Thermal impact response of a thermoelastic solid with a finite crack

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Abstract

A transient stress analysis for the problems of a thermoelastic medium containing a finite crack experiencing a sudden change in temperature over the surface of the crack is studied employing coupled thermoelasticity theory. By assuming that heat is suddenly transferred across the surface of a crack according to Newton's Law of Cooling and using Laplace and Fourier's method of integral transforms, the problem is reduced to a system of coupled dual integral equations. Further application of Tranter's method, in expressing unknown functions as an infinite series of Bessel functions, reduces the equations to an infinite set of linear algebraic equations whose solution in the Laplace transform domain is inverted numerically to yield the values of the dynamic stress-intensity factor, $k_1(t)$. The results reveal the significant influence of inertia but negligible coupling effects.

Keywords: *coupled thermoelasticity, transient, Newton's law, temperature, dynamic stress-intensity factor, finite crack*

1. Introduction

In past decades, the failure of brittle materials containing initial defects under dynamic loadings has been a main concern to many investigators (Kassir & Sih, 1975; Chen & Sih, 1977). The process leading to catastrophic failure in structural components may occur suddenly, or over a period of time, depending on the magnitude of the local stress field around the crack tip which is measured by the stress intensity-factor. Under rapidly applied thermal loading and fluctuating thermal disturbance, the stress intensity factor can be considerably amplified, and in many instances may trigger crack extension and eventual failure even before initial yielding takes place. Although thermal problems have been solved by many investigators in the past, most of them are usually based on conventional treatment neglecting the inertia term in the equation of motion and the thermoelastic coupling term in the heat conduction equation. This hypothesis of a quasi-static process is known to yield useful results without significant errors in practical engineering applications; however, when a temperature field exhibits steep time-gradients, the inertia and coupling effects may be significant and the coupled theory of thermoelasticity should be employed (Sternberg & Chakravorty, 1959; Takeuti & Furukawa, 1981).

Several transient and steady-state problems have been solved in the past using the coupled theory of thermoelasticity (Boley & Weiner, 1960; Chadwick, 1960). This theory takes into account the interdependence of temperature and displacement fields in which one can not exist without the other. Transient problems involving applied normal stress and/or heat flux across the surfaces of a finite crack was solved by Kassir, Phurkhao and Bandyopadhyay (1986). Their approach was to use the Fourier and the Laplace transforms to reduce the problem to a standard Fredholm integral equation whose solution in a regular time domain was numerically inverted to yield a dynamics stress-intensity factor. Georgiadis, Brock and Rigatos (1998) employed Green's function for cracked surfaces under a pair of line heat sources using coupled thermoelastodynamic equations. Recently, a transient problem concerning a penny-shaped crack was investigated by Ampunsuk and Phurkhao (2005) by utilizing the Hankel and Laplace transforms. The reported results reveal similar increases in the dynamic stress intensity-factor, rising to the peak quickly and then decreasing to the static value while, the coupling has a negligible effect.

Phurkhao and Kassir (1991) investigated the diffraction problems of thermally-induced thermo-elastic waves by a line of a finite crack using the integral transform technique and obtained the frequency-dependent dynamic stress intensity factor. In a recent paper, Phurkhao (2010) employed the same technique to solve an analogous problem concerning the propagation of thermoelastic waves induced by mechanical excitation.

While a considerable amount of work has been done on the topic of thermal impact, to the

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best of the author's knowledge, there are no solutions for an elastic medium with a finite crack being subjected to a sudden change in temperature over the crack surfaces when taking the inertia and coupling effects into consideration.

2. Objectives

The purpose of this study is to determine the transient response of a thermoelastic solid containing a finite crack flaw using the theory of coupled thermoelasticity. The main focus is the determination of the stress-intensity factor, the influence of inertia and the coupling effect when the crack experiences an abrupt change in temperature and the transfer of heat across the cracked surface according to Newton's law of cooling. The Laplace and Fourier integral transforms (Sneddon, 1951) are employed to reduce the mixed-boundary value problem to a system of dual integral equations (Sneddon, 1966). Furthermore, to facilitate the numerical work, the system of dual integral equations are then transformed to an infinite set of linear algebraic equations by the application of Tranter's method (Tranter, 1956).

Two typical materials, lead and copper, with high and low coupling constants, respectively, are selected for numerical computation and determination of the coupling effect inherent in the theory. The state of plane strain is assumed in this investigation.

3. Materials and methods

Neglecting body forces and internal heat sources, the governing equations of the linear coupled theory of thermoelasticity for isotropic elastic material in the non-dimensional space and time variables $(x, y, z; t)$ are (Boley and Weiner, 1960; Chadwick, 1960)

$$\beta^2\nabla^2\mathbf{u}+(1-\beta^2)\nabla e-\nabla\theta=\ddot{\mathbf{u}}, \tag{1}$$

$$\nabla^2\theta-\dot{\theta}-\varepsilon\dot{e}=0\ , \tag{2}$$

where, a dot over a function indicates differentiation with respect to the non-dimensional time t. The symbols $\boldsymbol{\nabla}$ and ∇^2 designate the usual del and Laplacian operators; β stands for the ratio of the isothermal velocities of the shear (c_2) and the dilatational waves (c_1) in the medium, and ε denotes the coupling constant. They are given as

$$\left.\begin{aligned} &c_1=\sqrt{(\lambda+2\mu)/\rho}, \qquad c_2=\sqrt{\mu/\rho},\\ &\varepsilon=\frac{(3\lambda+2\mu)^2\alpha^2T_0}{(\lambda+2\mu)\rho c_\varepsilon}. \end{aligned}\right\} \tag{3}$$

Here, α and c_ε denote, respectively, the coefficient of linear thermal expansion and the specific heat at constant volume. λ and μ represent the Lame's elastic constants, and ρ is the mass density of the medium. Moreover, T_0 and e designate the reference stress-free temperature (absolute) and the dilatation of the medium, respectively.

Equations (1) and (2) relate the normalized displacement vector $\mathbf{u}=(u_x,u_y,u_z)$ to the normalized temperature θ. The actual space and time variables $(x',y',z';t')$, the displacement vector $\mathbf{U}=(U_x,U_y,U_z)$ and the temperature T are related to the dimensionless variables by the following relations

$$\left.\begin{aligned} &x=c_1x'/\kappa, \quad y=c_1y'/\kappa, \quad z=c_1z'/\kappa,\\ &t=c_1^2t'/\kappa, \quad \theta=\left(T-T_0\right)/T_0,\\ &\mathbf{u}=\left(c_1/\kappa\right)\mathbf{U},\ \ e=\operatorname{div}\mathbf{u}, \end{aligned}\right\} \tag{4}$$

where, $\kappa(=k/\rho c_\varepsilon)$ and $T-T_0$ denotes, respectively, diffusivity **a**nd the deviation of temperature from the reference T_0.

In order to determine the equations governing the propagation of the dilatational and shear waves in the xy-plane under the state of plane strain, it is necessary to decompose the displacement vector $\mathbf{u}$ into two potential functions ϕ and ψ in the following form

$$u_x=\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial y}, \quad u_y=\frac{\partial\phi}{\partial y}-\frac{\partial\psi}{\partial x}, \quad u_z=0\ , \tag{5}$$

and insert them into (1) and (2). It follows that the potentials ϕ and θ associated with the dilatational waves are governed by

$$\nabla^2\phi-\theta=\ddot{\phi}, \tag{6}$$

$$\nabla^2\theta-\dot{\theta}-\varepsilon\nabla^2\dot{\phi}=0\,; \tag{7}$$

while, $\beta^2\nabla^2\psi=\ddot{\psi}$ (8)

governs the shear wave.

The non-dimensional stress components (normalized by μ), in particular, the components σ_y and τ_{xy} can be easily expressed in terms of the potentials ϕ, ψ and θ by

$$\sigma_y=\nabla^2\phi-2\beta^2\frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial y}\right)-\theta\ , \tag{9}$$

$$\tau_{xy}=\beta^2\left(2\frac{\partial^2\phi}{\partial x\partial y}+\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2\psi}{\partial x^2}\right). \tag{10}$$

In addition, the equations (6)-(8) must be supplemented by the specified boundary and initial conditions.

In this study, it is assumed that a finite crack of actual length $2a'$ situated in an infinite elastic medium along the x-axis from $-a'$ to $+a'$ with $a(=c_1a'/\kappa)$ being the normalized crack length, initially at rest, undeformed and at a reference temperature T_0, is subjected to a transfer of heat across the crack surfaces due to a sudden change in temperature θ_0. Considering the symmetry of the problem with respect to the x-axis, it is, therefore, sufficient to consider the problem within the region $y \geq 0$. The appropriate boundary conditions on $y = 0$ plane and $t > 0$ are

$$\tau_{xy} = 0, \quad \text{all } x, \tag{11}$$

$$\sigma_y = 0, \quad |x| \leq a, \tag{12}$$

$$u_y = 0, \quad |x| > a, \tag{13}$$

$$\frac{\partial \theta}{\partial y} + h\left[\theta - \theta_0 H(t)\right] = 0, \quad |x| \leq a, \tag{14}$$

$$\frac{\partial \theta}{\partial y} = 0, \quad |x| > a. \tag{15}$$

In (14), $H(t)$ denotes the Heaviside step function; while, $h(=\kappa h'/c_1 k)$ stands for the normalized coefficient of surface heat transfer (Carslaw & Jaeger, 1959). In addition to (11)-(15), all components of displacements, stresses and temperature fields must vanish at remote distances from the crack region.

Assuming initially all components of stresses, displacements and temperature are zero, application of the Laplace transform pair

$$\left.\begin{aligned} f^*(p) &= \int_0^\infty f(t)e^{-pt}dt \\ f(t) &= \frac{1}{2\pi i}\int_{Br} f^*(p)e^{pt}dp \end{aligned}\right\} \tag{16}$$

to (6)-(8) and (11)-(15) yields the governing equations

$$\nabla^2\phi^* - \theta^* = p^2\phi^*, \tag{17}$$

$$\nabla^2\theta^* - p\theta^* - \varepsilon p \nabla^2\phi^* = 0, \tag{18}$$

$$\beta^2\nabla^2\psi^* = p^2\psi^*. \tag{19}$$

and the boundary conditions

$$\tau^*_{xy} = 0, \quad \text{all } x \tag{20}$$

$$\sigma^*_y = 0, \quad |x| \leq a \tag{21}$$

$$u^*_y = 0, \quad |x| > a \tag{22}$$

$$\frac{\partial \theta^*}{\partial y} + h\left[\theta^* - \theta_0/p\right] = 0, \quad |x| \leq a \tag{23}$$

$$\frac{\partial \theta^*}{\partial y} = 0, \quad |x| > a \tag{24}$$

in the Laplace transform plane.

Equations (17)-(19) admit the following integrals bounded for large y

$$\phi^* = \frac{2}{\pi}\int_0^\infty \left\{A_1 e^{-\gamma_1 y} + A_2 e^{-\gamma_2 y}\right\}\cos(sx)ds \tag{25}$$

$$\theta^* = \frac{2}{\pi}\int_0^\infty \left\{\left(\varsigma_1^2 - p^2\right)A_1 e^{-\gamma_1 y} + \left(\varsigma_2^2 - p^2\right)A_2 e^{-\gamma_2 y}\right\}\cos(sx)ds \tag{26}$$

$$\psi^* = \frac{2}{\pi}\int_0^\infty A_3 e^{-\gamma_3 y}\sin(sx)ds. \tag{27}$$

Here, $A_j = A_j(s,p), j = 1,2,3$ are unknown transform parameters, and the exponents $\gamma_j, j = 1,2,3$ are defined by

$$\gamma_j = (s^2 + \varsigma_j^2)^{1/2} \geq 0, \quad 0 < s < \infty \tag{28}$$

$$\varsigma_{1,2}^2 = \left[1 + p + \varepsilon \pm \sqrt{(1+p+\varepsilon)^2 - 4p}\right]p/2, \tag{29}$$

$$\varsigma_3^2 = p^2/\beta^2. \tag{30}$$

Now, condition (20) in conjunction with (10), (25) and (27) are readily shown to imply that

$$A_3 = \frac{-2s(\gamma_1 A_1 + \gamma_2 A_2)}{2s^2 + \varsigma_3^2}. \tag{31}$$

The remaining boundary conditions, namely, (21)-(24) yields a set of simultaneous dual integral equations governing the remaining unknown functions A_1 and A_2

$$\int_0^\infty \left\{a_{11}A_1 + a_{12}A_2\right\}\cos(sx)ds = 0, \quad 0 \leq x \leq 1, \tag{32}$$

$$\int_0^\infty \left\{a_{21}A_1 + a_{22}A_2\right\}\cos(sx)ds = \pi h\theta_0/2p, \quad 0 \leq x \leq 1, \tag{33}$$

$$\int_0^\infty \left\{b_{11}A_1 + b_{12}A_2\right\}\cos(sx)ds = 0, \quad x > 1, \tag{34}$$

$$\int_0^\infty \left\{b_{21}A_1 + b_{22}A_2\right\}\cos(sx)ds = 0, \quad x > 1. \tag{35}$$

Here, for simplicity, the normalized crack dimension a has been taken as a unity; while, a_{ij} and b_{ij} being functions of s and p are given in the Appendix.

To proceed toward establishment of the system of simultaneous dual integral equations in the form suitable for determination of the solution, the following abbreviations are introduced

$$V_1(s,p) = \sqrt{s}\left\{b_{11}A_1 + b_{12}A_2\right\}, \tag{36}$$

$$V_2(s,p) = \sqrt{s}\left\{b_{21}A_1 + b_{22}A_2\right\}, \tag{37}$$

in conjunction with the identity (Magnus & Oberhettinger, 1949)

$$\cos(sx) = \sqrt{\frac{\pi sx}{2}}J_{-1/2}(sx) \tag{38}$$

to reduce the equations (32)-(35) to

$$\int_0^\infty \left\{c_{11}V_1 + c_{12}V_2\right\}J_{-1/2}(sx)ds = 0, \quad 0 \leq x \leq 1, \tag{39}$$

$$\int_0^\infty \{c_{21}V_1 + c_{22}V_2\} J_{-1/2}(sx)ds = \frac{h\theta_0}{p}\sqrt{\frac{\pi}{2x}}\ , 0 \le x \le 1, \tag{40}$$

$$\int_0^\infty V_1 J_{-1/2}(sx)ds = 0\,, \qquad x > 1\,, \tag{41}$$

$$\int_0^\infty V_2 J_{-1/2}(sx)ds = 0\,, \qquad x > 1\,. \tag{42}$$

Here, $J_{-1/2}$ is the usual Bessel function of the first kind of order -1/2, and $c_{ij} = c_{ij}(s, p)$ for $i, j = 1, 2$ are known functions given in the Appendix.

Following the method outlined by Tranter (1956) and Erdogan and Bahar (1964), the unknown functions $V_1(s, p)$ and $V_2(s, p)$ will then be represented in the form

$$V_1 = \sqrt{\frac{\pi}{2}} h\theta_0 \frac{s^{1-\beta_1}}{p^3} \sum_{m=0}^{\infty} \chi_{1,2m+1}(p) J_{-\frac{1}{2}+2m+\beta_1}(s)\,, \tag{43}$$

$$V_2 = \sqrt{\frac{\pi}{2}} h\theta_0 \frac{s^{1-\beta_2}}{p^3} \sum_{m=0}^{\infty} \chi_{2,2m+1}(p) J_{-\frac{1}{2}+2m+\beta_2}(s)\,. \tag{44}$$

It follows that equations (41) and (42) are automatically satisfied when use is made of the Sonine-Schafheitlin integral property (Magnus & Oberhettinger, 1949), i.e.,

$$\int_0^\infty s^{1-\beta_i} J_{k_i}(s) J_{-1/2}(sx)ds = 0\,, \quad x > 1\,, \tag{45}$$

$$\mathrm{Re}(2m+1) > 0 \quad k_i = -1/2 + 2m + \beta_i \quad \beta_i > 0$$

In (43) and (44), $\chi_{i,l}(p), i = 1, 2$ are unknown coefficients and $\beta_i, i = 1, 2$ are parameters which must be chosen so as to make the involved integrals exist.

Upon substitution of (43) and (44) into the remaining equations (39) and (40) and interchanging the order of summation and integration yields

$$\sum_{j=1}^{2}\sum_{m=0}^{\infty} \chi_{j,l}(p) \int_0^\infty s^{1-\beta_j} c_{1j}(s, p) J_{k_j}(s) J_{-1/2}(sx)ds = 0\,, \tag{46}$$

$$\sum_{j=1}^{2}\sum_{m=0}^{\infty} \chi_{j,l}(p) \int_0^\infty s^{1-\beta_j} c_{2j}(s, p) J_{k_j}(s) J_{-1/2}(sx)ds = p^2 x^{-1/2}, \quad k_j = -1/2 + 2m + \beta_j. \tag{47}$$

Further, equations (46) and (47) are then multiplied, respectively, by

$$x^{1/2}(1-x^2)^{\beta_1 - 1}\Im_k(-1/2 + \beta_1, 1/2, x^2)$$

and

$$x^{1/2}(1-x^2)^{\beta_2 - 1}\Im_k(-1/2 + \beta_2, 1/2, x^2)\,,$$

$k = 0, 1, 2, ..$

with $\Im_k$ being the Jacobi polynomial, and integrated with respect to x from 0 to 1. The resulting equations, when the following properties (Tranter, 1956) are utilized

$$\int_0^\infty x^{1/2}(1-x^2)^{\beta_i - 1}\Im_k(-1/2 + \beta_i, 1/2, x^2) J_{-1/2}(sx)dx = \frac{2^{\beta_i - 1}\Gamma(1/2)\Gamma(k+\beta_i)}{\Gamma(k+1/2)} s^{-\beta_i} J_{-1/2+2k+\beta_i}(s). \tag{48}$$

$$\left.\begin{aligned} \sum_{m=0}^{\infty} \chi_{1,l}(p) \int_0^\infty J_r(s) J_l(s) \frac{ds}{s} &= \frac{\chi_{1,r}(p)}{2r} \\ \sum_{m=0}^{\infty} \chi_{2,l}(p) \int_0^\infty J_{r-1/2}(s) J_{l-1/2}(s) \frac{ds}{s} &= \frac{\chi_{2,r}(p)}{(2r-1)} \end{aligned}\right\} \tag{49}$$

with $\beta_1 = 3/2$ and $\beta_2 = 1$, assume the forms

$$\frac{\chi_{1,r}(p)}{2r} + \sum_{j=1}^{2}\sum_{m=0}^{\infty} \chi_{j,l}(p) K_{1j}(p) = B_{1,r}(p)\,, \tag{50}$$

$$\frac{\chi_{2,r}(p)}{2r-1} + \sum_{j=1}^{2}\sum_{m=0}^{\infty} \chi_{j,l}(p) K_{2j}(p) = B_{2,r}(p)\,, \tag{51}$$

$$k = 0, 1, 2, \ldots,\ r = 2k+1,\ l = 2m+1$$

where

$$\left.\begin{aligned} B_{1,r}(p) &= 0, \qquad r = 1, 3, 5, ... \\ B_{2,r}(p) &= \frac{\Gamma(k+1/2)p^2}{\Gamma(1/2)\Gamma(k+1)} \int_0^1 \Im_k(1/2, 1/2, x^2)dx \\ &= \begin{cases} 0, & r > 1 \\ p^2, & r = 1 \end{cases} \end{aligned}\right\} \quad 52)$$

and

$$\left.\begin{aligned} K_{11}(p) &= \int_0^\infty s^{-1}\left[s^{-1}c_{11}(s, p) - 1\right] J_l(s) J_r(s)ds \\ K_{12}(p) &= \int_0^\infty s^{-3/2} c_{12}(s, p) J_{l-1/2}(s) J_r(s)ds \\ K_{21}(p) &= \int_0^\infty s^{-3/2} c_{21}(s, p) J_l(s) J_{r-1/2}(s)ds \\ K_{22}(p) &= \int_0^\infty s^{-1}\left[c_{22}(s, p) - 1\right] J_{l-1/2}(s) J_{r-1/2}(s)ds \end{aligned}\right\} \tag{53}$$

The asymptotic behavior of the following functions

$$s^{-1}c_{11}(s, p) - 1 = O(s^{-2})\,,\ c_{12}(s, p) = O(s^{-1})\,,$$

$$c_{21}(s, p) = O(s^{-1})\,,\ c_{22}(s, p) - 1 = O(s^{-1})$$

for $s \to \infty$ is observed, and for $s \to 0$ all the integrands are of the order 1; thus, all integrals in (53) exist at both upper and lower limits and pose no difficulties in numerically computational works.

In order to obtain the dynamic stress intensity factor, it is necessary to consider only the singular stress field near the crack tips; in particular, the stress component σ_y on the $y=0$ plane. With a view toward this goal, the stress intensity factor in the Laplace transform domain will be derived from the stress component

$$\sigma_y^* = \frac{2}{\pi}\int_0^\infty \{a_{11}A_1 + a_{12}A_2\}\cos(sx)ds \, . \qquad (54)$$

Now, substituting for $A_j(s,p)$ in terms of $V_j(s,p)$ for $j=1,2$ via (36)-(37) and making use of (43)-(44), it is readily confirmed that the transformed stress containing the singular term for $x \to 1$ assumes the form

$$\sigma_y^* = \sqrt{\frac{2}{\pi}}h\theta_0\int_0^\infty G(p,s)\cos(sx)ds + ..., $$

(55)

where

$$G(s,p) = \frac{(\beta^2-1)}{p}\sum_{m=1}^\infty \chi_{1,2m-1}(p)J_{2m-1}(s) \, . \qquad (56)$$

Further, interchanging the order of integration and summation in conjunction with the recurrence formula (Magnus & Oberhettinger, 1949)

$$J_{n+1}(s) = \frac{2n}{s}J_n(s) - J_{n-1}(s) \, , \qquad (57)$$

and retaining only the singular term, the equation (55) reduces to

$$\sigma_y^* = -\sigma_0\frac{\Phi(p)}{p}\int_0^\infty J_1(s)\cos(sx)ds + ... \qquad (58)$$

where

$$\Phi(p) = \sum_{m=1}^\infty (-1)^{m+1}\chi_{1,2m-1}(p) \, , \qquad (59)$$

and

$$\sigma_0 = -\sqrt{\frac{2}{\pi}}h\theta_0(\beta^2-1) \, . \qquad (60)$$

Near the crack tip, the singular part of the expression in (58) can be extracted by noting that as $x \to 1^+$ and $r = x-1$ (Erdelyi, 1954)

$$\int_0^\infty J_1(s)\cos(sx)ds = 1 - \frac{1}{\sqrt{2r}} \, . \qquad (61)$$

In view of this observation, equation (58) can be written in the following form

$$\sigma_y^*(x,0,p) = \frac{k_1^*(p)}{(2r)^{1/2}} + ... \qquad (62)$$

with $k_1^*(p)$ denoting the stress-intensity factor in the p-plane is given by

$$k_1^*(p) = \sigma_0\frac{\Phi(p)}{p} \, . \qquad (63)$$

Equations (50)-(51) can be solved numerically for $\chi_{j,l}(p)$ by truncating the series a finite number of terms, and the values of $\Phi(p)/p$ are then inserted into the Laplace inversion integral

$$\frac{k_1(t)}{\sigma_0} = \frac{1}{2\pi i}\int_{Br}\frac{\Phi(p)}{p}e^{pt}dp \, , \qquad (64)$$

to yield the stress-intensity factor $k_1(t)$ in the time domain. The numerical method of Laplace inversion outlined by Miller and Guy (1966) will be utilized to evaluate the integral over the Bromwich path in (64). The remaining section of this paper contains the numerical results of $k_1(t)/\sigma_0$ appearing in (64).

4. Results and discussion

Two materials, lead and copper having high and low values of coupling constants, respectively, were selected for numerical computation and application of the solution presented. The elastic constants listed in Table 1 are taken from (Phurkhao and Kassir, 1991). The solution of the infinite set of linear algebraic equations, (50)-(51) was determined numerically by truncating the infinite series solution to a finite number of terms to yield the values of the functions $\chi_{1,l}(p)$ and $\chi_{2,l}(p)$ at several discrete points $p=(1+n)\delta, n=1,2,...$ along the real axis. The real and positive number δ must be selected such that $f(t)$ can be best approximated within a particular range of time. It is observed that retaining 6-8 terms in the series, in all cases examined, the series converge rapidly. These values were then inserted into (64) to determine the corresponding dynamic stress-intensity factors in the regular time domain. The numerical Laplace inversion was then carried out by the scheme outlined by Miller and Guy (1966) which has been employed successfully for several transient problems in the past (Chen & Sih, 1977; Kassir et al., 1986). The inversion formula is given in the series expansion of the Lengendre functions orthogonal within the interval (-1,1) by

$$f(t) \approx \sum_{k=0}^N C_k P_k(2e^{-\delta t}-1) \, . \qquad (65)$$

Here, $f(t)$ is the inverse of the Laplace transform $f^*(p)$, and C_k are coefficients which can be determined from the orthogonalilty properties of the Jacobi polynomial. In this numerical work, it is found that N=6 and the values of $\delta = 0.125, 0.35$ yield a good approximation. The results of $k_1(t)/\sigma_0$ versus non-dimensional time $t(=c_1^2t'/\kappa)$ are displayed graphically in Figure. 1. It is observed that the dynamic stress-intensity factors exhibit a common character, rising very quickly with time and subsequently approaching the steady-state solution for a sufficiently long time. For lead, the increase in the stress-intensity factor over the static value is approximately 30% with the elapsed time of 3.3×10^{-9} sec.; while, for copper the increase is lower, about 15%, and occurs at approximately the same elapsed time.

The influence of mechanical coupling inherent in the theory was also investigated by setting $\varepsilon = 0$ in the numerical computations, and the results are shown graphically by dotted curves in Figure 1. There is no significant difference noted for both materials.

Table 1 Material properties (measured at $21^{\circ}C$)

Symbol	Unit	Copper	Lead
k	$Cal/s.cm.^{\circ}K$	0.93	0.084
κ	cm^2/s	1.14	0.25
α	$cm/cm^{\circ}C$	16.5E-6	29.3E-6
ε	-	0.017	0.0729
c_1	cm/s	4.36E5	2.145E5
E	dyn/cm^2	11.45E11	1.63E11
ν	-	0.32	0.446
ρ	gm/cm^3	8.93	11.34

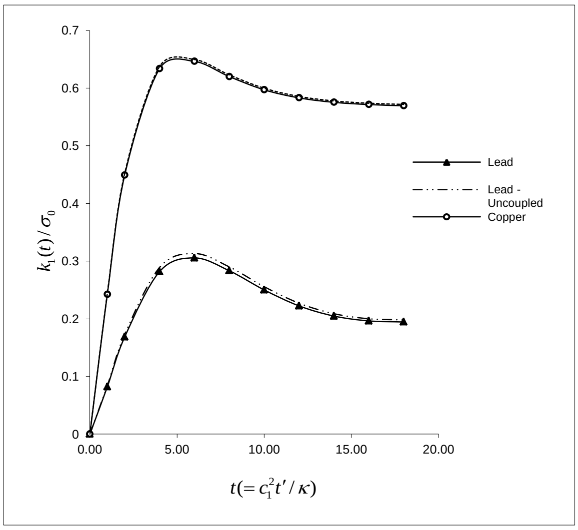

Figure 1 Dynamic stress-intensity factor as function of time

5. Conclusion

A class of mixed boundary-value problems involving a finite crack embedded in an elastic medium with its surfaces suddenly exposed to a change in temperature was investigated in this paper. The main objective is to determine the influences of inertia and thermoelastic coupling effects upon the stress-intensity factor when the heat exchange across the crack surface is according to Newton's law of cooling. Integral transform methods were utilized to reduce the problem to a system of simultaneous dual integral equations. Moreover, by expressing the unknown functions in the form of an infinite series of Bessel functions, the problem was reduced to solving a system of an infinite set of linear algebraic equations. Significant increase in stress intensity-factors due to an inertia effect has been noticed for the two selected materials, but the coupling effect is negligible for both materials. Based on the previous investigations (Kassir et al., 1986; Phurkhao & Kassir, 1991; Ampunsuk & Phurkhao, 2005; Phurkhao, 2010) and the present results, it can be concluded that, for normal materials in practical engineering applications, the stress induced by the coupling term in the heat conduction equation is not substantially significant. The straining in the solid has no significant effect upon the change in temperature.

The results found in this study are crucial in determining the stability of a crack under severe thermal impact loading. Moreover, the method outlined in this study may be applied in some other areas of a similar nature; for example, a fluid infiltrated poroelastic crack problem in rock materials where fluid pressure plays the same role as temperature.

6. References

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Appendix

In this appendix, some of the abbreviations employed in the paper are defined and collected for easy reference.

$$a_{1j}(s,p) = 2\beta^2 s^2 - \frac{4\beta^2 s^2 \gamma_3 \gamma_j}{2s^2 + \varsigma_3^2} + p^2, \quad j = 1,2$$

$$a_{2j}(s,p) = (p^2 - \varsigma_j^2)(\gamma_j + h), \quad j = 1,2$$

$$b_{1j}(s,p) = \frac{-2\gamma_j}{2s^2 + \varsigma_3^2}, \quad j = 1,2$$

$$b_{2j}(s,p) = (p^2 - \varsigma_j^2)\gamma_j, \quad j = 1,2$$

$$c_{11}(s,p) = \frac{1}{(\beta^2 - 1)p^2 \Delta}\left(a_{11}b_{22} - a_{12}b_{21}\right)$$

$$c_{12}(s,p) = \frac{1}{(\beta^2 - 1)p^2 \Delta}\left(-a_{11}b_{12} + a_{12}b_{11}\right)$$

$$c_{21}(s,p) = \frac{1}{\Delta}\left(a_{21}b_{22} - a_{22}b_{21}\right)$$

$$c_{22}(s,p) = \frac{1}{\Delta}\left(a_{22}b_{11} - a_{21}b_{12}\right)$$

$$\Delta = b_{11}b_{22} - b_{12}b_{21}$$