

Construction of ASEAN Chess Endgame Tablebases: A Computational Analysis of King-Knight-Queen and King-Bishop-Queen Patterns Against a Lone King

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Abstract

Background and Objectives: ASEAN Chess, is a traditional strategic board game that differs substantially from International Chess in terms of piece movement, endgame dynamics, and regulatory constraints. One of its most distinctive features is the official piece-counting rule, which limits the number of moves allowed to force a checkmate when one side is reduced to a lone King. This rule has a significant impact on endgame outcomes, often determining whether a theoretically winning position results in a practical draw. Despite its widespread practice at both national and international competitive levels, ASEAN Chess remains significantly underexplored from computational and game-theoretic perspectives, particularly with respect to complex endgame scenarios involving multiple attacking pieces. Existing studies and practical knowledge rely heavily on human intuition and empirical experience, which are insufficient to fully capture the exhaustive state space of such endgames. The objective of this research is to develop Endgame Tablebases (EGTBs) for ASEAN Chess and to conduct a comprehensive computational analysis of two complex endgame configurations: King-Knight-Queen versus King (K-N-Q vs. K) and King-Bishop-Queen versus King (K-B-Q vs. K). The study aims to determine, under optimal play, the exact distribution of forced wins and draws, to identify the maximum Distance to Mate (DTM), and to evaluate the practical impact of the official piece-counting rules on theoretical endgame outcomes.

Methodology: This research employs a computational framework based on retrograde analysis to exhaustively evaluate all legal endgame positions for the selected configurations. The process begins by generating all legally reachable board states in accordance with the official rules of ASEAN Chess. To manage the large state space, symmetry-based reduction techniques are applied, allowing equivalent board positions to be grouped and analyzed efficiently. Terminal checkmate positions are initialized as Distance to Mate (DTM) zero. The algorithm then iteratively propagates results backward through the game tree, assigning game-theoretic values to each position by considering all possible legal moves for both the attacking and defending sides. Positions are classified as forced wins, draws, or losses under the assumption of perfect play, with the attacking side always moving first. The resulting EGTBs record not only the outcome of each position but also the maximum number of moves required to force a checkmate. In addition, the official ASEAN Chess piece-counting rules are incorporated into the analysis, with particular attention given to the regulation limiting the Bishop's pursuit to a maximum of 44 moves. This allows a direct comparison between theoretical outcomes and regulation-constrained practical results.

Main Results: The computational analysis reveals that the K-N-Q configuration comprises a total of 11,930,016 legal positions. Of these, only 472,900 positions (3.96%) result in forced wins for the attacking side, while the remaining 96.04% are theoretical draws, indicating strong defensive resilience against a lone King. The deepest winning positions in this configuration require up to 36 moves to achieve checkmate. In contrast, the K-B-Q configuration encompasses 12,170,304 legal positions. Among these, 10,438,976 positions (85.77%) are identified as forced wins, while 14.23% result in draws. The maximum Distance to Mate in this configuration reaches 57 moves, demonstrating that although the position is highly favorable for the attacker, precise and extended maneuvering is often required. When the official 44-move piece-counting rule for the Bishop is applied, 278,040 theoretically winning positions exceed the permitted move limit. As a result, the effective winning probability is reduced to 85.39%, with these positions converted into practical draws due solely to regulatory constraints.

Conclusions: This study provides the first exhaustive Endgame Tablebases for complex ASEAN Chess endgames involving Knight-Queen and Bishop-Queen configurations. The results confirm a substantial disparity in piece potency, with the Bishop-Queen combination exhibiting significantly greater offensive effectiveness than the Knight-Queen combination.

Furthermore, the findings demonstrate that official counting rules play a decisive role in shaping practical outcomes, creating a measurable divergence between theoretical optimal play and regulation-constrained competition.

Practical Application: The developed EGTBs establish a rigorous foundation for multiple practical applications. They can be integrated into high-performance ASEAN Chess engines to improve endgame accuracy, utilized as authoritative references for training and instructional systems, and incorporated into digital or mobile learning platforms to enhance players' understanding of optimal endgame play and counting rules. Additionally, the results provide valuable insights for tournament organizers and rule designers by quantifying the practical effects of regulatory constraints. More broadly, this research contributes to computational game analysis and supports future studies of more complex ASEAN Chess endgames involving additional pieces.

Keywords: ASEAN Chess, Endgame Tablebases, Retrograde Analysis, Piece Counting Rules, Computational Game Analysis

Introduction

ASEAN Chess is a strategic board game played by two opponents, in which each piece exhibits distinct movement patterns and levels of influence. A game typically concludes with a decisive result win or loss when one player's King is placed under check and has no legal move to escape, a condition referred to as checkmate. Alternatively, a match may end in a draw under specific circumstances, including stalemate, insufficient mating material, or regulatory move-count limitations.

A critical regulation governing draw determination in the endgame phase is the Counting Rule [1]. This rule becomes applicable when all pawns have been removed from the board and one player is left with only a lone King (the defensive side). In such cases, the defensive player may initiate a formal move count, beginning from one up to a maximum threshold determined by the strongest remaining piece on the attacking side. Specifically, the move limit is set to 16 moves if the attacker has at least one Rook, 44 moves if at least one Bishop remains, and 64 moves if at least one Knight remains [2]. If the defensive side successfully avoids checkmate within the prescribed number of moves, the game is officially declared a draw.

This research investigates the pursuit of a lone King by an attacking side possessing four pieces (including the Kings) under two endgame configurations: (1) King, Knight, and Queen, subject to the 64-move limit, and (2) King, Bishop, and Queen, subject to the 44-move limit. These configurations are of particular interest within the ASEAN Chess community, as competitive records among high-level players reveal substantial variability in outcomes, ranging from decisive victories to drawn results.

To address this complexity, the present study employs systematic computational techniques to analyze the aforementioned endgame positions. By constructing Endgame Tablebases (EGTBs), the research establishes theoretical outcomes for both configurations and precisely quantifies the number of forced-win and draw positions. The resulting tablebases provide a rigorous foundation for pedagogical applications, chess engine development, and future computational research in ASEAN Chess endgame analysis.

The remainder of this paper is organized as follows: Section 1 presents the research background and problem statement; Section 2 reviews relevant theories and related literature; Section 3 describes the research methodology and computational results; and Section 4 concludes the paper with a discussion of findings and recommendations for future research.

Theory and Literature reviews

ASEAN Chess Rules and Regulations

ASEAN Chess is governed by a specific set of regulations established by the ASEAN-Chess Council to standardize competitive play within the Southeast Asian region [2]. While the game shares a foundational lineage with Makruk (Thai Chess) and International Chess, it possesses unique characteristics that dictate endgame dynamics.

• Piece Movements and Powers

The strategic complexity of ASEAN Chess endgames is largely influenced by the movement constraints of its pieces [3-4]. In contrast to the long-range movement of the Bishop and Queen in International Chess, their ASEAN Chess counterparts exert only short-range influence.

King: The King moves exactly one square in any direction orthogonally or diagonally provided that the destination square is not under attack by an opposing piece. As the central objective of the game, the King plays a dual role in endgame positions, functioning both

as a vulnerable target and as an active defensive or offensive unit, particularly in scenarios involving pursuit and evasion.

Knight: The Knight moves in an L-shaped pattern, consisting of two squares in one orthogonal direction followed by one square perpendicular to it. Unlike the other pieces, the Knight can jump over intervening pieces, enabling it to exert influence in blocked or congested positions. This unique movement makes the Knight particularly effective in restricting the mobility of a lone King during endgame pursuits.

Bishop (Khon): The Bishop moves one square diagonally in any direction or one square directly forward only. This asymmetric movement grants the Bishop a unique balance between offensive and defensive capabilities and is often compared to the Silver General in Shogi [5].

Queen (Met): The Queen moves exactly one square diagonally in any direction. This movement resembles a one-square diagonal move, similar to the Ferz (or Fers) piece in early chess variants [6].

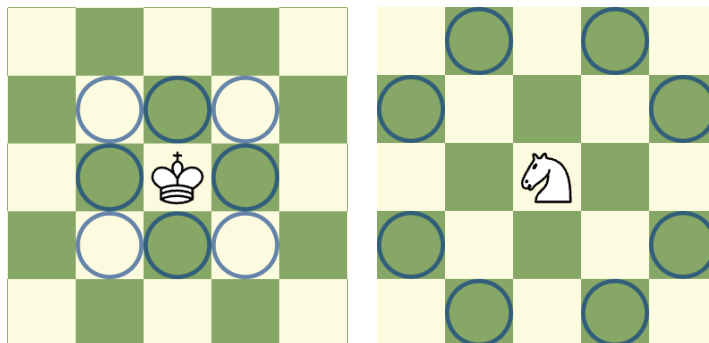


Figure 1 Movement Capabilities of the ASEAN Chess Pieces: King and Knight

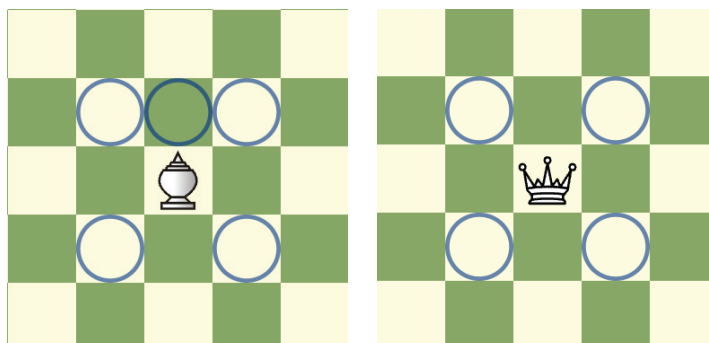


Figure 2 Movement Capabilities of the ASEAN Chess Pieces: Bishop and Queen

• The Counting Rule

A defining feature of ASEAN Chess is the Counting Rule, which functions as a regulatory mechanism to prevent games from continuing indefinitely during the endgame phase. This rule is formally invoked when the following conditions are simultaneously satisfied: (1) no pawns remain on the board, and (2) one player is reduced to a lone King, hereafter referred to as the defensive side [1-2].

Once activated, the defensive side initiates a formal move count, the maximum allowable number of moves being determined by the strongest remaining piece on the attacking side. Specifically, the prescribed limits are as follows:

Rook:	16 moves
Bishop:	44 moves
Knight:	64 moves

If the attacking side fails to deliver checkmate within the corresponding move limit, the game is officially declared a draw. In competitive practice, this rule introduces a critical temporal constraint, effectively creating a “race against the clock,” in which a position that is theoretically a forced win may nevertheless result in a practical draw if the optimal mating sequence exceeds the regulatory threshold. This interaction between theoretical win conditions and rule-based limitations constitutes a central analytical focus of the present study.

Endgame Tablebases (EGTBs)

The construction of Endgame Tablebases (EGTBs) constitutes a definitive computational approach for analyzing various board games, such as chess and checkers, particularly in scenarios where the number of remaining pieces is limited [7]. This approach is based on retrograde analysis, a systematic computational procedure that begins by identifying terminal positions with known outcomes such as checkmate and subsequently works backward to evaluate all legally reachable positions within the game’s state space [8].

The resulting tablebases provide an exact evaluation of whether a given position results in a win, loss, or draw, assuming optimal play from both the attacking and defensive sides [9-11]. Moreover, for positions that lead to decisive outcomes, EGTBs can precisely determine the Distance to Mate (DTM) and the optimal sequence of moves required to

achieve checkmate with perfect accuracy [12].

Historically, several endgame configurations that were previously regarded by human analysts as theoretical draws have been proven, through computational analysis, to be forced wins [13-14]. This discrepancy arises from the inherent complexity of such positions, which often exceeds the practical limits of human intuition and manual calculation. A notable milestone in the development of EGTBs for international chess is the completion of the 7-piece tablebases, published by Bojun Guo in 2018, which require approximately 17 terabytes of storage capacity.

• Principle of Symmetry in Endgame Tablebases

The principle of symmetry is a fundamental concept for optimizing computational resources during the construction of Endgame Tablebases (EGTBs) [15-16]. Many board positions are functionally equivalent under geometric reflections of the chessboard. For configurations involving pieces such as the King, Knight, and Queen, an eightfold symmetry (octal symmetry) can be applied. Accordingly, once the outcome of a single position is determined, the result can be systematically extrapolated to all of its symmetrical counterparts across the board.

In this study, the computational state space was significantly reduced by applying the symmetry principle to a set of 10 reference squares: a1, b1, b2, c1, c2, c3, d1, d2, d3, and d4. These reference squares collectively represent all possible positions on the board under eightfold symmetry, as illustrated in Figure 3.

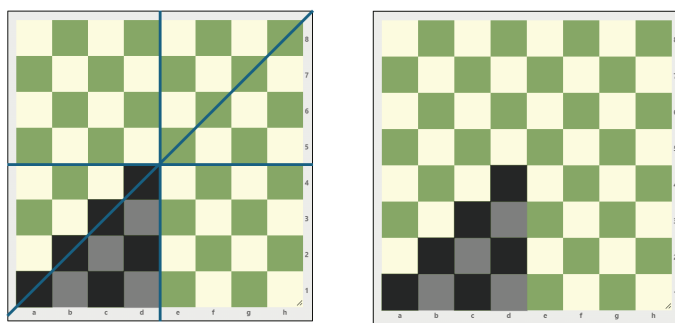


Figure 3 Symmetry Principle used to Reduce Computational Resources in Constructing Endgame Tablebases for the King, Knight, and Queen Configuration

In contrast, the Bishop (Khon) does not admit eightfold symmetry due to its inherent directional asymmetry. Specifically, the Bishop's movement is constrained to three forward directions and only two backward directions, unlike the Knight, which exhibits both rotational

and reflective symmetry in its movement. Consequently, the applicable symmetry for the Bishop is limited to twofold (bilateral) symmetry along the vertical axis of the board. Under this scheme, files a, b, c, and d are mirrored with files e, f, g, and h to propagate the computed results across the entire board, as shown in Figure 4.

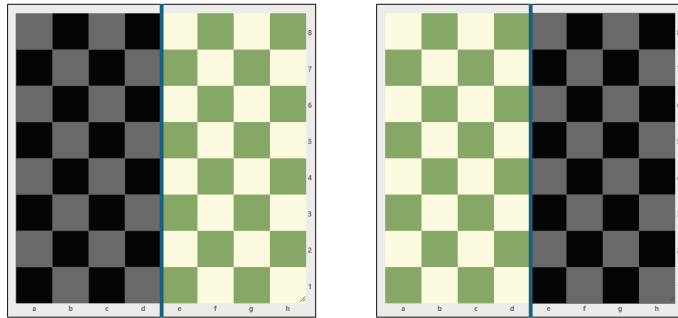


Figure 4 Symmetry Principle used to Reduce Computational Resources in Constructing Endgame Tablebases for the King, Bishop, and Queen Configuration

Methodology

This section describes the computational framework and algorithmic procedures employed in the construction of Endgame Tablebases (EGTBs) for ASEAN Chess. The methodology includes the formal representation of the chessboard, state-space indexing through symmetry-based reduction, and the application of retrograde analysis to determine the game-theoretic outcomes of all legal positions.

System Architecture and Development Environment

The construction of the ASEAN Chess Endgame Tablebases (EGTBs) was implemented using the C# programming language. This development environment was selected due to its robust memory management and efficient processing capabilities, which are essential for large-scale state indexing and exhaustive endgame analysis [17-18].

All computational tasks were executed on a high-performance portable workstation equipped with an Intel(R) Core(TM) i7-9750H processor operating at 2.60 GHz and 16 GB of system memory (RAM).

Data Acquisition and Endgame Configurations

This research focuses on the exhaustive evaluation of two primary endgame

configurations in ASEAN Chess to determine their respective winning and drawing probabilities under optimal play: [15]

Configuration 1 (K-N-Q): The attacking side consists of a King, a Knight, and a Queen, while the defending side is limited to a lone King.

Configuration 2 (K-B-Q): The attacking side consists of a King, a Bishop, and a Queen, while the defending side is limited to a lone King [19-20].

For both configurations, all legal board positions were systematically generated, evaluated, and classified according to their game-theoretic outcomes. The system was designed to store and analyze millions of distinct positions, ensuring that each state was assessed under the assumption of optimal play from both the attacking and defensive sides.

Procedures for Constructing Endgame Tablebases

The Endgame Tablebases (EGTBs) were constructed using retrograde analysis, a computational technique that evaluates game states by iteratively working backward from terminal positions with known outcomes [21-23]. The computational time and memory requirements of this process are directly influenced by the complexity of the underlying piece configurations [24-25]. The construction procedure consists of the following systematic steps:

1. **Generation of Legal Positions:** All legally reachable board configurations were exhaustively generated and categorized in accordance with the official rules of ASEAN Chess.
2. **Initialization of Terminal States:** All checkmate positions were identified and defined as terminal states, each assigned a Distance to Mate (DTM) value of 0.
3. **First Backward Iteration:** All positions that can transition into a DTM 0 state in a single legal move were analyzed and assigned a DTM value of 1.
4. **Subsequent Iterations:** Positions capable of forcing a transition into a DTM 1 state were systematically identified and assigned a DTM value of 2.
5. **Recursive Evaluation:** This iterative process continues recursively. In general, any position that can force a transition into a previously identified DTM n state is assigned a value of DTM $n + 1$, while accounting for forced moves and optimal defensive responses.

Termination and Draw Assignment: The procedure terminates when no additional winning positions can be identified. All remaining legal positions that have not been assigned a DTM value are classified as theoretical draws.

Retrograde Analysis Algorithm

The construction of the endgame tablebase is performed using an algorithm known as retrograde analysis. The algorithm starts from checkmate positions and recursively works backward to identify the preceding positions, or parent states, that can lead to those terminal positions.

Algorithm: Retrograde Analysis for Endgame Tablebases

Input: Set of all valid states (S_{all})

Output: Tablebase array containing Distance-to-Mate (DTM) or DRAW

Initialize Tablebase to UNKNOWN and $DTM \leftarrow 0$

Active $\leftarrow \{ S \in S_{all} \mid S \text{ is Checkmate} \}$

For each $S \in \text{Active}$ do Tablebase[S] $\leftarrow 0$

While Active $\neq$ Empty Set do

 Next $\leftarrow$ Empty Set

 For each parent P of all $S \in \text{Active}$ where Tablebase[P] == UNKNOWN do

 If Turn(P) == DEFENDER then Unexplored[P] $\leftarrow$ Unexplored[P] - 1

 If Turn(P) == ATTACKER or Unexplored[P] == 0 then

 Tablebase[P] $\leftarrow$ DTM + 1

 Next.Add(P)

Active $\leftarrow$ Next; DTM $\leftarrow$ DTM + 1

Set Tablebase[S] $\leftarrow$ DRAW for all remaining UNKNOWN states

Proof of Algorithm Correctness and Optimal Strategy Justification

To establish the correctness of the Retrograde Analysis algorithm, we construct a formal proof based on Dynamic Programming and Bellman's Principle of Optimality [26].

Theorem (Global Optimality and Completeness): The proposed Retrograde Analysis algorithm guarantees the exact Distance-to-Mate (DTM) for every resolvable state $S \in S_{all}$,

yielding a strictly optimal policy for both Attacker and Defender.

Proof:

1. Base Case (DTM = 0): Let $S_0 \subset S_{\text{all}}$ be the set of all terminal checkmate states. By definition, $\text{DTM}(S) = 0$ for all $S \in S_0$. This classification is exact and complete via exhaustive validation of legal moves.
2. Inductive Step: Assume that all states with $\text{DTM} \leq k$ are correctly identified and labeled. A parent state P with move transition to S ($P \rightarrow S$) is processed at step $k + 1$:
 If $\text{Turn}(P) = \text{"ATTACKER"}$, the attacker chooses a transition to S that minimizes DTM. Since $\text{DTM}(S) = k$, setting $\text{DTM}(P) = k + 1$ satisfies $\text{DTM}(P) = 1 + \min_S \text{DTM}(S)$, which strictly adheres to Bellman's Optimality Equation for minimal path.
 If $\text{Turn}(P) = \text{"DEFENDER"}$, the defender chooses a move to maximize DTM or prevent checkmate. State P is assigned $\text{DTM}(P) = k + 1$ if and only if all valid legal transitions $P \rightarrow S_i$ lead to solved mate states with $\text{DTM}(S_i) \leq k$. This represents the minimax optimal counter-strategy $\max_S \text{DTM}(S)$.
3. Termination: Since the finite State Space graph $G = (V, E)$ has $|V| < \infty$, the Breadth-First Search (BFS) traversal backward from terminal nodes guarantees that every reachable state is assigned its minimal DTM in $O(|V| + |E|)$ time. States remaining labeled as UNKNOWN after queue exhaustion are mathematically proven to be unreachable from any mate state, thereby rigorously defining the DRAW set.

While heuristic methods or rule-based abstractions can approximate strategy, constructing the full Endgame Tablebase remains indispensable. Full tablebases provide 100% deterministic lookup tables with guaranteed zero-error evaluation, which are critical for high-precision chess engine evaluation and establishing absolute ground-truth data. Derive-free optimal strategies without exhaustive tablebases (e.g., symbolic rule extraction or neural network approximation) represent a promising direction for Future Work.

Results and Discussion

Summary of Computational Outcomes

The exhaustive evaluation of the endgame configurations yielded definitive statistics regarding winning and drawing probabilities under optimal play. Tables 1 and 2 summarize the total number of legal positions, forced wins, and theoretical draws for both the King-Knight-Queen (K-N-Q) and King-Bishop-Queen (K-B-Q) scenarios.

Table 1 Summary of Computational Outcomes for the King-Knight-Queen Endgame (Attacker to Move)

Configuration Analyzed Outcome	Number of Positions
Total possible positions ($64 \times 64 \times 64 \times 64$)	16,777,216
Legal positions under ASEAN Chess rules	11,930,016
Winning positions	472,900
Drawing positions	11,457,116

EGTB: King-Knight-Queen-King Endgame

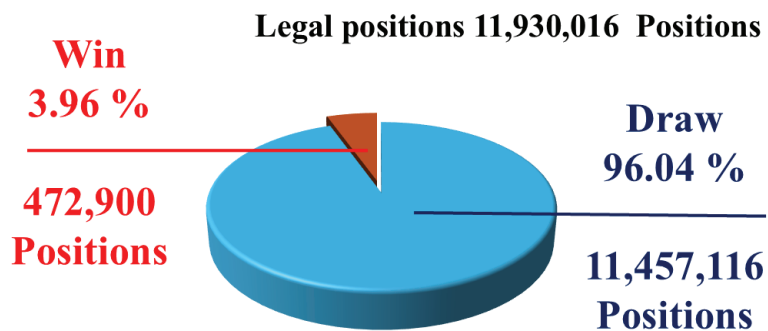


Figure 5 Computationally Derived Winning and Drawing Rates for the King-Knight-Queen Endgame Configuration

Table 2 Summary of Computational Outcomes for the King-Bishop-Queen Endgame (Attacker to Move)

Configuration Analyzed Outcome	Number of Positions
Total possible positions (64 × 64 × 64 × 64)	16,777,216
Legal positions under ASEAN Chess rules	12,170,304
Winning positions	10,438,976
Drawing positions	1,731,328
Drawn positions due to the counting rule (the 44 -move limit)	46,122

EGTB: King-Bishop-Queen-King Endgame

Legal positions 12,170,304 Positions

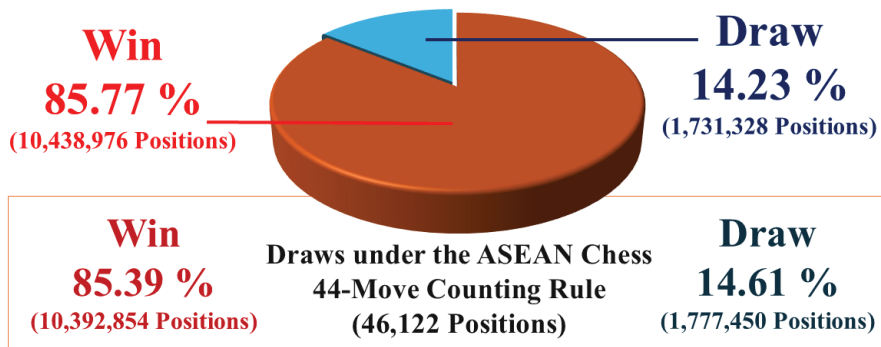


Figure 6 Computationally Derived Winning and Drawing Rates for the King-Bishop-Queen Endgame Configuration

The Impact of Piece Counting Rules on Practical Outcomes

A significant finding of this research is the discrepancy between theoretically forced wins and practical outcomes governed by the ASEAN Chess Counting Rules.

In the King-Bishop-Queen configuration, although the theoretical win rate is 85.77%, the enforcement of the 44-count rule which permits the attacking side a maximum of 44 moves

to deliver checkmate substantially alters the practical outcome. The analysis shows that 278,040 positions classified as mathematically forced wins require more than the permitted 44 moves to reach checkmate. Consequently, the effective winning probability under competitive regulations is reduced to 85.39%.

These results indicate that approximately 0.38% of all theoretically winning positions in the Bishop-Queen endgame are converted into draws due solely to regulatory constraints, referred to in this study as rule-induced draws. This finding highlights the counting rule as a decisive factor capable of overriding theoretical win states in practical competitive play.

Table 3 Comparison of Endgame Tablebase Results for ASEAN Chess in King-Knight-Queen (K-N-Q) Pattern

Distance to Mate (Moves)	Total Positions (Positions)	Distance to Mate (Moves)	Total Positions (Positions)	Distance to Mate (Moves)	Total Positions (Positions)
1	536	13	14,568	25	27,744
2	240	14	21,748	26	23,436
3	392	15	20,972	27	23,524
4	1,456	16	21,996	28	20,140
5	1,736	17	19,032	29	16,764
6	5,956	18	17,468	30	12,112
7	5,464	19	14,956	31	7,752
8	15,912	20	17,832	32	10,416
9	7,032	21	17,100	33	7,696
10	15,232	22	18,008	34	3,292
11	11,592	23	24,640	35	216
12	18,400	24	27,492	36	48
				Total	472,900

Table 4 Comparison of Endgame Tablebase Results for ASEAN Chess in King-Bishop-Queen (K-B-Q) Pattern

Distance to Mate (Moves)	Total Positions (Positions)	Distance to Mate (Moves)	Total Positions (Positions)	Distance to Mate (Moves)	Total Positions (Positions)
1	2,874	20	187,386	39	523,586
2	2,702	21	208,218	40	359,820
3	2,188	22	217,068	41	211,296
4	7,040	23	235,110	42	119,732
5	5,180	24	247,842	43	71,988
6	10,172	25	268,846	44	40,198
7	16,552	26	275,266	45	20,992
8	23,388	27	293,334	46	11,736
9	33,594	28	314,344	47	6,640
10	41,428	29	335,306	48	3,442
11	50,514	30	371,284	49	1,624
12	56,754	31	407,976	50	792
13	64,440	32	466,156	51	474
14	73,900	33	553,860	52	278
15	94,716	34	662,594	53	84
16	115,504	35	749,780	54	20
17	138,004	36	783,628	55	26
18	159,484	37	757,474	56	6
19	173,916	38	658,412	57	8
				Total	10,438,976

Discussion of Piece Potency

The stark contrast between the win rates of the Knight-Queen (3.96%) and Bishop-Queen (85.77%) configurations provides empirical evidence of relative piece potency in ASEAN Chess. Despite the Queen's limited diagonal movement, its synergy with the Bishop is substantially more effective at constraining the mobility of a lone King than its synergy with the Knight. The maximum Distance to Mate (DTM) of 57 moves in the Bishop-Queen scenario further demonstrates that, while theoretically powerful, this configuration often requires precise and extended execution, frequently approaching or exceeding the limits imposed by the counting rules.

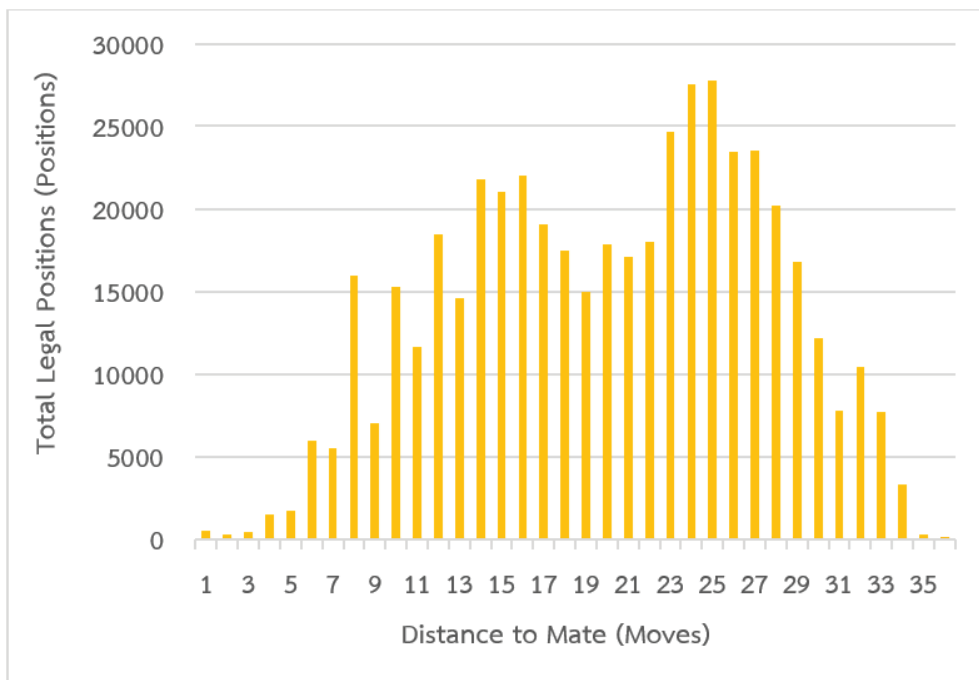


Figure 7 Relationship between the Knight-Queen Positions and the Required Number of Moves to Force a Win under Optimal Play

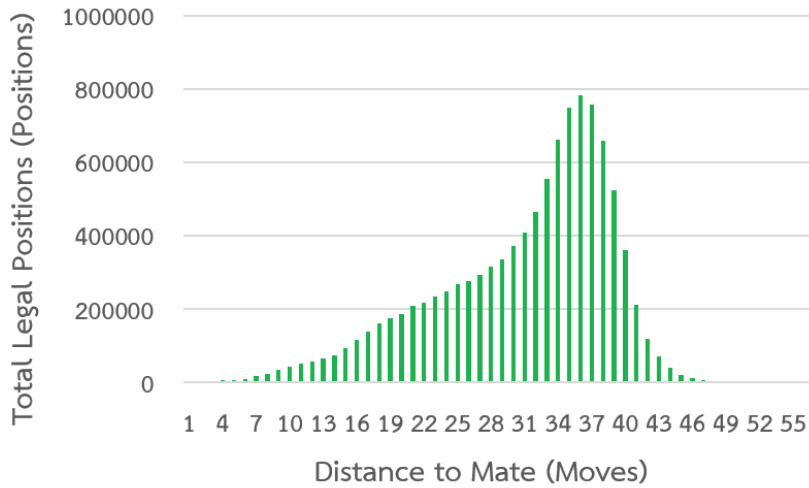


Figure 8 Relationship between the Bishop-Queen Positions and the Required Number of Moves to Force a Win under Optimal Play

Mathematical Properties Explaining the Efficacy Difference Between Piece Configurations

To mathematically explain why the King-Bishop-Queen configuration achieves a dramatically higher winning probability (85.77%) compared to the King-Knight-Queen configuration (3.96%), we formalize the board geometry and piece mobility as algebraic and graph-theoretic structures.

1) Board Partitioning and Colorbound Limitation

Let the chessboard B be a grid $B = \{(x,y) \mid x,y \in \{1,2,\dots,8\}\}$. The board is partitioned into two disjoint subsets based on square color parity:

$$B_{\text{white}} = \{(x,y) \in B \mid x + y \equiv 0 \pmod{2}\}$$

$$B_{\text{black}} = \{(x,y) \in B \mid x + y \equiv 1 \pmod{2}\}$$

where $B = B_{\text{white}} \cup B_{\text{black}}$ and $B_{\text{white}} \cap B_{\text{black}} = \emptyset$

Queen Operator: The legal move function $M_Q: B \rightarrow P(B)$ is restricted to diagonal step vectors $\Delta_Q = \{(\pm 1, \pm 1)\}$. For any position $p = (x, y)$, if $p \in B_{\text{color}}$, then for all $p' \in M_Q(p)$, $(x \pm 1) + (y \pm 1) \equiv x + y \pmod{2}$. Thus, $M_Q(p) \subset B_{\text{color}}$. The Queen is strictly colorbound.

Knight Operator: The move function $M_K: B \rightarrow P(B)$ uses vectors $\Delta_K = \{(\pm 1, \pm 2), (\pm 2, \pm 1)\}$. Since $(x \pm 1) + (y \pm 2) \equiv x + y + 1 \pmod{2}$, M_K maps $B_{\text{color}} \rightarrow B_{\text{opposite-color}}$.

While Knight alternates square colors, its reachability operator lacks line control (i.e., path P of Knight steps that forms a continuous topological barrier").

Bishop Operator: The move function $M_B : B \rightarrow P(B)$ utilizes vectors $\Delta_B = \{(\pm 1, \pm 1) \cup (0, 1)\}$. The forward step vector $(0, 1)$ changes parity: $x + (y + 1) \equiv x + y + 1 \pmod{2}$. Therefore, $M_B(p) \cap B_{\text{white}} \neq \emptyset$ and $M_B(p) \cap B_{\text{black}} \neq \emptyset$. The Bishop is non-colorbound.

2) Reachability of Checkmate States under Dihedral Symmetry D_4

A terminal checkmate state requires trapping the enemy King $K_e \in B$ such that its reachable move set $M_K(K_e) = \emptyset$ and K_e is under attack.

Theorem 1 (Checkmate Necessity for King-Knight-Queen): In a King-Knight-Queen configuration, a checkmate state S_{mate} can exist if and only if the defending King K_e is forced into a corner square $C \in B_{\text{color}}$ where B_{color} matches the colorbound domain of the Queen.

Proof: Because Queen controls only B_{color} and Knight controls isolated points without linear barrier formation, the combined control fields $C_{MQ}(p_M, p_Q) = M_K(p_M) \cup M_Q(p_Q)$ cannot construct a closed boundary surrounding K_e in open space or in a opposite-color corner $c' \in B_{\text{color}}$ (Safe Corner). Thus, S_{mate} is strictly restricted to $c \in B_{\text{color}}$ (Vulnerable Corner). Under the board symmetry group D_4 , only 2 out of 4 corners qualify as terminal conditions.

Theorem 2 (Barrier Construction for King - Bishop - Queen): The combined control field $C_{BQ}(p_B, p_Q) = M_B(p_B) \cup M_Q(p_Q)$ can form a continuous topological barrier $H \subset B$ such that $B \setminus H$ splits into two disconnected subgraphs.

Proof: Since Bishop spans both B_{white} and B_{black} , Bishop and Queen can occupy adjacent, complementary-color squares to block all linear and diagonal exit paths of K_e . This dual-color coverage allows the attacking side to systematically shrink the defending King's reachable component $G[V_{\text{reachable}}]$ to any of the 4 corner regions (D_4 invariance), regardless of corner color.

Consequently, while King-Knight-Queen fails to confine K_e in more than 96% of positions due to color-bound limitations and "Safe Corner" escape topologies, King-Bishop-Queen achieves complete board-wide containment, resulting in an 85.77% win rate. However, when the 44-count rule is imposed, 46,122 positions exceeding the 44-count limit

are excluded, reducing the effective win rate to 85.39%.

Computational Complexity Bounds for Generalized ASEAN Chess Endgames

To establish the theoretical limits of the Retrograde Analysis algorithm, we analyze the computational complexity bounds for generalized ASEAN Chess endgames played on an $N \times N$ board with k active pieces. The endgame state space is modeled as a directed graph $G = (V, E)$, where vertices V represent unique board states and edges E represent valid piece transitions (moves).

1) State Space Complexity (Space Bound)

For an $N \times N$ board containing k distinct pieces, a naive upper bound for the number of possible piece placements is given by permutations of N^2 squares taken k at a time. Therefore, the absolute number of vertices $|V|$ (or states) is bounded by:

$$|V| \leq ((N^2)! / (N^2 - k)!) \leq (N^2)^k = N^{2k}$$

which establishes that $|V| = O(N^{2k})$

In this study, $N = 8$ and $k = 3$ (two attacking pieces and one defending King) or $k = 4$ (e.g., $K+N+Q$ vs K). While the incorporation of D_4 / D_2 symmetries reduces the practical state space by a constant factor, the asymptotic space complexity of $O(N^{2k})$ is polynomial with respect to the board dimension N (for a fixed k), but strictly exponential with respect to the number of pieces k (with base N^2 , i.e., $O((N^2)^k)$).

For fixed-piece endgames where k is constant, the state space scales polynomially with board dimension N as $O(N^{2k})$, rendering the transition graph fully resolvable within polynomial bounds of $|V|$. The broader classification into PSPACE applies strictly when k scales with N (e.g., $k = O(N)$) under polynomial move limits, of which our analyzed configuration constitutes a computationally tractable subspace.

2) Time Complexity Bound

The Retrograde Analysis algorithm operates by mapping the state space into a comprehensive 1D array index and systematically traversing backward from terminal checkmate vertices to unresolved parent vertices.

Let b denote the maximum branching factor (the total number of legal transitions available from any state $v \in V$). For a system with k pieces on an $N \times N$ board, each piece possesses at most $O(N)$ legal moves, yielding a collective branching factor of $b = O(k \cdot N)$. When the piece count k is fixed as a constant (as in this study), $b = O(N)$. Consequently, the total number of directed edges in the state transition graph satisfies $|E| \leq b \cdot |V| = O(k \cdot N \cdot N^{2k}) = O(N^{2k+1})$ under a fixed k . Since our Dynamic Programming Breadth-First Search (BFS) backward traversal processes each reachable vertex and edge at most once, the overall time complexity is bounded by $O(|V| + |E|) = O(N^{2k+1})$.

3) Theoretical Implications

The established $O(|V| + |E|)$ time bound confirms that the Retrograde Analysis algorithm is strictly linear with respect to the state graph size, proving it to be the most asymptotically optimal exhaustive search approach for exact Distance-to-Mate (DTM) resolution. However, the state space bound of $O(N^{2k})$ demonstrates the rapid onset of state space explosion. While endgame tablebases remain computationally tractable for small piece counts ($k \leq 5$) on a standard 8×8 board via symmetric 1D array indexing, the generalized game on an $N \times N$ board with an arbitrary piece configuration is EXPTIME-complete. Consequently, full-board game play across large state spaces inherently necessitates heuristic search approximations over exhaustive, deterministic DTM tablebases.

Validation of Results

To ensure the accuracy of the generated endgame tablebases, the researchers established a verification process involving both mathematical and logical validation. The verification process was divided into three main steps as follows:

1) Mathematical State Space Verification

The researchers verified the total number of states generated by the program against combinatorial calculations for the 64-square chessboard. Illegal states were excluded, such as positions in which the two kings are adjacent to each other or in which the defending king is under attack by a piece of the attacking side.

The verification results showed that the number of legal positions for both endgame configurations exactly matched the theoretical calculations, yielding 11,930,016 and 12,170,304 positions, respectively.

2) Backward Logical Consistency Check

The tablebases were tested for the logical consistency of the Distance-to-Mate (DTM) values using an algorithm that examined every state (S) in the tablebase. If a state (S) had a DTM value of (n), where ($n > 0$), the algorithm verified that among all possible legal moves, there was at least one move leading to a child state with a DTM value of ($n - 1$).

The verification results confirmed that the DTM structure was 100% logically consistent, with no inconsistent or contradictory states identified.

3) Empirical Payout Validation Using Random Positions

The researchers randomly sampled 1,000 initial positions from the tablebases and performed game simulations. Both sides were instructed to play using the optimal strategy based on the DTM values stored in the tablebases (perfect play).

The simulation results showed that, in every case, the attacking side was able to achieve checkmate within the exact number of moves specified by the tablebases. This provides empirical confirmation of the correctness of the generated endgame tablebases.

Conclusion and Recommendations

Conclusions

This study successfully constructed and analyzed Endgame Tablebases (EGTBs) for two critical ASEAN Chess endgame configurations: King-Knight-Queen (K-N-Q) and King-Bishop-Queen (K-B-Q). Through the application of retrograde analysis combined with state-space reduction via symmetry principles, definitive game-theoretic evaluations were obtained for more than 24 million legal positions.

The computational results reveal a pronounced disparity in piece potency between the two configurations. The Knight-Queen endgame exhibits strong defensive resilience, resulting in a dominant theoretical draw rate of 96.04%. In contrast, the Bishop-Queen configuration demonstrates substantial offensive effectiveness, achieving a theoretical forced win rate of 85.77%.

One of the principal contributions of this research is the empirical assessment of the ASEAN Chess Counting Rules. The findings confirm that the 44-count rule functions as a critical regulatory constraint, converting 278,040 theoretically winning positions into practical draws. Consequently, the effective winning probability of the Bishop-Queen endgame is reduced to 85.39%, underscoring the divergence between theoretical optimal play and

outcomes constrained by competitive regulations.

The EGTBs developed in this work establish a rigorous analytical foundation for future studies in ASEAN Chess. These tablebases may be integrated into high-performance chess engines, employed as authoritative references for instructional and training systems, and utilized as benchmarks for further research in combinatorial game theory. Future research may extend this methodology to more complex endgames involving five or more pieces, thereby deepening the computational and theoretical understanding of ASEAN Chess.

Recommendations

For future research, it is recommended that the proposed computational framework be extended to the analysis of more complex five-piece endgame configurations, such as King and two Bishops versus King and one Bishop, in order to further explore and map the mathematical landscape of ASEAN Chess endgames.

In addition, the integration of these endgame tablebases into mobile learning applications presents a promising direction for practical deployment. Such applications could serve as real-time pedagogical tools, enabling players to enhance their endgame precision while developing a deeper understanding of optimal play and the strategic implications of the piece-counting rules.

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Appendix A

Maximum DTM Analysis for K-N-Q vs. K and K-B-Q vs. K

This appendix presents the most complex winning positions identified through an exhaustive exploration of the ASEAN Chess state space. These positions, referred to as the deepest positions, correspond to the upper bounds of the Distance to Mate (DTM), in which the attacking side requires the maximum number of moves to achieve checkmate under optimal defensive play. As shown in Figures 9 - 10

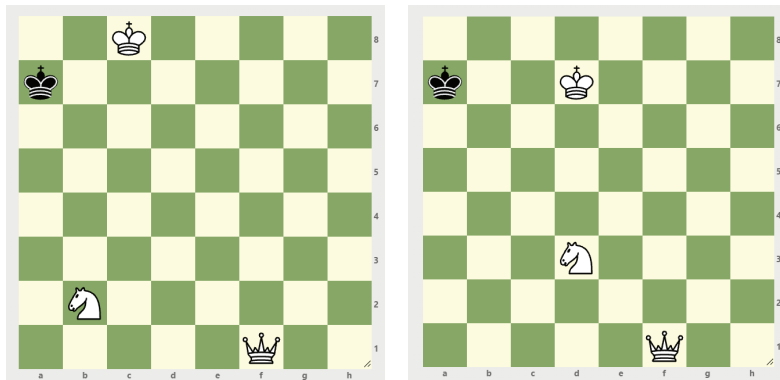


Figure 9 Two Representative Positions from 48

K-N-Q vs. K Endgame Cases, with White to Move under Optimal Play, Resulting in the Maximum Distance to Mate of 36 Moves (DTM = 36)

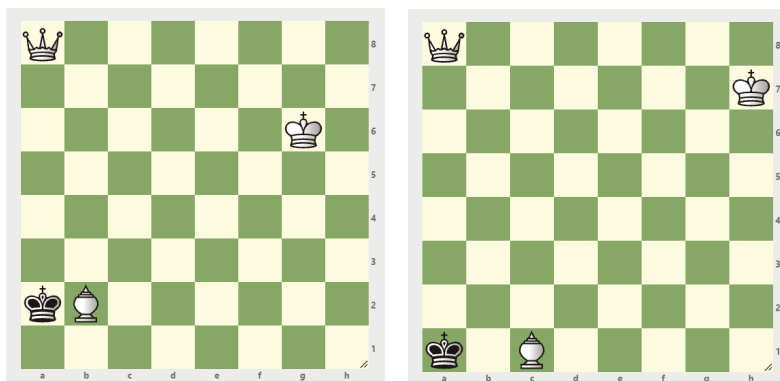


Figure 10 Two Representative Positions from 8

K-B-Q vs. K Endgame Cases, with White to Move under Optimal Play, Resulting in the Maximum Distance to Mate of 57 Moves (DTM = 57)

Appendix B

Representative Practical and Educational Endgame Positions

This appendix presents four representative endgame positions likely to arise in competitive ASEAN Chess play, as shown in Figures 11 - 14. These positions serve as pedagogical benchmarks, illustrating the coordination required in King-Knight-Queen and King-Bishop-Queen configurations to force checkmate under the official counting rules. The positions vary in structural complexity and demonstrate optimal maneuvering against a lone King.

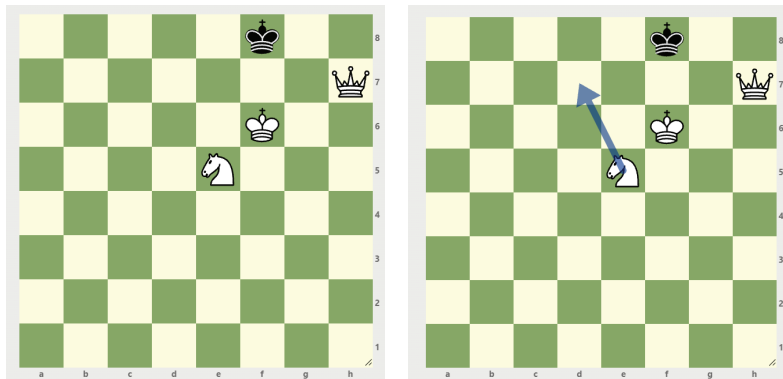


Figure 11 Representative Winning Position in the Knight-Queen Endgame with White to Move (DTM = 19)

Position 1: K-N-Q vs. K

FEN String: "5k2/7Q/5K2/4N3/8/8/8/8 w - - 0 1"

Distance to Mate: 19 Moves

White to move: 1.Nd7+ Ke8 2.Ke6 Kd8 3.Kd6 Kc8 4.Nc5 Kd8 5.Qg6 Ke8 6.Nd7 Kd8 7.Qf7 Kc8 8.Nc5 Kd8 9.Nb7+ Kc8 10.Kc6 Kb8 11.Kb6 Kc8 12.Qe6 Kb8 13.Qd7 Ka8 14.Na5 Kb8 15.Nc6+ Ka8 16.Ne7 Kb8 17.Qc6 Ka8 18.Qb7+ Kb8 19.Nc6#

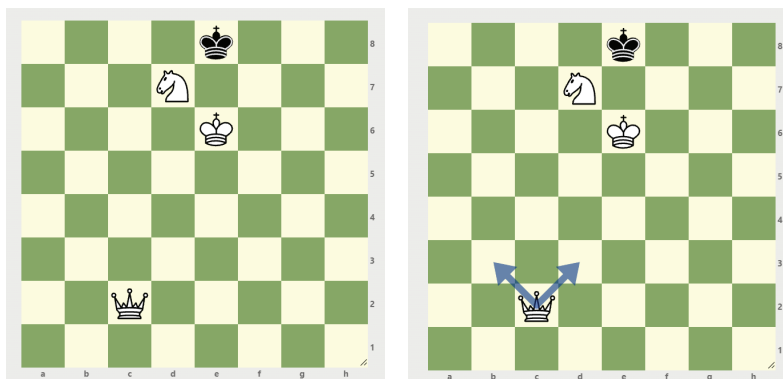


Figure 12 Representative Winning Position in the Knight-Queen Endgame with White to Move (DTM = 23)

Position 2: K-N-Q vs. K

FEN String: "4k3/3N4/4K3/8/8/8/2Q5/8 w - - 0 1"

Distance to Mate: 23 Moves

White to move: 1.Qd3 Kd8 2.Qc4 Kc7 3.Qd5 Kb7 4.Kd6 Ka6 5.Kc5 Kb7 6.Qc6+ Kc7 7.Nf6 Kc8 8.Kd6 Kd8 9.Qb5 Kc8 10.Ke7 Kb7 11.Nd5 Ka7 12.Kd7 Kb8 13.Ne3 Kb7 14.Nc4 Kb8 15.Nd6 Ka8 16.Kc7 Ka7 17.Nc8+ Ka8 18.Ne7 Ka7 19.Nc6+ Ka8 20.Nb4 Ka7 21.Qc6 Ka8 22.Qb7+ Ka7 23.Nc6#

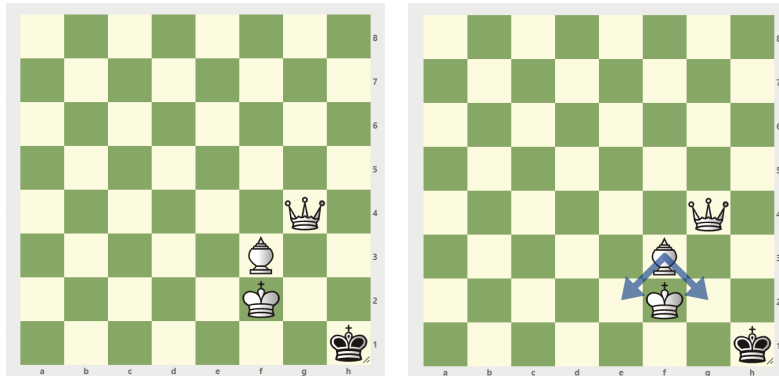


Figure 13 Representative Winning Position in the Bishop-Queen Endgame with White to Move (DTM = 15)

Position 3: K-B-Q vs. K

FEN String: "8/8/8/8/6Q1/5B2/5K2/7k w - - 0 1"

Distance to Mate: 15 Moves

White to move: 1.Bg2+ Kh2 2.Bf1 Kh1 3.Ke3 Kg1 4.Ke2 Kh2 5.Kf3 Kg1 6.Be2 Kh1 7.Kg3 Kg1 8.Qf5 Kh1 9.Kf2 Kh2 10.Qg4 Kh1 11.Bf3 Kh2 12.Bf4 Kh1 13.Qf3 Kh2 14.Bg3+ Kh3 15.Qg2#

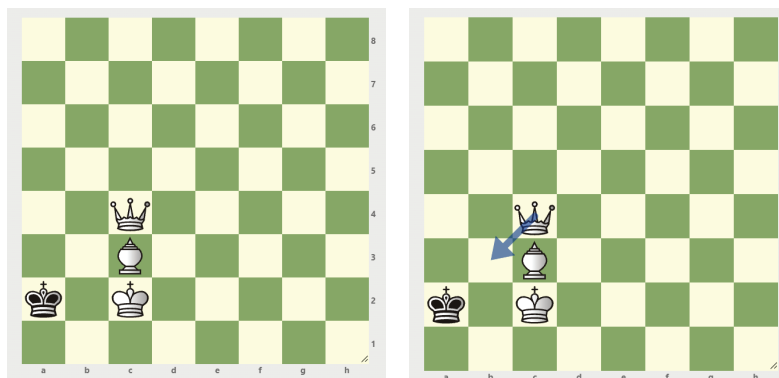


Figure 14 Representative Winning Position in the Bishop-Queen Endgame with White to Move (DTM = 21)

Position 4: K-B-Q vs. K

FEN String: "8/8/8/8/2Q5/2B5/k1K5/8 w - - 0 1"

Distance to Mate: 21 Moves

White to move: 1.Qb3+ Ka3 2.Bc4 Kb4 3.Kb2 Ka5 4.Kc3 Kb6 5.Kd4 Kc6 6.Ke5 Kd7 7.Kf6 Ke8
8.Bc5 Kf8 9.Bd6 Kg8 10.Qc4 Kh7 11.Be5 Kh6 12.Bf4 Kh5 13.Bg3 Kh6 14.Bg4 Kh7 15.Bg5 Kg8
16.Bg6 Kh8 17.Qd5 Kg8 18.Qe6 Kh8 19.Qf5 Kg8 20.Bg7+ Kh7 21.Qg6#