot{u}\} = \{\dot{U} \quad \dot{V} \quad \dot{W}\}^T$ describe the LCPESMA plate's density and velocity vector components, respectively.

External work done because of mechanical loading

When the external mechanical load q_m acts, the potential energy of that work is calculated by

$$\Pi_3 = -W_q = 0.5 \int_A \{q_m\} \{w\}^T dA \quad (20)$$

Finite element model

This study employs a 2-D iso-parametric element comprising nine nodes and seven degrees of freedom per node, subject to C^0 continuity. For the plate element, the displacement field of the j^{th} node will be (Reddy, 2005)

$$\begin{aligned} \{q\} &= \sum_{j=1}^N [N_j] \{q_j\}; \text{ and} \\ \{y\} &= \sum_{j=1}^N [N_j] \{y_j\}; \quad \{x\} = \sum_{j=1}^N [N_j] \{x_j\} \end{aligned} \quad (21)$$

where $\{x_j\}$, $\{y_j\}$, N_j , and $\{q_j\}$ are the coordinates, shape functions, and nodal DoF, respectively. The plate element has N nodes.

The plate element's strain energy

The global strain energy is determined by combining the energies of all the elements. Equation (15), with the finite element model of Equation (21), can be further expressed as

$$U_p = \sum_{e=1}^{NE} U_p^{(e)} = \frac{1}{2} \{q\}^T [K_l] \{q\} \quad (22)$$

NE refers to the number of elements.

with $[K_l] = [K_b] + [K_s]$

where $[K_s]$, $[K_b]$, $[K_l]$, and $\{q\}$ are the shear stiffness matrix, bending stiffness matrix, global stiffness matrix, and global displacement vector, respectively.

Analysis of elastically supported foundation

The sum of the finite element model in equation (21) for each element in equation (17) can be used to determine the strain energy by the supported elastic foundation, as under

$$U_f = \sum_{e=1}^{NE} U_f^{(e)} = 0.5 \{q\} [K_{fl}] \{q\} \quad (23)$$

where $[K_{fl}]$ is defined as the linear foundation stiffness matrix.

Thermal buckling load of plate element

After adding up all the elements, the potential energy resulting from the thermal load of LCPESMA, equation (18), may also be expressed using the finite element model, equation (21), as

$$V = \sum_{e=1}^{NE} V^{(e)} = 0.5 \lambda \{q\}^T [K_g] \{q\} \quad (24)$$

Correspondingly, $[K_g]$ and λ represent the global geometric stiffness matrix and the thermal buckling load.

The geometric stiffness matrix for the element is expressed as

$$K_g^{(e)} = 0.5 \int_A B_g^{(e)T} [N_o] B_g^{(e)} dA \quad (25)$$

where $[N_o] = \begin{bmatrix} N_{Tx} & N_{Txy} \\ N_{Txy} & N_{Ty} \end{bmatrix}$ with

$$\begin{aligned} \begin{bmatrix} N_{Tx} \\ N_{Ty} \\ N_{Txy} \end{bmatrix} &= \begin{bmatrix} N_{Tsx} \\ N_{Tsy} \\ N_{Tsx} \end{bmatrix} - \begin{bmatrix} N_{Tmx} \\ N_{Tmy} \\ N_{Tmxy} \end{bmatrix} \text{ here} \\ \begin{bmatrix} N_{Tmx} \\ N_{Tmy} \\ N_{Tmxy} \end{bmatrix} &= \sum_{k=1}^{NE} \int_{-h/2}^{h/2} [Q_k^m] \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k \Delta T V_m dz; \text{ and} \\ \begin{bmatrix} N_{Tsx} \\ N_{Tsy} \\ N_{Tsx} \end{bmatrix} &= \sum_{k=1}^{NE} \int_{-h/2}^{h/2} Q_k^s \begin{Bmatrix} V_s \\ V_s \\ V_s \end{Bmatrix}_k dz \end{aligned} \quad (26)$$

where and are the reduced elasticity matrix of conventional graphite epoxy matrix materials and recovery stress matrix, respectively.

The kinetic energy of the plate element

Likewise, the vibrating LCPESMA's kinetic energy can be stated through equations (21) and (19) as follows:

$$T = \sum_{e=1}^{NE} T^{(e)} = \{\dot{q}\}^T [M] \{\dot{q}\} \quad (27)$$

whereas the global consistent mass matrix is specified by $[M]$.

Potential energy due to external work done

Equation (20) can be formulated through the finite element model Equation (21), as

$$V_w = \sum_{e=1}^{NE} V_w^{(e)} = -\{q\}^T \{P_M\} \quad (28)$$

where,

$$\{P_M\} = (0 \quad 0 \quad q_m \quad 0 \quad 0 \quad 0 \quad 0)^T \quad (29)$$

The mass matrix, stiffness matrices, and load vectors can be calculated by altering the expressions from the local (x, y) to the natural (ξ, η) coordinate system using Gauss quadrature integration.

Governing equation of motion

Using the Hamilton principle, the excitation frequency governing equation for ESLCPESMA can be derived. In conservative systems, the Lagrange equation takes the form of Adhikari and Singh, (2020).

$$\frac{\partial}{\partial t} \left(\frac{\partial T}{\partial \dot{q}} \right) + \frac{\partial (U_p - V - V_w)}{\partial q} + \frac{\partial (U_f)}{\partial q} = 0 \quad (30)$$

Substituting equations. (22, 23, 24 and 27, 28) in Equation (30) and modifying, gives

$$[M]\{\ddot{q}\} + [K]\{q\} = \{F\} \quad (31)$$

where $\{F\} = \{P_M\}$

Equation (31) represents the thermally induced force vibration equation, assuming the ESLCPESMA vibrates in its principal mode during each iteration. Equation (31) becomes a thermally free vibration problem after assuming $\{F\}$ to be zero, which is a standard eigenvalue problem that can be solved. In the deterministic approach, the mean values of input system parameters are used, which yields only the mean response and does not account for the effects of random system properties. The deterministic methods cannot be used to solve Equation (31) in a random investigation. In the deterministic approach, the average input parameters are used, discarding the impact of random input properties on the response and limiting the analysis to the mean output on statistics like the expected mean, standard deviation, CoV, and safety factor. The second-order statistics of

excitation natural frequency and instability response are determined using the SFEM, which combines the finite element method with the mean-centred SOPT.

Parametric instability analysis

The parametric thermal instability equation and regions can be derived by considering dynamic instability regions using the Mathieu-Hill equation through Bolotin's approach, as expressed by Yusof *et al.*, (2020).

$$[[K] - (\alpha \pm 0.5\beta)N_{cr}[K_g] - 0.25\omega^2[M]]\{q\} = 0 \quad (32)$$

where α , β , and ω are static stability factors, dynamic stability factors, and excitation natural frequency, respectively. β represents the conditions on either side of the mean positions of the instability regions.

Eigenvalues identify the boundary frequencies of instability regions (disturbance or resonance frequencies). The instability zones of the ESLCPESMA subjected to the periodic axial thermal loads are displayed by fluctuations in the β - ω plane. The parametric instability analysis of ESLCPESMA's flow chart is illustrated in Figure 2.

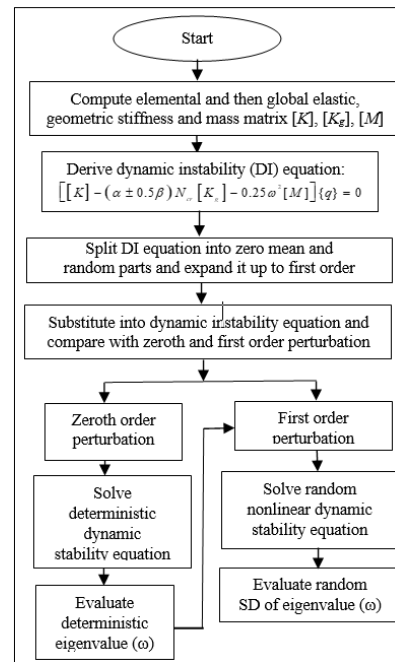


Figure 2. The flow chart of the parametric instability analysis

Solution technique: stochastic analysis

In contrast to the deterministic approach, where all random system parameters are assumed to be fixed from their mean values, stochastic analysis provides a more realistic assessment of structural performance with inherent random system input parameters. To achieve higher reliability and safety in structural designs, it is crucial to consider the impact of random input variables on structural performance. The perturbation methods are computationally more efficient than sampling-based approaches such as MCS and polynomial chaos (PC) for minor random variations. As described, the random governing dynamic instability equation (Equation 32) can be solved by a stochastic approach using the mean-centred perturbation second-order perturbation technique. This approach is beneficial when dealing with random variations in the dynamic instability factor. Characterisation frequency during iterations and time do not significantly affect the stiffness matrix (Shegokar & Lal, 2016). A systematic solution procedure for stochastic dynamic instability analysis using the SFEM, which quantifies random variability in system parameters, is illustrated in Figure 2.

Second-order perturbation method

The operational input random system variables in Equation (32) can be roughly represented by Taylor series expansions around their mean values up to the second order. The excitation frequency (ω), consisting of various input random variables (b_i) can be expressed in terms of a general excitation frequency function (ω), which is described as

$$\omega = \omega(b_1, b_2, \dots, b_i) \quad (33)$$

The Taylor series expansion of a general function $\omega(b_1, b_2, \dots, b_i)$ in terms of its mean input random variables ($\mu_{b_1}, \mu_{b_2}, \dots, \mu_{b_i}$) is expressed under

$$\omega(b_1, b_2, \dots, b_i) = \omega(\mu_{b_1}, \mu_{b_2}, \dots, \mu_{b_i}) + \sum_{i=1}^n (b_i - \mu_{b_i}) \omega_{,b_i} + 0.5 \sum_{i=1}^n \sum_{j=1}^n (b_i - \mu_{b_i}) (b_j - \mu_{b_j}) \omega_{,b_i b_j}^2 \quad (34)$$

The mean values of the b_i 's are used to determine the derivatives. The linear terms can truncate the series to determine the first-order estimated mean of ω or $E(\omega) \approx \omega(\mu_{b_1}, \mu_{b_2}, \dots, \mu_{b_i})$. The variance of the first order of the function, ω can be denoted as $\text{Var}(\omega')$.

$$\text{Var}(\omega') \approx \sum_{i=1}^n \sum_{j=1}^n E_i E_j \text{Cov}(b_i, b_j) \quad (35)$$

The two partial derivatives of the response with respect to input random variables $\frac{\partial \omega}{\partial b_i}$ and $\frac{\partial \omega}{\partial b_j}$ are denoted by E_i and E_j , respectively, and are assessed using the mean values of the b_i 's. For uncorrelated random variables, equation (35) can be further written as

$$\text{Var}(\omega') \approx \sum_{i=1}^n \sum_{j=1}^n E_i^2 \text{CoV}(b_i, b_j) = \sum_{i=1}^n \sum_{j=1}^n \omega_{,b_i} \text{CoV}(b_i, b_j) \omega_{,b_j} \quad (36)$$

The amplification factor E_i corresponds to the uncertainties in each random variable b_i . It is possible to describe further the second-order mean, ω denoted $E(\omega'')$, as

$$E(\omega'') \approx \omega(\mu_{b_1}, \mu_{b_2}, \dots, \mu_{b_i}) + 0.5 \sum_{i=1}^n \sum_{j=1}^n \omega_{,b_i} \text{CoV}(b_i, b_j) \omega_{,b_j} \quad (37)$$

The second-order mean is sufficient for most engineering applications because of the third and fourth moments of the b_i 's (Haldar & Mahadevan, 2000). The dynamic transverse central deflection's first-order sensitivity to random factors is computed using a central difference method

$$\{\omega_{,b_i}\} = \frac{\{\omega(b_i + \text{CoV} \times b_i) - \omega(b_i - \text{CoV} \times b_i)\}}{(2 \times \text{CoV} \times b_i)} \quad (38)$$

The CoV, in an input system with n random variables, can be determined by the covariance matrix, $\text{CoV}(b_i, b_j)$ which is calculated based on the correlation coefficients ρ_{ij} between the random variables b_i and b_j (Haldar & Mahadevan, 2000)

$$\text{CoV}(b_i, b_j) = \sum_{i=1}^n \sum_{j=1}^n [C'] \quad (39)$$

Where

$$[C'] = \begin{bmatrix} 1 & \rho_{b_1, b_2} & \dots & \rho_{b_1, b_i} \\ \rho_{b_2, b_1} & 1 & \dots & \rho_{b_2, b_i} \\ \dots & \dots & \dots & \dots \\ \rho_{b_i, b_1} & \rho_{b_i, b_2} & \dots & 1 \end{bmatrix}$$

The standard deviation ω , denoted as $\text{SD}(\omega')$ can be written as

$$\text{SD}(\omega') = \sqrt{\text{Var}(\omega')} \quad (40)$$

The square root of a given variance is equal to the standard deviation. This study introduces the

second-order statistics of ESLCPESMA's disturbing frequency, including expected mean and standard deviation, using SOPT. The sampling-based MCS is used only for validation due to computationally expensive problems with many samples.

Results and discussions

In the proposed research, the stochastic FEM-based parametric instability analysis of LCPESMA is evaluated using a C0-based FEM with HSDT, incorporating uncertain system parameters, via SOPT in MATLAB. Various support conditions are considered to examine their influence on the parametric instability regions, such as:

All edges are supported (SSSS)

$$v_0 = w_0 = \phi_y = \theta_y = 0 \text{ at } x = 0, a$$

$$u_0 = w_0 = \phi_x = \theta_x = 0 \text{ at } y = 0, b$$

All edges are clamped and supported (CCCC)

$$u_0 = v_0 = w_0 = \phi_x = \theta_x = \phi_y = \theta_y = 0$$

$$\text{at } x = 0, a \text{ and } y = 0, b$$

The foundation parameters (k_1 and k_2), excitation frequency (ω), and dimensionless critical buckling temperature (T_{cr}) (unless otherwise stated) are defined as

$$T_{cr} = N_{cr} \alpha_0 \Delta T \times 1000 \quad \omega = \varpi \left(\frac{b^2}{h} \right) \sqrt{\frac{\rho_m}{E_{2m}}}, \quad k_1 =$$

$$K_1 \frac{b^4}{E_2 h^3}, \quad k_2 = K_2 \frac{b^2}{E_2 h^3}$$

where N_{cr} , α_0 , ΔT , and ϖ are the dimensional critical buckling temperature, thermal expansion coefficient ($1 \times 10^{-6}/^\circ\text{C}$), applied uniform temperature distribution, and dimensional excitation frequency, respectively. The parameter 'm' represents a matrix. For the investigation of the expected mean and SD of thermal instability analysis LCPESMA, which is made of Nitinol-based SMA fibres, the following material attributes for the present investigation are used (Kumar and Singh, 2009)

Material: Graphite/Epoxy composite

$E_{1m} = 155 \text{ GPa}$, $E_{2m} = 8.07 \text{ GPa}$, $G_{12m} = G_{13m} = 4.55 \text{ GPa}$, $G_{23m} = 3.25 \text{ GPa}$, $\nu_{12m} = 0.22$, $\alpha_0 = 10^{-6}/^\circ\text{C}$, $\alpha_{1m} = -0.07 \times \alpha_0$, $\alpha_{2m} = 30 \times \alpha_0$, $\rho_m = 1600 \text{ kg/m}^3$. Subscript 's' is used for SMA fibres and 'm' for matrix and conventional fibres. Further, ν_{12} , α_l , E_l , E_2 , $V_m (= I - V_s)$, and G_{12} , are Poisson's ratio, thermal expansion coefficient, elastic modulus in

the longitudinal and transverse direction, volume fraction of graphite epoxy composite matrix, and shear modulus. Young modulus, E_s , and recovery stress σ_s for Nitinol-based SMA fibres, depending on the temperature increment, are adopted from Kumar and Singh, (2009).

where $G_s = 24.86 \text{ GPa}$, $\alpha_s = 10.26 \times 10^{-6}/^\circ\text{C}$, $\nu_{12s} = 0.33$, $E_0 = 0.1 \text{ GPa}$, $\rho_s = 6450 \text{ kg/m}^3$.

According to the SMA fibers' pre-strain (ϵ), the increase in temperature recovery stress is calculated as per Kumar and Singh, (2009). The multiphase LCP used in the study is reinforced with SMA, a conventional fibre, and an epoxy matrix. The following uncorrelated input random system parameters (b_i) are considered in the sequence form as $b_1=a$, $b_2=h$, $b_3=E_{1m}$, $b_4=E_s$, $b_5=\sigma_r$, $b_6=V_s$, $b_7=T$, $b_8=\beta$, $b_9=k_1$, and $b_{10}=k_2$.

The present research outline of SFEM has two sections. The first section calculates the expected mean excitation frequency and instability regions for one group. In contrast, in the second section, the SD of the excited frequency concerning assumed input random variables for excitation frequency and instability regions is evaluated. The two sections are solved one after the other for the calculation of the mean and SD, dimensionless excited natural frequency of the ESLCPESMA using SOPT in conjunction with FEM. The SD results of excitation frequency are provided considering a CoV of the system characteristics equal to 0.3, i.e., 30% variation of input random system characteristics from their mean values, while considering the SOPT limitation (Halder and Mahadevan, 2000). It is noted that all the results are generated without considering foundation parameters unless otherwise specified.

Validation and convergence of deterministic dynamic instability investigation

It is noted that all results are generated for the static stability factor (α) assumed to be zero. To ensure the required number of elements in the present FEM, the convergence study of instability regions of laminated square CCCC-supported composite plate reinforced with SMA fibre [0/S/0] for $a/h=50$, $T=25$, $E_s=22 \times 10^9$, $\sigma_r=6.0 \text{ MPa}$, and $V_s=0.3$ has been carried out with the variation of dynamic instability factors with parametric frequency as indicated in Figure 3.

As the number of elements increases, the excitation frequency converges after 25 elements (nel). Hence, for further computation of results, nel equal to 25 is used.

The efficacy of the current FEM model is illustrated by the validation evaluation of the thermal buckling temperature of a laminated square $[\pm 45]_3$ SSSS plate with various plate thickness

ratios (a/h) as shown in Figure 4. For the given a/h , the present results using C^0 FEM are validated against those of Lee, (1997) using FSDT. Both outcomes agree well.

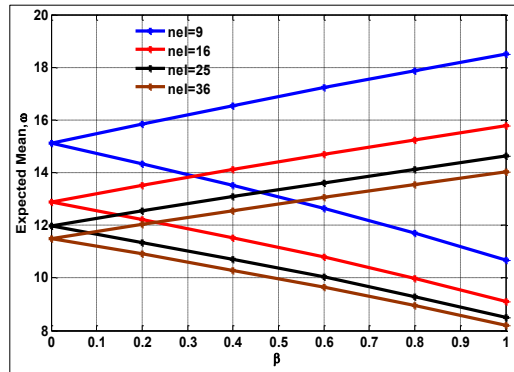


Figure 3. Convergence study of the principal instability region

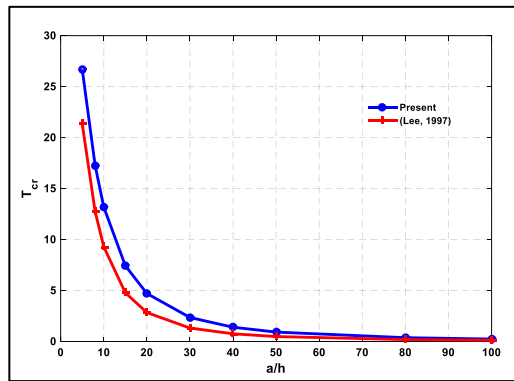


Figure 4. Validation study of thermal buckling temperature

Table 1 presents the validation study of the deterministic dimensionless thermal buckling temperature for a three-layer SMA composite

[0/S/0], with various a/h ratios and V_s where $\varepsilon = 1\%$, and $\sigma_r = 6.0$ MPa as reported by Kumar and Singh, (2009). The current simulation, which employs the C^0 finite element approach, agrees well with Kumar and Singh, (2009) for a range of a/h ratios and V_s . The buckling temperature rises as the volume fraction index rises, and falls as the plate thickness ratio increases.

Table 1. Validation study of thermal buckling temperature (T_{cr}) of square LCPESMA for [0/S/0]

V_s	Present Simulation			(Kumar and Singh, 2009)		
	$a/h=50$	$a/h=80$	$a/h=100$	$a/h=50$	$a/h=80$	$a/h=100$
0.1	0.259	0.107	0.069	0.37	0.17	0.085
0.2	0.655	0.269	0.175	0.72	0.31	0.200
0.3	1.260	0.517	0.336	1.18	0.52	0.374

The second validation evaluation uses (Shen *et al.*, 2003) to validate the fundamental frequency of SSSS-supported laminated square plates with three different combinations of thermal loading conditions ($E_{11}/E_{22} = 40$, $G_{23}/E_{22} = 0.5$, $G_{12}/E_{22} = G_{13}/E_{22} = 0.6$, $\nu_{12} = 0.25$, $\alpha_{22} = 11.4 \times 10^{-6}/^\circ\text{C}$, $\alpha_{11} = 1.14 \times 10^{-6}/^\circ\text{C}$).

Table 2 illustrates that the present simulation utilizing the C^0 finite element method agrees reasonably well with the semi-analytical method of Shen *et al.*, (2003) for varying values of ΔT and a/h . The dimensionless fundamental frequency lowers as the temperature and plate thickness ratio rise because the plate becomes less rigid.

Figure 5 depicts the results of validating the principal instability region of a [0/90/90/0] laminated SSSS-supported LCPESMA for $a/h = 25$ and $\alpha = 0.0$. Good agreements are found between the present simulated results using HSDT with a C^0 -based FEM of (Moorthy *et al.*, 1990) and those using FSDT.

Table 2. Validation study of the fundamental frequency $\omega = \bar{\omega} \left(\frac{b^2}{h} \right) \sqrt{\frac{\rho}{E_2}}$ of laminated composite plates under three sets of thermal loading conditions

Lay-up (k_1, k_2)	a/h	$\Delta T = 0^\circ\text{C}$		$\Delta T = 100^\circ\text{C}$		$\Delta T = 200^\circ\text{C}$	
		(Shen <i>et al.</i> , 2003)	Present	(Shen <i>et al.</i> , 2003)	Present	(Shen <i>et al.</i> , 2003)	Present
(0, 0)	50	18.68	18.76	16.41	16.66	13.82	14.26
	20	17.48	17.73	17.17	17.13	16.57	17.05
	10	14.70	15.24	14.59	15.15	14.49	15.06
(0/90/0)	05	10.26	10.88	10.22	10.84	10.18	10.81

For $\alpha = 0.4$ and $a/h = 25$, the principal instability region of a $[0/90/90/0]$ laminated composite plate is depicted in Figure 6 as the subject of a validation investigation for boundary conditions. The current simulated results using HSDT theory of Sahoo and Singh, (2018) are based on FEM-based layer-wise theory. The instability regions and excitation frequency of clamped-supported plates are higher than those of supported plates due to the higher number of boundary constraints.

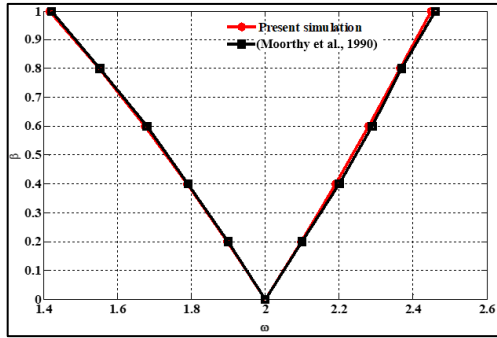


Figure 5. Validation of the dynamic stability region

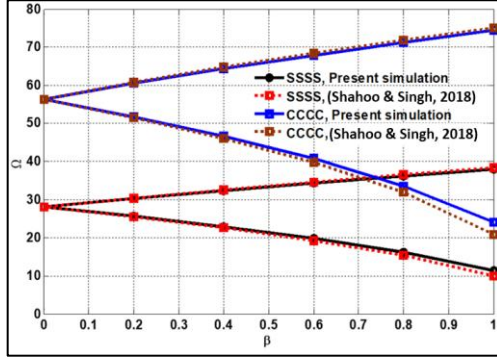
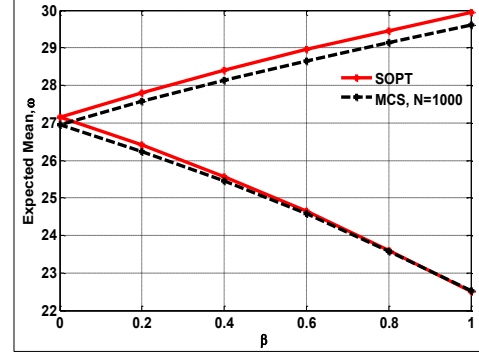


Figure 6. Validation of boundary conditions

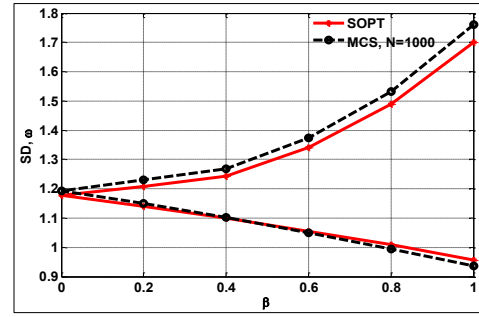
Figure 7 (a)-(b) demonstrates the validation study of expected mean and corresponding SD $\{\text{CoV}, b_i (i=4, 6, 7) = 0.3\}$, of a CCCC supported square LCPESMA with dynamic stability factor (β) for $a/h=100$, $V_s=0.2$, $[0/S/0]$, $\varepsilon=1\%$, $T=25^\circ\text{C}$, $E_s = 22 \times 10^9 \text{ MPa}$, and $\sigma_r = 6 \times 10^6 \text{ MPa}$. For different β , the expected mean and corresponding SD of excitation frequency employing sampling-based MCS and stochastic-based SOPT are in good agreement.

For the sample of random number generations, the Gaussian normal distribution with 1000 random samples is considered based on convergence. The sampling-based MCS, which requires a large

number of samples and various values of β , necessitates a considerable amount of computation time; it is primarily used for validation.



(a)



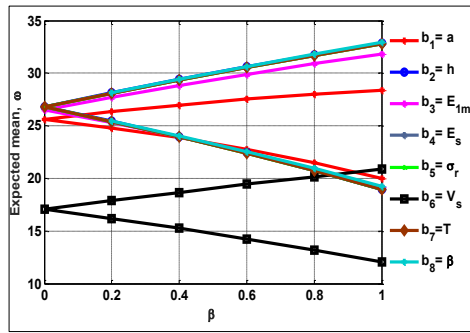
(b)

Figure 7. (a) Validation study of stochastic expected mean excitation frequency (b) Validation study of stochastic SD excitation frequency

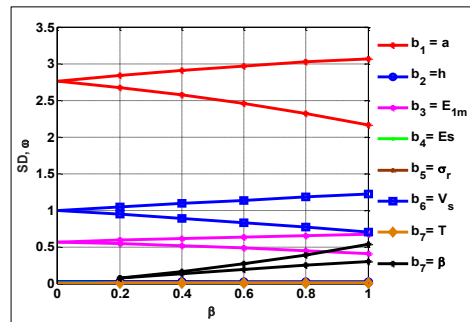
Figure 8 (a)-(b) demonstrates the impact of specific random system parameters $\{\text{CoV}, b_i (i=1-8) = 0.1\}$, on expected mean and SD of an SSSS supported square LCPESMA with dynamic stability factor (β) for $a/h=50$, $V_s=0.3$, $[0/S/0/0]$, $[0/90/0/90]$, $\varepsilon=1\%$, $T=25^\circ\text{C}$, $E_s=22 \times 10^9 \text{ MPa}$, and $\sigma_r=6 \times 10^6 \text{ MPa}$.

For different β values, the expected mean of excitation frequency is higher for β , T , σ_r , E_s , h , and E_{lm} . For different β , the SD of excitation frequency is higher for β , T , σ_r , E_s , h , and E_{lm} . Hence, from a reliability perspective, these random parameters should be strictly controlled and accurately measured.

Figure 9 (a)-(b) demonstrates the impact of number and positions of SMA fibers on expected mean and SD of random system parameters $\{\text{CoV}, b_i (i=4, 6, 7) = 0.3\}$, on expected mean and variance of SD of a SSSS supported square LCPESMA with dynamic stability factor (β) for $a/h = 50$, $V_s=0.2$, $\varepsilon = 1\%$, $T=90^\circ\text{C}$, $E_s = 81.5 \times 10^9 \text{ MPa}$, and $\sigma_r = 131.3 \times 10^6 \text{ MPa}$.

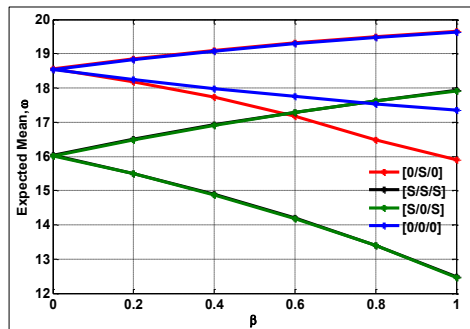


(a)

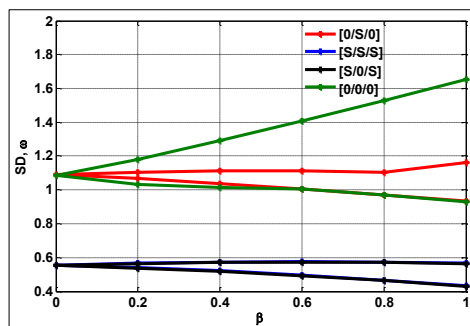


(b)

Figure 8. (a) Effect of individual random parameters on mean excitation frequency (b) Effect of individual random parameters on SD excitation frequency



(a)

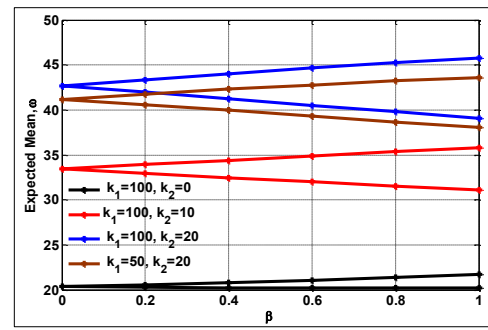


(b)

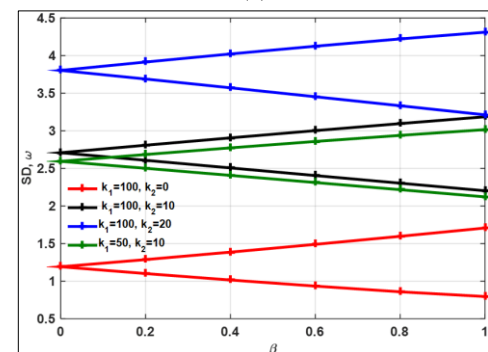
Figure 9. Effect of SMA fibres on mean excitation frequency

As the number of SMA layers rises, the expected mean and SD of the excitation frequency fall along with the rise in the instability region. The position of the SMA fibre at the top and bottom proved to be the lowest SD and instability region. Hence, SMA layers applied at the top and bottom positions provide maximum reliability and safety. With the increase of the SMA layer, the reliability and protection of the structure increase.

Figure 10(a) and Figure 10 (b) demonstrates how foundation parameters (k_i) affect the expected mean and corresponding SD {CoV, b_i ($i=9, 10, 11$) = 0.3}, of a CCCC supported square ESLCPESMA with dynamic stability factor (β) for $a/h=80$, $V_s=0.3$, $[0/S/0]$, $\varepsilon=1\%$, $T=25^\circ\text{C}$, $E_s=22\times 10^9\text{MPa}$, and $\sigma_r=6\times 10^6\text{MPa}$. For different random foundation parameters, the effect of individual change in expected mean and SD for k_2 is highest. The shear foundation should be accounted for properly for higher reliability and safety. With the increase in foundation parameters, the excitation frequency increases because of the increased stiffness of the foundation.



(a)



(b)

Figure 10. (a) Effect of foundation parameters on the mean excitation frequency (b) Effect of foundation parameters on SD excitation frequency

Figure 11 (a)-(b) demonstrates the impact of three frequency modes on the expected mean and corresponding SD $\{\text{COV}, b_i (i=4, 6, 7) = 0.1\}$, of a CCCC-supported square LCPESMA with dynamic stability factor (β) for $a/h = 80$, $V_s = 0.1$, $[0/S/0]$, $\epsilon = 1\%$, $T = 25^\circ\text{C}$, $E_s = 22 \times 10^9 \text{ MPa}$, and $\sigma_r = 6 \times 10^6 \text{ MPa}$. Different frequency modes: With an increase in frequency mode, the expected mean of the excitation frequency increases.

The SD of excitation frequency is highest for the 3rd mode and lowest for the 2nd mode. From an assurance and security perspective, a lower frequency mode of excitation will be preferred.

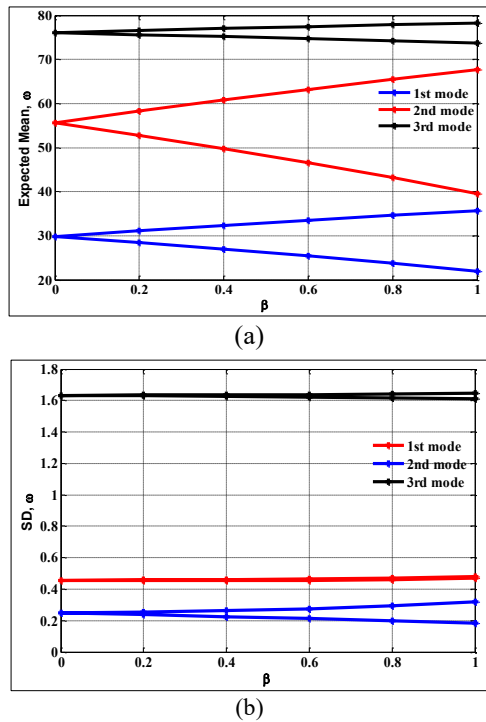


Figure 11. (a) Effect of frequency mode on expected mean excitation frequency (b) Effect of frequency mode on SD of excitation frequency

Figure 12(a) and Figure 12(b) demonstrates the impact of temperature and percentage recovery strain on expected mean and corresponding SD $\{\text{CoV}, b_i (i=4, 6, 7) = 0.1\}$, of a CCCC-supported square LCPESMA with dynamic stability factor (β) for $a/h=80$, $V_s=0.2$, $[S/S/S]$.

The expected mean excitation frequency rises as the temperature rises. Meanwhile, with the increase in percentage strain, the change in mean excitation frequency alongside instability regions is negligible. The SD of excitation frequency and corresponding instability region rise with the

temperature rise, and the instability region decreases with increases in strain. Considering reliability, the minimum temperature with the minimum percentage strain, and the lower frequency mode of excitation, the preferred option will be the one with the lowest frequency. It is noted that all results generated for the static stability factor (α) are assumed to be zero. The first mode is considered unless stated otherwise.

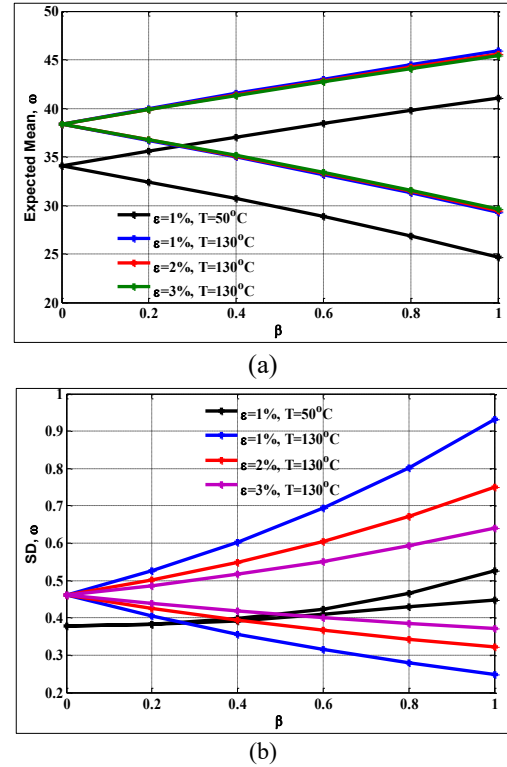


Figure 12. (a) Effect of temperature and % strain on the mean frequency and instability regions (b) Effect of temperature and % strain on SD of frequency and instability regions

Conclusions

This study analyses second-order statistics, such as SD and expected mean, in response to random system parameters like thermally induced excitation frequency and instability, within an ESLCPESMA structure. A computationally efficient C^0 FEM based on HSDT and the SOPT is employed to achieve these results. Based on the analytical investigation, the following conclusions are drawn.

As the dynamic instability factor rises, the expected mean and corresponding SD of excitation frequency with random system parameters rise, and the possibility of higher resonance conditions

increases. Hence, a lower dynamic instability factor would be preferred for higher safety.

The influence of random changes in β , T , σ_r , E_s , h , and E_{1m} is most substantial over several random system parameters. Hence, these random parameters should be appropriately accounted for to ensure higher reliability and safety.

As the number of SMA layers rises, the expected mean and SD of the excitation frequency fall along with the rise in the instability region. The reliability of the structures increases, and the possibility of resonance conditions decreases with both the increase of SMA layers and the placement of SMA layers in a balanced position.

The effect of the shear foundation is highly susceptible, and there are possibilities of higher resonance conditions than the Winkler foundation parameters. Hence, a perfect assessment of the shear foundation is highly desirable for the reliability and safety of the ESLCPESMA.

With the increase in the frequency mode, the expected mean and SD of excitation frequency increase, and the possibility of higher resonance conditions increases. Hence, a lower-order frequency mode is chosen for a better reliability perspective.

For higher reliability, a minimum temperature with minimum percentage strain and a lower frequency mode of excitation are preferred.

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Nomenclature

a, b, h	Length, width, and thickness	ΔT	Temperature increments
X, Y, Z	Cartesian coordinates	E_s, G_s, ν_s, ρ_s	Young's and shear moduli, Poisson's ratio, and density of SMA
u, v, w	Midplane deformations	E_m, G_m, ν_m, ρ_m	Young's and Shear moduli, Poisson's ratio, and density of matrix
$\sigma_x, \sigma_y, \sigma_z$	Cartesian stresses	K_g	Geometric stiffness matrix
$\varepsilon_x, \varepsilon_y, \varepsilon_z$	Cartesian strains	K_l	Linear stiffness matrix
V_s, V_m	Volume fraction of SMA and matrix	$\alpha_x, \alpha_y, \alpha_{xy}$	Thermal expansion coefficients
T_{cr}	Thermal buckling temperature	q	Displacement vector
ω	Excitation frequency	K_1, K_2	Shear and spring foundation stiffness matrices
$\bar{\omega}$	Dimensional frequency	k_1, k_2	Shear and spring foundation parameters