

## Nonlinear dual-excited and steam-valving control of synchronous generators via immersion and invariance

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### Abstract

In this paper, with immersion and invariance (I&I) techniques, a nonlinear dual-excited and steam-valving control of synchronous generators is proposed for the transient stability and voltage regulation enhancement of an electrical power system after the occurrence of a large disturbance. The proposed nonlinear controller is used to not only achieve power angle stability, frequency and voltage regulation but also ensure that the closed-loop system is transiently and asymptotically stable. In order to show the effectiveness of the proposed controller design, the simulation results illustrate that the proposed controller can not only keep the system transiently stable but also simultaneously accomplish the system stability improvement and voltage regulation following temporary faults and permanent faults.

**Keywords:** Dual-excited synchronous generator; steam-valving control; transient stability; Immersion and Invariance methodology.

### บทคัดย่อ

บทความนี้เสนอ การควบคุมที่มีการกระตุ้นสองชุดและวาล์วไอน้ำที่ไม่เป็นเส้นของเครื่องกำเนิดไฟฟ้าเชิงโรตอร์ด้วยวิธีการฝังและความเข้มแข็งเพื่อเพิ่มเสถียรภาพชั่วคราวและการควบคุมแรงดันไฟฟ้าเมื่อมีสัญญาณรบกวนขนาดใหญ่บนสายส่งไฟฟ้า ตัวควบคุมที่ไม่เป็นเชิงเส้นจากวิธีการฝังและความเข้มแข็งถูกนำมาใช้เพื่อบรรลุเสถียรภาพของมุมกำลังและการควบคุมความถี่และแรงดันพร้อมทั้งรับประกันได้ว่า ระบบวงรอบปิดมีเสถียรภาพแบบชั่วคราวและแบบเชิงเส้นกำกับ เพื่อแสดงถึงประสิทธิภาพของการออกแบบตัวควบคุม ผลการจำลองด้วยคอมพิวเตอร์แสดงให้เห็นว่า ตัวควบคุมที่นำเสนอสามารถทำให้ระบบมีเสถียรภาพแบบชั่วคราว และยังสามารถบรรลุเสถียรภาพของมุมกำลังพร้อมทั้งการควบคุมความถี่และแรงดันของระบบไฟฟ้ากำลังภายใต้สัญญาณรบกวนที่เกิดขึ้นชั่วคราวและถาวร

**คำสำคัญ:** Dual-excited synchronous generator; steam-valving control; transient stability; Immersion and Invariance methodology.

### 1. Introduction

Due to power systems with the rapid increase of the size and complexity, power system stability, including power angle stability along with frequency and voltage regulation, is of great importance. In general, power system operation is faced with the difficult task of maintaining stability when small or large disturbances occur in the power system. Therefore, the excitation control (Bazanella & Conceicao, 2004; Dib, Kenne, & Lamnabhi-Lagarigue, 2009; Galaz, Ortega, Bazanella, & Stankovic, 2003; Lu, Sun, & Wei, 2001; Paul & Gerardo, 2004) and steam-valving control (Jian, Chen, Liu, Liu, & Jing, 2011; Lu, Sun & Wei, 2001; Sun & Zhao, 2010) recently

become two main ways to not only further stabilize and control dynamic models of generators, but also achieve the desired control objectives.

From references above, the excitation control and steam-valving control have independently been employed to not only improve power system operations but also provide an opportunity for system stability improvement of the power system. In order to further improve the system stability- particularly transient stability of power systems, the coordination of generator excitation and steam-valving control has attracted much attention in literature for years. Wang & Mao (2009) have shown that with the help of differential geometry theory and variable structure

control theory, a nonlinear variable structure single-excited and steam-valving control can provide satisfactory dynamic performance and good robustness. Guo, Hill, & Wang (2000) have shown that a nonlinear decentralized robust control (both single excitation and steam valve control loop) is able to make the closed-loop systems transiently stable when a fault occurs, and is robust with regard to uncertain network parameters - attenuating the persistent disturbances. A nonlinear single-excitation control of synchronous generators with steam valve control (Li & Wang, 2006) has been proposed to attenuate external disturbances and deal with unknown parameters via a Hamiltonian function methodology. Very recently, based on the Hamiltonian function methodology, a family of robust adaptive single-excited and steam-valving control (Xu, & Hou, 2012) for synchronous generators has been proposed and can offer a degree of freedom to accomplish further desired control performances. Besides, a number of nonlinear control design techniques for the single-excited and steam-valving control of synchronous generators in literature have been proposed to stabilize and control dynamic models of power systems, and provide additional benefits beyond single-excited controller or steam-valving controller. However, in the references above, note that the d-axis field voltage is assumed to be constant, resulting in reducing the fourth-order dynamic system into a third-order dynamic system. Therefore, in order to offer greater flexibility the combination of d-axis and q-axis field voltages (called dual-excitation) is a promising way to improve system stability. Moreover, the dual excitation of synchronous generators has d-axis and q-axis field windings and each field voltage can be independently adjusted, leading to more flexibility for achieving control objectives.

So far, both linear and nonlinear controllers (Aggarwal, & Hogg, 2012; Daniels, & Lee, 1976; Huang, Tu, & Chen, 1997; Wang, & Lin, 2011) of dual-excited synchronous generators have been developed to improve the power system stabilization and dynamic performance.

To the best of our knowledge, relatively little prior work has been devoted to the combination of dual-excited and steam-valving control of synchronous generators. In Chen, Ji, Wang, & Xi (2006), using coordinated passivation techniques the stability enhancement for single-machine power systems has been investigated and

shown to provide better dynamic performance than a feedback linearization methodology.

This paper continues this line of investigation and particularly extends our work reported in Kanchanaharuthai (2013) by using the concept of immersion and invariance (I&I) technique to design a nonlinear control law for system stability and voltage regulation enhancement of a Single-Machine Infinite Bus (SMIB) power system. The I&I technique is based on selecting a target dynamical system that can capture the desired behavior of the closed-loop system to be controlled. The control objective of this method is to find a stabilizing control law not only to ensure that the closed-loop system behaves asymptotically the same as the pre-specified target system, i.e. achieve asymptotic model matching, but also to achieve the desired closed-loop system performance requirements in Section 2. However, in our prior work, there is still a drawback in control design procedure due to practically immeasurable variables, particularly two generator transient voltage sources that are employed in the resulting control law. The I&I controller proposed in this paper is based upon all measurable state variables and designed to not only simultaneously achieve power angle stability along with frequency and voltage regulation, but also keep the system transiently stable. Simulation results are provided for an SMIB with a Synchronous Generator (SG), including both dual-excited and steam-valving control connected to the infinite bus.

The rest of this paper is organized as follows. In Section 2, the problem formulation is provided. In Section 3, power system models considered is briefly given. The I&I design methodology used to construct a nonlinear control law is stated in Section 4. In Section 5, simulation results are given while we conclude in Section 6.

## 2. Problem statement

In this paper we are interested in studying the transient stability of a nonlinear power system including generator dual excitation and steam-valving controller. The nonlinear system considered can be written in the general form as

$$\dot{x}(t) = f(x) + g(x)u(x), \quad (1)$$

where  $x \in \mathbb{R}^n$  is the state variable,  $u \in \mathbb{R}^m$  is the control action, and  $f(x)$  and  $g(x)$  are assumed to be smooth functions.

The problem of interest in this paper is the following: given a stable equilibrium point  $x_e$ , find a stabilizing state feedback controller law  $u(x)$  so that the closed-loop system satisfies

1. The desired equilibrium point  $x_e$  is asymptotically and transiently stable.
2. Power angle stability, voltage and frequency regulation are simultaneously achieved.

In the next section, we provide simplified non-linear models of power system elements and use these to design a state feedback control law that meets the requirements given above.

### 3. Power system models

In this section, the dynamic models of the synchronous generator are briefly provided. A dynamic model of a synchronous generator (SG) can be obtained as

$$\begin{aligned}\dot{\delta} &= \omega - \omega_s \\ \dot{\omega} &= \frac{1}{M} (P_m - P_e - D(\omega - \omega_s)) \\ \dot{P}_m &= -\frac{P_m - P_{me}}{T_{H\Sigma}} + \frac{C_H}{T_{H\Sigma}} u_G \\ \dot{E}'_q &= -\frac{X_{d\Sigma}}{X'_{d\Sigma} T'_{d0}} E'_q + \frac{(X_{d\Sigma} - X'_{d\Sigma})}{X'_{d\Sigma} T'_{d0}} V_\infty \cos \delta + \frac{u_{fd}}{T'_{d0}} \\ \dot{E}'_d &= -\frac{X_{q\Sigma}}{X'_{q\Sigma} T'_{q0}} E'_d - \frac{(X_{q\Sigma} - X'_{q\Sigma})}{X'_{q\Sigma} T'_{q0}} V_\infty \sin \delta + \frac{u_{fq}}{T'_{q0}}\end{aligned}\quad (2)$$

with

$$\begin{aligned}P_e &= \frac{E'_q V_\infty \sin \delta}{X'_{d\Sigma}} + \frac{E'_d V_\infty \cos \delta}{X'_{q\Sigma}} \\ &\quad + \frac{(X'_{d\Sigma} - X'_{q\Sigma}) V_\infty^2 \sin 2\delta}{2X'_{d\Sigma} X'_{q\Sigma}},\end{aligned}$$

where  $\delta$  is the power angle of the generator,  $\omega$  denotes the relative speed of the generator,  $D \geq 0$  is a damping constant,  $E'_q$  and  $E'_d$  is the q-axis, and d-axis internal transient voltages, respectively.  $X'_{d\Sigma}$  and  $X'_{q\Sigma}$  are the d-axis and q-axis transient reactances, respectively.  $P_e$  is the electrical power delivered by the generator to the voltage at the infinite bus  $V_\infty$ ,  $\omega_s$  is the

synchronous machine speed,  $\omega_s = 2\pi f$ ,  $H$  represents the per unit inertial constant,  $f$  is the system frequency and  $M = 2H / \omega_s$ .

$X'_{d\Sigma} = X'_d + X_T + X_L$  is the reactance consisting of the direct axis transient reactance of SG, the reactance of the transformer  $X_T$ , and the reactance of the transmission line  $X_L$ . Similarly,

$X_{d\Sigma} = X_d + X_T + X_L$  is identical to  $X'_{d\Sigma}$  except that  $X_d$  denotes the direct axis

reactance of SG.  $T'_{d0}$  and  $T'_{q0}$  are the d-axis and q-axis transient short-circuit time constants.  $u_{fd}$

and  $u_{fq}$  are the d-axis and q-axis field voltage

control inputs, respectively.  $P_{me}$  is the initial

value of mechanical power,  $C_H$  is the assigned

coefficient of high-pressure cylinder.  $T_{H\Sigma}$  is the

equivalent time constant of steam valve control

systems.  $u_G$  is the steam-valving control input.

(See Lu, Sun, & Wei (2001) for further details).

In practice, the generator transient

voltages  $E'_d$  and  $E'_q$  are often physically not

measurable while only  $P_m$  is always monitored, so

we consider two new variables, namely,  $P_{eq}$  and

$P_{ed}$  as

$$\begin{aligned}P_e &= P_{eq} + P_{ed}, \\ P_{eq} &= \frac{E'_q V_\infty \sin \delta}{X'_{d\Sigma}} + \frac{(X'_{d\Sigma} - X'_{q\Sigma}) V_\infty^2 \sin 2\delta}{2X'_{d\Sigma} X'_{q\Sigma}}, \\ P_{ed} &= \frac{E'_d V_\infty \cos \delta}{X'_{q\Sigma}}\end{aligned}$$

Differentiating the electrical powers  $P_{eq}$

and  $P_{ed}$ , respectively, in (2) and then defining the

variables

$$x_1 = \delta, x_2 = \omega - \omega_s, x_3 = P_m,$$

$x_4 = P_{eq}, x_5 = P_{ed}$ . The dynamic model of the

power system including dual-excited and steam-valving control can be expressed as the general

form (1) and (4) below.

$$\begin{aligned}
 f(x) &= \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \\ f_5(x) \end{bmatrix} \\
 &= \begin{bmatrix} x_2 \\ \frac{1}{M}(x_3 - x_4 - x_5 - Dx_2) \\ -\frac{1}{T_{H\Sigma}}(x_3 - P_{me}) \\ (-a_q + x_2 \cot x_1)x_4 + \frac{b_q V_\infty \sin 2x_1}{2X'_{d\Sigma}} \\ -(a_d + x_2 \tan x_1)x_5 - \frac{b_d V_\infty \sin 2x_1}{2X'_{q\Sigma}} \end{bmatrix}, \\
 x &= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, u(x) = \begin{bmatrix} \frac{C_H u_f}{T_{H\Sigma}} \\ \frac{u_{fd}}{T'_{d0}} \\ \frac{u_{fq}}{T'_{q0}} \end{bmatrix}, \\
 g(x) &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_3(x) & 0 & 0 \\ 0 & g_4(x) & 0 \\ 0 & 0 & g_5(x) \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \frac{V_\infty \sin x_1}{X'_{d\Sigma}} & 0 \\ 0 & 0 & \frac{V_\infty \cos x_1}{X'_{q\Sigma}} \end{bmatrix},
 \end{aligned}$$

(3)

where

$$\begin{aligned}
 a_q &= \frac{X_{d\Sigma}}{X'_{d\Sigma} T'_{d0}}, a_d = \frac{X_{q\Sigma}}{X'_{q\Sigma} T'_{q0}}, b_q = \frac{(X_{d\Sigma} - X'_{d\Sigma})V_\infty}{X'_{d\Sigma} T'_{d0}}, \\
 b_d &= \frac{(X_{q\Sigma} - X'_{q\Sigma})V_\infty}{X'_{q\Sigma} T'_{q0}}, m = \frac{X'_{d\Sigma} - X'_{q\Sigma}}{2X'_{d\Sigma} X'_{q\Sigma}} V_\infty^2.
 \end{aligned}$$

The region of operation is defined in the set.

$$D = \{x \in S \times \square \times \square \times \square \times \square \mid 0 < x_1 < \frac{\pi}{2}\}.$$

The open loop operating equilibrium is denoted by

$$\begin{aligned}
 x_e &= [x_{1e}, x_{2e}, x_{3e}, x_{4e}, x_{5e}]^T \\
 &= [\delta_e, 0, P_{me}, P_{eqe}, P_{ede}]^T.
 \end{aligned}$$

#### 4. Immersion and invariance

The I&I method for stabilizing nonlinear systems was proposed in Astolfi & Oreta (2003), and further developed as summarized in Astolfi, Karagiannis, & Oreta (2007). The method is based on the notion of invariant manifolds and system immersion. This methodology carries out from transforming the original state of the system  $x(t)$  into two new states, namely  $\xi(t)$  and  $z(t)$ . The dimension of state  $\xi(t)$  becomes strictly less than the dimension of state  $x(t)$ . The new reduced state  $\xi(t)$  is called the target dynamics and the transformation employed to get these states defines the invariant manifold. The state  $z(t)$  is called the off-the-manifold state and complements the dimension of  $\xi(t)$ . The resulting control law is designed to ensure that the original state  $x(t)$  is bounded, that the manifold is rendered invariant, and that the off-line-manifold state  $z(t)$  converges asymptotically to the origin. Besides, the original state  $x(t)$  will converge to a desired equilibrium point with a dynamic behavior converging to that of the target dynamical system. Roughly speaking, the I&I concept relies upon selecting a target dynamical system that is capable of capturing the desired behavior of the closed-loop system to be controlled. This method is applicable to practical control design problems for many types of systems, refer to Astolfi et al. (2007) for further details. For transient stability and voltage

regulation enhancement of power systems with excitation control, see Dib, Kenne, & Lamnabhi-Lagarrigue (2009).

The following results, discovered in those papers, are used to design this proposed nonlinear coordinated controller for power systems including dual-excitation and steam-valving controllers

**Theorem 1:** Consider the nonlinear system<sup>1</sup> (Astolfi & Oreta, 2003; Astolfi et al., 2007)

$$\dot{x}(t) = f(x) + g(x)u(x), \quad (4)$$

with state  $x \in \mathbb{R}^n$  and control input  $u \in \mathbb{R}^m$ , and an assignable equilibrium point  $x_e \in \mathbb{R}^n$  to be stabilized. Let  $s < n$ , and assume that there exist smooth mappings

$$\alpha: \mathbb{R}^s \rightarrow \mathbb{R}^s, \pi: \mathbb{R}^s \rightarrow \mathbb{R}^n, c: \mathbb{R}^n \rightarrow \mathbb{R}^m, \\ \phi: \mathbb{R}^n \rightarrow \mathbb{R}^{n-s}, \varphi: \mathbb{R}^{n \times (n-s)} \rightarrow \mathbb{R}^m,$$

such that the following hold.

**(H1) (Target system)** The system

$$\dot{\xi} = \alpha(\xi), \quad (5)$$

with state  $\xi \in \mathbb{R}^s$ , has an asymptotically stable equilibrium at  $\xi_e \in \mathbb{R}^s$  and  $x_e = \pi(\xi_e)$

**(H2) (Immersion condition)** For all  $\xi \in \mathbb{R}^s$

$$f(\pi(\xi)) + g(\pi(\xi))c(\pi(\xi)) = \frac{\partial \pi(\xi)}{\partial \xi} \alpha(\xi) \quad (6)$$

**(H3) (Implicit manifold)** The following set identity holds.

$$\mathbf{M} := \{x \in \mathbb{R}^n \mid x = \pi(\xi), \exists \xi \in \mathbb{R}^s\} \\ = \{x \in \mathbb{R}^n \mid \phi(x) = 0\} \quad (7)$$

**(H4) (Manifold attractivity and trajectory boundedness)**

All trajectories of the system

$$\dot{z} = \frac{\partial \phi(x)}{\partial x} [f(x) + g(x)\varphi(x, z)], \quad (8) \\ \dot{x} = f(x) + g(x)\varphi(x, z)$$

are bounded and satisfy  $\lim_{t \rightarrow +\infty} z(t) = 0$ .

Then,  $x_e$  is a globally asymptotically stable equilibrium of the closed loop system

$$\dot{x}(t) = f(x) + g(x)\varphi(x, \phi(x)). \quad (9)$$

Taken from Astolfi et al., (2007) Theorem 1 above can be interpreted as follows. Given the nonlinear system (4) and the selected target dynamical systems (5), find if possible, a manifold  $\mathbf{M} = \{x \in \mathbb{R}^n \mid x = \pi(\xi), \xi \in \mathbb{R}^s\}$  that can be rendered invariant and asymptotically stable and for which the restriction of the closed-loop system to  $\mathbf{M}$  is described by the target system (H1). Note that the control input  $u$  that makes the manifold invariant is not unique, it is only uniquely defined on  $\mathbf{M}$ , that is,  $\varphi(\pi(\xi), 0) = c(\pi(\xi))$ . Based on (H4) in order to drive the off-the-manifold coordinate  $z$  to zero and keep the system trajectories bounded, one of all possible controls is selected

#### 4.1 I&I Controller Design

##### 4.1.1 Target system

In order to design a stabilizing controller and verify the condition according to Theorem 1, we start with selecting the target dynamics as the mechanical subsystems (e.g., a simple damped pendulum system)

$$\dot{\xi}_1 = \xi_2, \dot{\xi}_2 = -\frac{\partial V(\xi_1)}{\partial \xi_1} - R(\xi)\xi_2, \quad (10)$$

where  $V(\xi_1)$  and  $R(\xi)$  represent the potential energy and a damping function of the pendulum systems, respectively, both of which are selected. The pendulum system, considered with a stable equilibrium point  $\xi_e = (\xi_{1e}, 0)^T$ , has the potential energy  $V(\xi_1)$  satisfying the two following

assumptions: (i)  $\frac{\partial V(\xi_{1e})}{\partial \xi} = 0$  (ii)  $\frac{\partial^2 V(\xi_{1e})}{\partial^2 \xi} > 0$

and the damping function verifying  $R(\xi_e) \geq 0$

and the energy function  $H(\xi) = \frac{1}{2} \xi_2^2 + V(\xi_1)$ .

It is easy to choose the potential energy  $V(\xi_1)$  along conditions as

$$V(\xi_1) = -\beta \cos \tilde{\xi}_1, \tilde{\xi}_1 = \xi_1 - \xi_{1e}, \exists \beta > 0$$

<sup>1</sup> It is assumed that all functions and mappings are  $\mathbb{C}^\infty$  throughout this paper

#### 4.1.2 Immersion condition

As the desired target system has been selected, a mapping  $\pi: \mathbf{S} \times \square \rightarrow \mathbf{S} \times \square \times \square \times \square$  is determined as follows.

$$\pi(\xi_1, \xi_2) := (\xi_1, \xi_2, \pi_3(\xi), \pi_4(\xi), \pi_5(\xi))^T$$

where  $\pi_3(\xi)$ ,  $\pi_4(\xi)$  and  $\pi_5(\xi)$  are selected. Besides, the condition of Theorem 1 gives the constraints, namely,  $\xi_{ie} = x_{ie}, i=1,2,\dots,5$ . We can choose  $\pi_3(\xi), \pi_4(\xi)$  and  $\pi_5(\xi)$  to satisfy condition (7), especially the second row as shown below.

$$\begin{aligned} & \begin{bmatrix} \xi_2 \\ \frac{1}{M}[\pi_3(\xi) - \pi_4(\xi) - \pi_5(\xi)] - \frac{D}{M}\xi_2 \\ f_3(\xi) \\ f_4(\xi) \\ f_5(\xi) \end{bmatrix} \\ & + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_3(\xi) & 0 & 0 \\ 0 & g_4(\xi) & 0 \\ 0 & 0 & g_5(\xi) \end{bmatrix} \begin{bmatrix} c_1(\pi(\xi)) \\ c_2(\pi(\xi)) \\ c_3(\pi(\xi)) \end{bmatrix} \\ & = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{\partial \pi_3}{\partial \xi_1} & \frac{\partial \pi_3}{\partial \xi_2} \\ \frac{\partial \pi_4}{\partial \xi_1} & \frac{\partial \pi_4}{\partial \xi_2} \\ \frac{\partial \pi_5}{\partial \xi_1} & \frac{\partial \pi_5}{\partial \xi_2} \end{bmatrix} \begin{bmatrix} \xi_2 \\ -\frac{\partial V(\xi_1)}{\partial \xi_1} - R(\xi)\xi_2 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{\partial \pi_3}{\partial \xi_1} & \frac{\partial \pi_3}{\partial \xi_2} \\ \frac{\partial \pi_4}{\partial \xi_1} & \frac{\partial \pi_4}{\partial \xi_2} \\ \frac{\partial \pi_5}{\partial \xi_1} & \frac{\partial \pi_5}{\partial \xi_2} \end{bmatrix} \begin{bmatrix} \xi_2 \\ -\beta \sin \tilde{\xi}_1 - \frac{\gamma_d + D}{M} \xi_2 \end{bmatrix}, \gamma_d \geq 0 \quad (11)$$

From the expression above, in order to simplify our derivations,  $\pi_4(\xi)$  and  $\pi_5(\xi)$  are chosen as follows,  $\pi_4(\xi) = x_{4e} - \gamma_d \xi_2$  and  $\pi_5(\xi) = x_{5e} + \gamma_d \xi_2$ . Consequently, we can compute  $\pi_3(\xi)$  as

$$\pi(\xi) = \beta M \sin \tilde{\xi}_1 + \gamma_d \xi_2 + \pi_4(\xi) + \pi_5(\xi) \quad (12)$$

As the mapping  $\pi(\xi)$  has been chosen, by using some lengthy, but straightforward, calculation from the third to fifth rows, respectively, we have the control input below that renders the manifold  $\mathbf{M}$  invariant.

$$\begin{pmatrix} \frac{C_H u_f}{T_{H\Sigma}}, \frac{u_{fd}}{T_{d0}'}, \frac{u_{fq}}{T_{q0}'} \end{pmatrix}^T = \begin{pmatrix} c_1(\pi(\xi)) \\ c_2(\pi(\xi)) \\ c_3(\pi(\xi)) \end{pmatrix}$$

#### 4.1.3 Implicit Manifold

From the results above, the mapping  $\pi(\xi)$  has been defined and the condition in (8) is verified. It is obvious that the mapping  $\phi(x)$  can be defined as

$$\phi(x) = \begin{pmatrix} x_3 - \pi_3(x_1, x_2) \\ x_4 - \pi_4(x_2) \\ x_5 - \pi_5(x_2) \end{pmatrix}. \quad (13)$$

#### 4.1.4 Manifold attractivity and trajectory boundedness:

In this subsection, a control law  $u = \varphi(x, z)$  is designed to ensure that all trajectories of the closed-loop system are bounded and converge to the manifold  $\mathbf{M}$ . Let  $z := \phi(x)$  be the off-the-line manifold coordinate, substituting  $\dot{x}_3, \dot{x}_4$  and  $\dot{x}_5$  into the expression below we have

$$\begin{aligned}
 \dot{z}_1 &= \dot{x}_3 - \dot{\pi}_3(x_1, x_2) = \frac{-(x_3 - x_{3e})}{T_{H\Sigma}} \\
 &\quad + \frac{C_H \varphi_1(x, z)}{T_{H\Sigma}} - \frac{\partial \pi_3}{\partial x_1} \dot{x}_1 - \frac{\partial \pi_3}{\partial x_2} \dot{x}_2 \\
 \dot{z}_2 &= \dot{x}_4 - \dot{\pi}_4(x_2) \\
 &= -f_4(x, z) + \frac{\varphi_2(x, z)}{T'_{d0}} - \frac{\partial \pi_4}{\partial x_2} \dot{x}_2 \\
 \dot{z}_3 &= \dot{x}_5 - \dot{\pi}_5(x_2) \\
 &= -f_5(x, z) + \frac{\varphi_3(x, z)}{T'_{q0}} - \frac{\partial \pi_5}{\partial x_2} \dot{x}_2
 \end{aligned}$$

In order to ensure that the trajectories of the off-the-manifold coordinate  $z$  are bounded and  $\lim_{t \rightarrow +\infty} z(t) = 0$  according to condition (10), we

take  $\dot{z}_i = -\gamma_i z_i, \gamma_i > 0, i = 1, 2$ . and then we get

$$\begin{aligned}
 \frac{C_H \varphi_1(x, z)}{T_{H\Sigma}} &= \frac{(x_3 - x_{3e})}{T_{H\Sigma}} + \frac{\partial \pi_3}{\partial x_1} \dot{x}_1 + \frac{\partial \pi_3}{\partial x_2} \dot{x}_2 \\
 &\quad - \gamma_1 z_1, \\
 \frac{\varphi_2(x, z)}{T'_{d0}} &= -f_4(x, z) + \frac{\partial \pi_4}{\partial x_2} \dot{x}_2 - \gamma_2 z_2 \\
 \frac{\varphi_3(x, z)}{T'_{q0}} &= -f_5(x, z) + \frac{\partial \pi_5}{\partial x_2} \dot{x}_2 - \gamma_3 z_3.
 \end{aligned}$$

#### 4.1.5 The control law:

We can compute the control laws as follows:

$$\begin{aligned}
 \frac{C_H u_G}{T_{H\Sigma}} &= \frac{C_H \varphi_1(x, \phi(x))}{T_{H\Sigma}} \\
 &= \frac{(x_3 - x_{3e})}{T_{H\Sigma}} + \frac{\partial \pi_3}{\partial x_1} \dot{x}_1 + \frac{\partial \pi_3}{\partial x_2} \dot{x}_2 \\
 &\quad - \gamma_1 (x_3 - \pi_3(x_1, x_2)), \\
 \frac{u_{fd}}{T'_{d0}} &= \frac{\varphi_2(x, \phi(x))}{T'_{d0}} = \frac{X'_{d\Sigma}}{V_\infty \sin x_1} \\
 &\quad \times \left[ -f_4(x) + \frac{\partial \pi_4}{\partial x_2} \dot{x}_2 - \gamma_2 (x_4 - \pi_4(x_2)) \right],
 \end{aligned}$$

$$\begin{aligned}
 \frac{u_{fq}}{T'_{q0}} &= \frac{\varphi_3(x, \phi(x))}{T'_{q0}} = \frac{X'_{q\Sigma}}{V_\infty \cos x_1} \\
 &\quad \times \left[ -f_5(x) + \frac{\partial \pi_5}{\partial x_2} \dot{x}_2 - \gamma_3 (x_5 - \pi_5(x_2)) \right],
 \end{aligned} \tag{14}$$

where  $\frac{\partial \pi_3}{\partial x_1} = -M \beta \cos(x_1 - x_{1e}), \frac{\partial \pi_3}{\partial x_2} = \gamma_d$  and

$$\frac{\partial \pi_4}{\partial x_2} = -\frac{\partial \pi_5}{\partial x_2} = -\gamma_d.$$

According to the condition (H4), it is also necessary to prove boundedness of the trajectories of the closed-loop system with the control law  $\varphi_i(x, \phi(x)), i = 1, 2$  and the off-the-manifold coordinate  $z$  as given by

$$\begin{aligned}
 \dot{x}_1 &= x_2, \\
 \dot{x}_2 &= \frac{1}{M} (x_3 - x_4 - x_5 - D x_2), \\
 \dot{x}_3 &= -\frac{1}{T_{H\Sigma}} (x_3 - x_{3e}) + \frac{C_H u_G}{T_{H\Sigma}}, \\
 \dot{x}_4 &= f_4(x) + g_4(x) \frac{u_{fd}}{T'_{d0}}, \\
 \dot{x}_5 &= f_5(x) + g_5(x) \frac{u_{fq}}{T'_{q0}}, \\
 \dot{z}_i &= -\gamma_i z_i, \quad i = 1, 2, 3.
 \end{aligned} \tag{15}$$

We begin with the fact that clearly  $x_1 \in \mathbb{S}$  is bounded and  $z_1, z_2$  and  $z_3$  are exponentially decaying functions- that is,  $z_i(t) = z_i(0)e^{-\gamma_i t}$ ,  $i = 1, 2, 3$  and also bounded. It follows that  $x_4 = z_2 + \pi_4(\xi_2)$ , and  $x_5 = z_3 + \pi_5(\xi_2)$  are bounded. Substituting  $x_3 = z_1 + \pi_3(x_1, x_2), x_4 = z_2 + \pi_4(x_2)$  and  $x_5 = z_3 + \pi_5(x_2)$  into the second equation of (15) and using the energy function  $W(x, z) = \frac{1}{2} x_2^2 + V(x_1) + \frac{1}{2} (z_1^2 + z_2^2 + z_3^2)$ ,

we have

$$\begin{aligned}
 \dot{W} &= -\frac{D+\gamma_d}{M}x_2^2 - \frac{x_2(z_1-z_2-z_3)}{M} - \gamma_1z_1^2 \\
 &\quad - \gamma_2z_2^2 - \gamma_3z_3^2, \\
 &\leq -\frac{D+\gamma_d}{M}x_2^2 + \frac{|x_2||z_1|}{M} - \frac{|x_2||z_2|}{M} \\
 &\quad - \frac{|x_2||z_3|}{M} - \gamma_1z_1^2 - \gamma_2z_2^2 - \gamma_3z_3^2, \\
 &\leq -\frac{D+\gamma_d+2}{M}x_2^2 + \Delta(z_1, z_2, z_3),
 \end{aligned}$$

where

$$\begin{aligned}
 \Delta(z_1, z_2, z_3) &= -\left(\gamma_1 - \frac{1}{2M}\right)z_1^2 \\
 &\quad - \left(\gamma_2 + \frac{1}{2M}\right)z_2^2 - \left(\gamma_3 + \frac{1}{2M}\right)z_3^2
 \end{aligned}$$

and the first inequality follows from Young's inequality, i.e.  $2ab \leq ca^2 + \frac{b^2}{c}$  to eventually

obtain the final inequality. From the last inequality above,  $\gamma_1, \gamma_2, \gamma_3$  are positive and  $D \geq 0, \gamma_d \geq 0$ ,

it can be seen that there exists a time  $t_f$  such that

$\Delta(z_1, z_2, z_3) = 0$ , for all  $t \geq t_f$  and eventually we

have  $\dot{W} \leq -\frac{(D+\gamma_d+2)}{M}x_2^2 \leq 0$ . Therefore, there

exists a ball around the operating equilibrium, strictly contained in  $D$  such that all trajectories starting in this set satisfy

$H(x_1, x_2) \leq H(x_1(0), x_2(0))$  - thus resulting in

boundedness of  $(x_1, x_2)$ . This also implies

boundedness of  $\pi_3(x_1, x_2)$ . Finally, boundedness

of  $x_3, x_4$  and  $x_5$  follows the fact that

$$x_3 = z_1 +$$

$$\pi_3(x_1, x_2), x_4 = z_2 + \pi_4(x_2), x_5 = z_3 + \pi_5(x_2)$$

Hence, boundedness of the trajectories of (15) and

$\lim_{t \rightarrow +\infty} z(t) = 0$  have been shown. We can establish

the main result summarizing the proposed I&I controller design in the following proposition.

**Proposition 1:** The closed-loop system (15) with the control laws (14) is locally asymptotically stable in  $x_e$

*Proof:* The proof of proposition 1 is based on the arguments as given above in (10)-(13).

## 5. Simulation results

In this section, simulation results from coordination of generator dual excitation and steam-valving control in SMIB power systems considered in previous sections are shown using power angle stability as well as voltage and frequency regulations to point out the transient stability enhancement and dynamic properties.

Consider the single line diagram as shown in Figure 1 with SG connected through parallel transmission line to an infinite-bus. Such generators deliver 1.2749 pu., power while the terminal voltage  $V_t$  is 1.0475 pu., and an infinite-bus voltage is 1.0 per unit. However, when there is a three-phase fault (a large perturbation) occurring at the point  $P$ , the midpoint of one of the transmission lines, this leads to rotor acceleration, voltage sag, and large transient-induced electro-mechanical oscillations.

We are, therefore, interested in the following question. After the fault is cleared from the network, will the system return to a post-fault equilibrium state?

In this paper, the following two fault sequences are of interest, namely temporary and permanent faults. Usually, there are four basic stages associated with transient stability studies (temporary and permanent faults) of a power system: (i) The system is in a pre-fault steady state (ii) A fault occurs at  $t_0$  (iii) The fault is isolated by opening the breakers at  $t_c$  (iv) The transmission line is recovered without the fault at  $t = t_r$  sec. Eventually, the system is in a post-fault state at  $t = t_f$  sec.

In this paper, two cases with different faults sequences are investigated as follows:

### Temporary fault

The system is in a pre-fault steady state, a fault occurs at  $t_0 = 0.5$  sec., the fault is isolated by opening the breaker of the faulted line at  $t_c = 1$  sec., the transmission line is recovered without the fault at  $t_r = 1.5$  sec. Afterward the system is in a post-fault state.

### Permanent fault

The system is in a pre-fault steady state, a fault occurs at  $t_0 = 0.5$  sec., the fault is isolated by permanently opening the breaker of the faulted

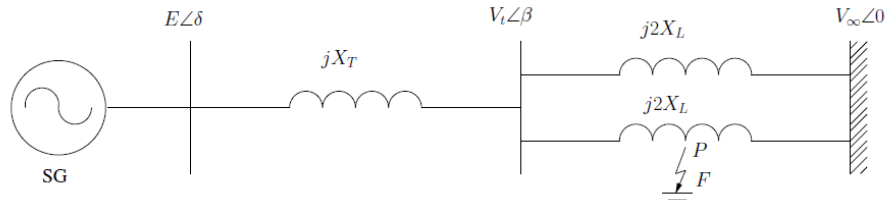


line at  $t_c = 1$  sec. The system is eventually in a post-fault state.

In this section, we investigate the effectiveness of the coordinated (dual-excitation/steam-valving control) controller for improving transient stability of a power system

through power angle stability, voltage, frequency, and power regulations.

These results are also compared with the existing nonlinear controller, namely the coordinated passivation controller (Chen, Ji, Wang, & Xi, 2006)



**Figure 1** A single line diagram of SMIB

The physical parameters (pu) and initial conditions  $(\delta_e, \omega_s, P_{me}, P_{eqe}, P_{ede})$  for this proposed power system model are given as.

$\omega_s = 2\pi f$  rad/s,  $D = 5$ ,  $H = 5$ ,  $f = 60$  Hz,  $V_\infty = 1\angle 0^\circ$ ,  $T'_{d0} = 10$ ,  $T'_{q0} = 4$ ,  $X_q = 1.6$ ,  $X'_d = 0.38$ ,  $X_d = 1.6$ ,  $X'_d = 0.23$ ,  $X_T = 0.13$ ,  $X_L = 0.17$ ,  $T_{H\Sigma} = 0.4$ ,  $\delta_e = 0.3445$  rad,  $\omega = \omega_s$ ,  $P_{me} = 1.2749$  pu,  $E'_{qe} = 1.0703$ ,  $E'_{de} = 0.522$ ,  $V_t = V_{ref} = 1.0475$  pu.

The tuning parameters of the coordinated controller selected to test in this paper are  $\gamma_1 = \gamma_2 = \gamma_3 = 100$ ,  $\beta = 1000$ , and  $\gamma_d = 0.5$ . From our simulation results, the following can be seen.

The transient stability of a power system with both generator dual excitation and steam-valving control can be effectively enhanced by using the nonlinear coordinated controller proposed as shown in Figures 2 and 3. Although there is a large sudden fault (temporary or permanent) on the network, the system is able to keep transiently stable.

Time trajectories of a power angle  $\delta$ , SG relative speed (frequency)  $\omega - \omega_s$ , the mechanical power  $P_m$  of the proposed controller and the coordinated passivation controller, respectively, are shown in Figures 2(a) and 3(a). After the fault is cleared from the network, from two fault cases above the power angles  $\delta \rightarrow \delta_e$ , and the SG relative speeds,  $\omega - \omega_s \rightarrow 0$ , and the mechanical power,  $P_m \rightarrow P_{me}$  settle to the pre-fault steady state as expected. Note also that, due

to the presence of the permanent fault on the network, power angle, SG relative speed, and the mechanical power of the I&I controller and the coordinated controller can go to the pre-fault state. In comparison with the coordinated passivation controller, time histories of the coordinated (dual excitation/steam-valving control) controller, particularly power angles and relative speeds, have obviously smaller overshoot along with faster reduction of oscillation. Regarding power and voltage regulation as shown in Figure 2(b) and 3(b), the coordinated controller provides clearly better transient responses over the coordinated passivation controller and quickly settles to their pre-fault steady state of active power. In particular, the voltage sag of the proposed controller is quickly stabilized in comparison with the coordinated passivation controller in terms of settling time, rise time, and smaller overshoots. Their voltage responses also return to the desired reference voltage  $V_{ref}$ , except for the permanent fault case, where there is a change on network structure  $X_2$ . Figure 2(c) illustrates time histories of off-the-manifold coordinates  $z_1$ ,  $z_2$  and  $z_3$ , showing the manifold  $\mathbf{M}$  implicitly described by  $\phi(x) = 0$ . These results indicate that the combination of dual excitation and steam-valving control can obviously further improve transient stability along with dynamic properties when compared to the coordinated passivation controller. In other words, in the permanent fault, Figures 3(a)-(b) illustrate power angle, SG relative speed, and the mechanical power can return to the pre-fault state excluding terminal voltage  $V_t$ . Also, the off-the-

manifold coordinates  $z_1$ ,  $z_2$  and  $z_3$  converge to zero, as expected. Independent of the steady-state operating point of the system and fault sequences above, the nonlinear coordinated controller can achieve the expected requirements and accomplish better dynamic properties (dynamic properties) of the closed-loop systems under a large sudden fault. From the simulation results above, it can be concluded that not only transient stability is enhanced but also power angle stability as well as frequency, power, and voltage regulations are simultaneously accomplished according to two expected requirements for the proposed controller.

## 6. Conclusions

In this paper, a nonlinear dual excitation and steam-valving controller, used to enhance the transient stability of a power system, has been proposed using I&I methodology. The proposed controller obtained in this work was further extended from the work of Kanchanaharuthai (2013) where some measurable states variables can be known. Simulation results have demonstrated that power angle stability along with voltage and frequency regulations are fulfilled by following the large (transient) disturbances on the network via I&I nonlinear model-based control design methodology. In particular, in spite of the occurrence of severe disturbances on the transmission line, the proposed coordinated controller proposed can not only maintain the transient stability but also accomplish better dynamic properties of the system than a coordinated passivation controller.

In this work, we are interested in the design of a stabilizing controller for the power system with dual-excited and steam-valving controller for SGs connected to an infinite bus. Usually, a large interconnected system can be represented by the infinite bus when its voltage and frequency remain constant under all conditions. Even though in a large-scale power system there many generators, it is often possible to reduce the power system to a set of machines of interest connected through an equivalent network (Thevenin equivalent circuit) as shown in Figure 1. Unfortunately, if the reduced order power system is not an adequate representation of the power system for transient stability studies, then we can extend the earlier results to multi-machine systems by following an idea from Dib, Ortega, & Hill

(2013). Also, throughout our results, we proposed a stabilizing control law for the SMIB power system that is lossless (for example, without transfer conductances between buses). Consequently, to further increase the efficacy of the resulting controller for successful applications in large-scale electric power systems, the nonlinear controller for power systems that are lossy should be developed. Also, many important control problems such as disturbance attenuation and adaptive stabilization problems for uncertain nonlinear power systems should be considered.

## 7. Acknowledgements

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## 8. Appendix

A. Coordinated Passivation Controller (Chen, Ji, Wang, & Xi, 2006)

For simplicity, let us define the state variable by

$$\tilde{x}_1 = \delta - \delta_e, \tilde{x}_2 = \omega - \omega_s, \tilde{x}_3 = P_m - P_{me},$$

$$y_1 = P_{eq} - P_{eqe}, y_2 = P_{eq} - P_{eqe} \text{ then the power}$$

systems considered in (3) is output strictly passive under the coordinated passivation controller are

$$\frac{C_H}{T_{H\Sigma}} u_G = \frac{\tilde{x}_3}{T_{H\Sigma}} - \frac{\omega_s}{H} (\tilde{x}_2 + c_1 \tilde{x}_1) - c_3 (\tilde{x}_3 - \alpha(\tilde{x}_1, \tilde{x}_2)) + \frac{\partial \alpha}{\partial \tilde{x}_1} \tilde{x}_2 + \frac{\partial \alpha}{\partial \tilde{x}_2} \dot{\tilde{x}}_2$$

$$\frac{V_\infty \sin \tilde{x}_1}{X_{d\Sigma}'} \frac{u_{fd}}{T_{d0}'} = -f_4(\tilde{x}, y) + \frac{2H}{\omega_s} (\tilde{x}_2 + c_1 \tilde{x}_1) + v_1$$

$$\frac{V_\infty \cos \tilde{x}_1}{X_{q\Sigma}'} \frac{u_{fq}}{T_{q0}'} = -f_5(\tilde{x}, y) + \frac{2H}{\omega_s} (\tilde{x}_2 + c_1 \tilde{x}_1) + v_2$$

where

$$\alpha(\tilde{x}_1, \tilde{x}_2) = -P_{me} - \frac{H}{\omega_s} (c_2 (\tilde{x}_2 + c_1 \tilde{x}_1) + \tilde{x}_1) - \frac{H}{\omega_s} \left( \left( c_1 - \frac{D}{H} \right) \tilde{x}_2 - P_{eqe} - P_{ede} \right)$$

and in this paper tuning parameters were chosen as  $c_1 = c_2 = c_3 = 20$ . Further, if  $v_i = -\beta_i y_i$

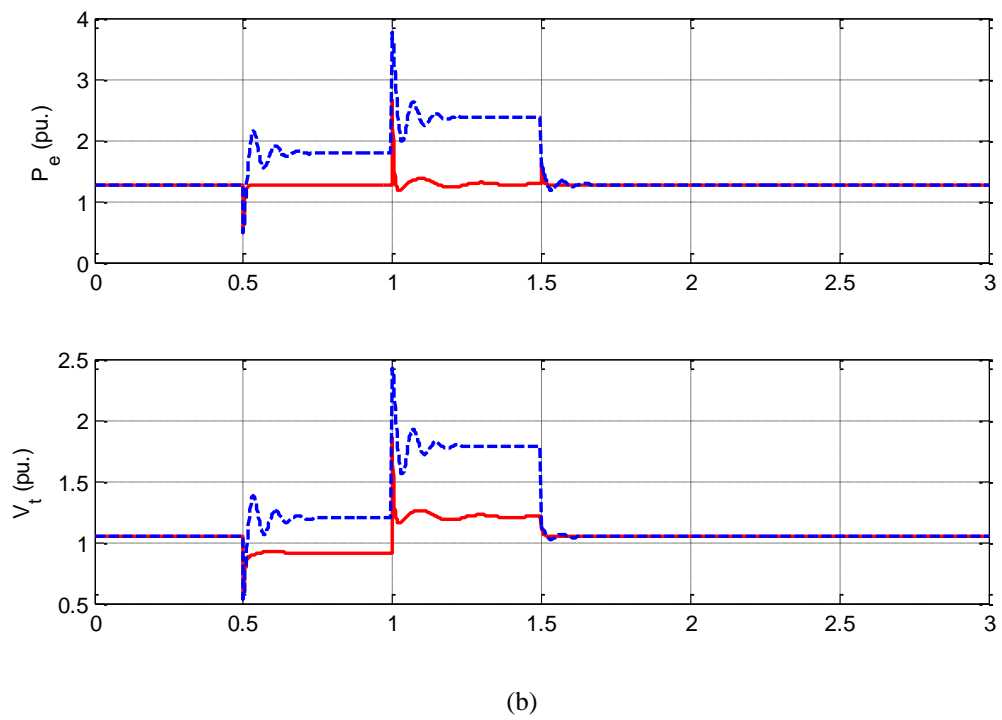
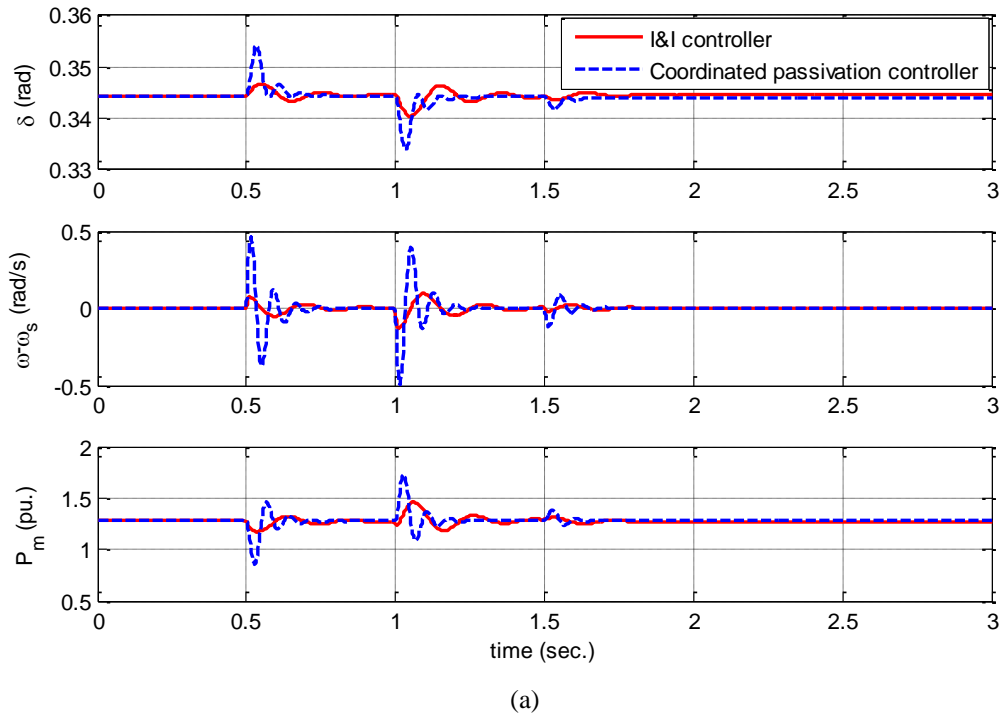
$(\beta_i > 0, i=1,2)$  and  $c_1 + c_2 + c_3 > D/H$ , the closed-loop system is asymptotically stable.

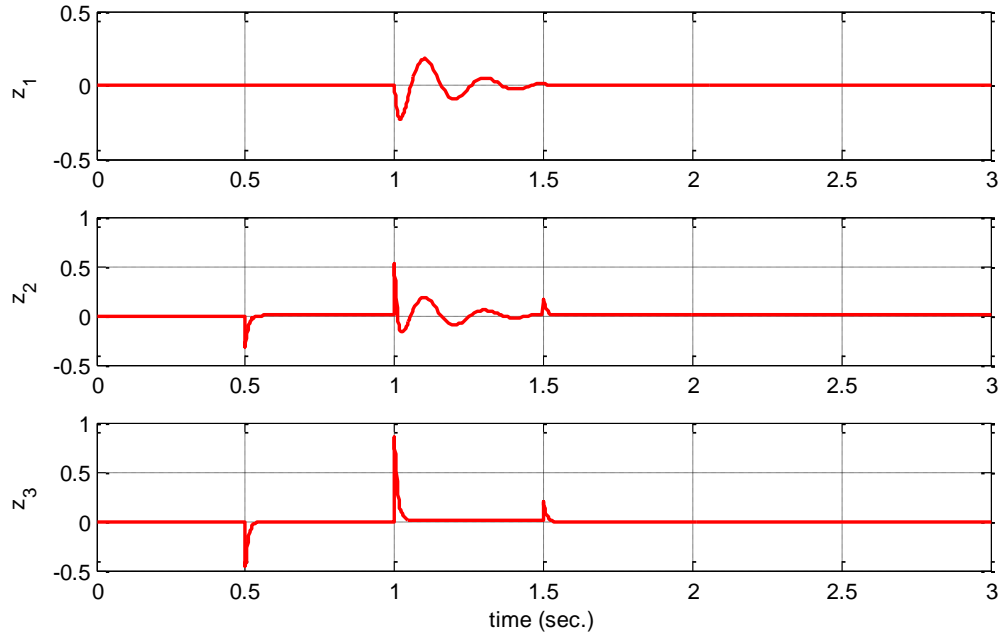
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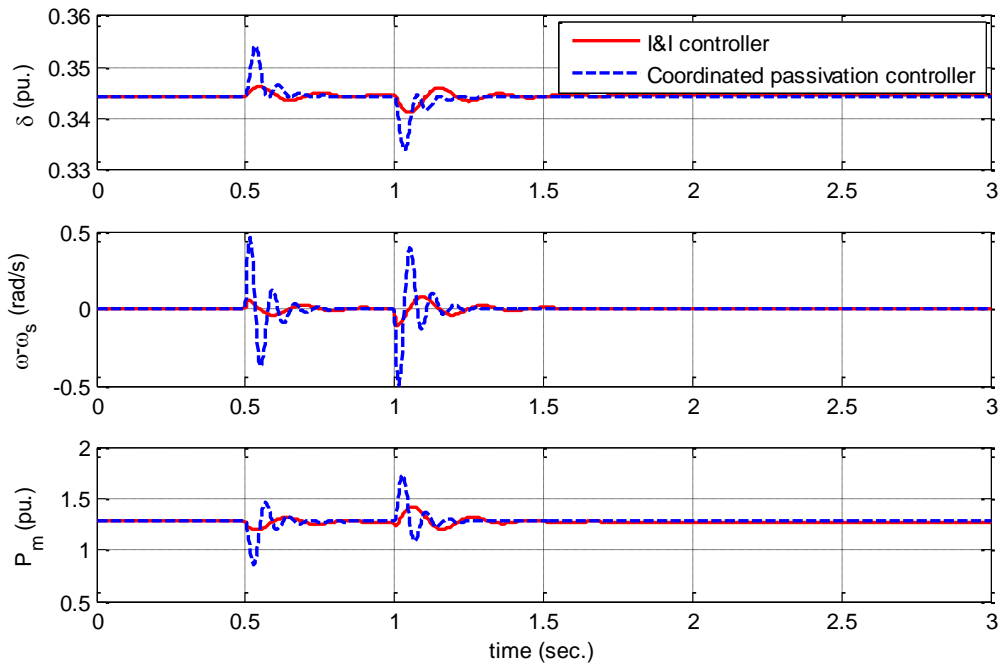
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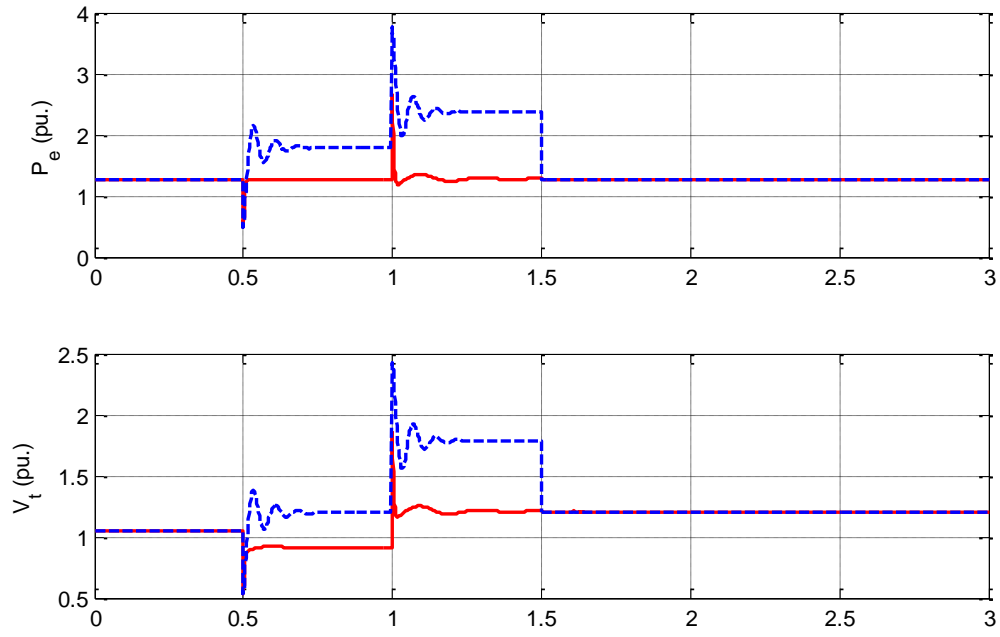


(c)

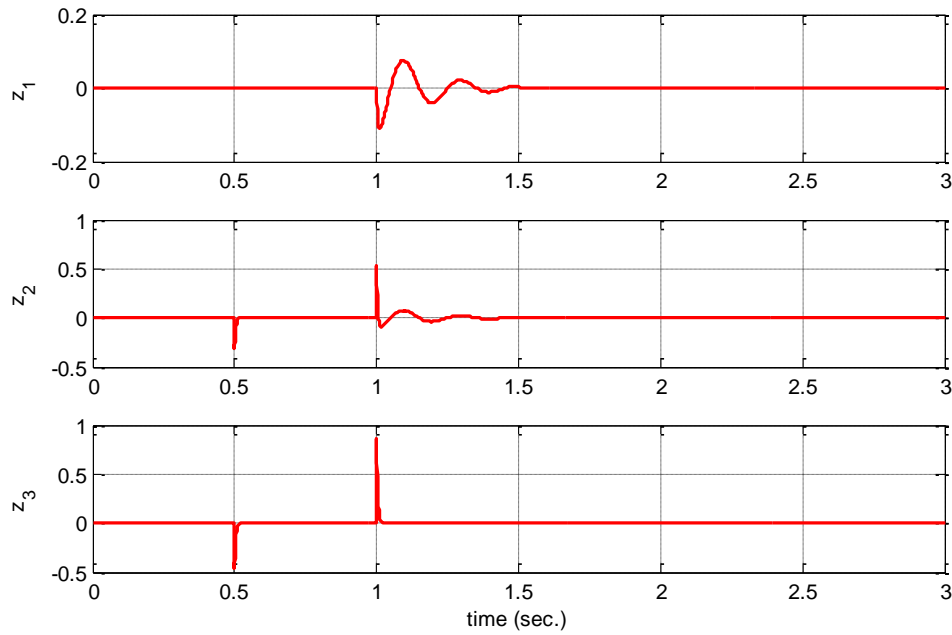
**Figure 2** Temporal fault case: Time histories of (a) power angle ( $\delta$ ), relative speed ( $\omega - \omega_s$ ) and mechanical power voltage ( $P_m$ ), (b) active power ( $P_e = P_{eq} + P_{ed}$ ) and terminal voltage ( $V_t$ ), (c) the off-the-line manifold coordinates  $z_1, z_2$  and  $z_3$ .



(a)



(b)



(c)

**Figure 3** Permanent fault case: Time histories of (a) power angle ( $\delta$ ), relative speed ( $\omega - \omega_s$ ) and mechanical power voltage ( $P_m$ ), (b) active power ( $P_e = P_{eq} + P_{ed}$ ) and terminal voltage ( $V_t$ ), (c) the off-the-line manifold coordinates  $z_1, z_2$  and  $z_3$