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A new fuzzy parameterized intuitionistic fuzzy soft multiset theory and group decision-making

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Abstract

Intuitionistic fuzzy soft sets (IFSSs) can effectively represent and simulate the uncertainty and diversity of judgment information offered by decision makers. In comparison to fuzzy soft sets (FSSs), IFSSs are highly beneficial for expressing vagueness and uncertainty more accurately. As a result, in this paper, we offer an approach for solving real-life group decision making problems (DMPs) with fuzzy parameterized intuitionistic fuzzy soft multisets (p-sets) by extending the fuzzy soft multiset (FSMS) based decision-making method (DMM). FSMS is a fantastic and useful tool to deal with DMPs and all the existing FSMS-based DMMs are good for solving DMPs, but in their methods, they used FSMS evaluated by only one decision maker, and the importance of membership degrees of parameters are not considered, so these methods are may not be useful in the modelling of group-DMPs, but the constructed method in this paper is very advantageous for solving real-life group-DMPs. To demonstrate the applicability of our DMM in helpful applications, certain real-life examples are used.

Keywords: *decision-making; fuzzy set; intuitionistic fuzzy set; multiset; soft set.*

1. Introduction

Soft set (SS) was first proposed by Molodtsov (1999) as a fundamental and useful mathematical method for dealing with complexity, unclear definitions, and unknown objects (elements). Since there are no limitations to the description of elements in SST, researchers may choose the type of parameters that they need, significantly simplifying DMPs and making it easier to make decisions in the absence of partial knowledge, it is more effective. While several mathematical tools for modeling uncertainties are available, such as operations analysis, probability theory, game theory, fuzzy set (FS), rough set (RS), and interval valued fuzzy set (IVFS), intuitionistic fuzzy set (IFS), each of these theories has inherent difficulties. Furthermore, all of these theories lack parameterization of the tools, which means they can't be used to solve problems, especially in the economic, environmental, and social realms. In the sense that it is clear of the aforementioned difficulties, SS (Molodtsov, 1999) stands out.

The SS (Molodtsov, 1999) is extremely useful in a variety of situations. Molodtsov (1999) developed the basic results of SS and successfully applied it to a variety of fields, including the smoothness of functions, operations analysis, Riemann integrations, game theory, probability, and so on. Maji, Biswas, and Roy (2003) went on to present several new concepts on SS, such as intersection, union, complements, and subset, etc. as well as a detailed discussion of the use of SS in DMPs. Ali, Feng, Liu, Min, and Shabir (2009) presented several operations on SSs and shown that certain De Morgan's rules hold in SSs to these new definitions. Thereafter, several researchers doing their innovative research work in this theory and applied in various field. Rajput, Thakur, and Dubey (2020) defined soft almost ββ-continuity in soft topological spaces. Dalkılıç (2021) introduced a novel approach to SS based DM under uncertainty.

Since Zadeh (1965) introduced the idea of FSs, several new approaches and theories for dealing with imprecision and ambiguity have been proposed. Maji, Biswas, and Roy (2001a; 2001b) described FSSs by combining SSs and FSs, which have a lot of potential for solving DMPs. The applications of FSS theory have been gradually concentrated by using these concepts. Lathamaheswari, Nagarajan, Kavikumar, & Broumi (2020) introduced the concept of triangular interval type-2 FSS and also, shown its applications. Petchimuthu, Garg, Kamacı, and Atagün (2020) defined generalized products of fuzzy soft matrices and the mean operators, as well as the applications of these concepts in MCGDM. Paik and Mondal, (2020) introduced a distance-similarity technique to solve FSs and FSSs based DMPs. Paik and Mondal (2021) had shown the representation and applications of FSSs in a type-2 environment. Močkoř and Hurtik (2021) used the concept FSSs in image processing applications. Gao and Wu (2021) defined filter and its applications in fuzzy soft topological spaces. Dalkılıç and Demirtaş (2021) introduced the idea of bipolar fuzzy soft D-metric spaces. Bhardwaj and Sharma (2021) described an advanced uncertainty measure using FSSs and shown its application in DMPs.

In some circumstances, generalizations of FS such as IFS (Atanassov, 1986) and IVIFS (Atanassov & Gargov, 1989) make representations of the objective world more convincing, functional, and exact, making it very promising. Many scholars have recently concentrated on both theoretical and applied research relating to the idea of IFS and IVIFS see (Iqbal, & Rizwan, 2019; Joshi, 2020; Lathamaheswari et al., 2020; Liu, & Jiang, 2020). As a generalization of FSs, Atanassov (1986) proposed the idea of IFSs. Maji, Biswas, and Roy (2001a; 2001b) developed the concept of IFSS as an important mathematical method for solving DMPs in an uncertain situation by combining SS with IFS, and Jiang, Tang, and Chen (2011) proposed an adjustable approach to IFSS dependent DMPs. Many scholars have recently concentrated on both theoretical and applied studies relating to the principle of IFSS. Wan, Wang, and Dong (2019) presented intuitionistic fuzzy preference relation and group DMM, thereafter, Wan, Xu, and Dong (2020) proposed an Atanassov IF programming approach for solving group DMPs with interval-valued Atanassov IF preference relations. Dong (2020) developed some theories and DMMs based on IVIFS. Wan and Dong (2021) introduced a new best-worst method extension based on intuitionistic fuzzy reference comparisons. Liu, Wan, and Dong (2021) proposed an axiomatic design-based mathematical programming tool for heterogeneous MCGDM with linguistic fuzzy truth degrees. Athira, John, and Garg (2020) presented a novel entropy measure of Pythagorean FSSs. Garg and Arora (2018, 2020a; 2020b) introduced the idea of bonferroni mean aggregation operators under IFSS environment with their applications in DMPs and also, proposed TOPSIS technique based on correlation coefficient for solving DMPs with IFSS information. Based on the Archimedean t-norm of the IFSS information, Garg and Arora (2021) proposed generalized Maclaurin symmetric mean aggregation operators. Garg (2021a; 2021b)/ introduced several novel exponential operation rules and operators for interval-valued q-rung orthopair FSs in group DMPs, as well as the idea of connection number based q-rung orthopair FSs and their application in DMPs.

A multiset (bag) (Yager, 1986) is a series of items in which there is a lot of repetition of elements. Yager (1986) discusses the bag structure's utility in relational databases and examples of bag applications in practice. Several authors have since looked into the wider range of properties and uses of bags. As a generalization of SS and bag, Alkhazaleh and others (Alkhazaleh, Salleh, & Hassan, 2011; Balami & Ibrahim 2013) introduced the idea of soft multiset (SMS) and its fundamental operations such as union, complement, and intersection, etc., and thereafter, Mukherjee and others (Tokat, & Osmanoglu, 2013; Mukherjee & Das, 2014) introduced the idea of topological space and investigated its connectedness and compactness of SMS. Recently, Riaz, Karaaslan, Nawaz, & Sohail. (2021) presented the idea of soft multi-RS topology as well as its applications in multi-criteria DMPs. Alkhazaleh and Salleh (2012) initiated the FSMS theory as speculation of SMS and focused on the use of FSMS-based DMPs. Mukherjee and Das (2015a; 2015b; 2015c) pointed out that the Alkhazaleh and Salleh (2012) methodology is insufficient for comprehending FSMS-based DMPs, and they introduced a new DMM to solve

FSMS based DMPs. Recently, Das (2018) introduced the theory of weighted-FSMS, and studied its applications in DMPs. Akin (2020) proposed an application of FSMSs to algebra. As a generalization of FSMS, Mukherjee and Das (2014; 2015a; 2015b; 2015c; 2016) introduced the idea of IFSMS theory. There were also some more articles devoted to this topic, such as Mukherjee & Das (2013; 2015a; 2015b; 2015c).

Related works

Alkhazaleh and Salleh (2012) initiated the FSMS theory as speculation of SMS (Alkhazaleh et al., 2011) and focused on the use of FSMS-based DMPs. Mukherjee and Das (2015a; 2015b; 2015c) pointed out that the Alkhazaleh-Salleh methodology (Alkhazaleh & Salleh, 2012) is insufficient for comprehending FSMS-based DMPs, and they introduced a new DMM to solve FSMS based DMPs. Recently, Das (2018) introduced the theory of weighted-FSMS, and studied its applications in DMPs. Balami, Gwary, and Terkimbir (2018) proposed an FSMS approach to DMPs and Akin (2020) proposed an application of FSMSs to algebra. As a generalization of FSMS, Mukherjee and Das (2014) introduced the idea of IFSMS and studied some topological properties on IFSMSs. There were also some more articles devoted to this topic, such as Mukherjee and Das (2015a; 2015b; 2015c) introduced the idea of relations on IFSMSs. Thereafter, Mukherjee and Das (2016) studied more results on IFSMSs and shown their applications in information systems. All the existing DMMs given in (Alkhazaleh, & Salleh, 2012; Mukherjee, & Das, 2015a; 2015b; 2015c; Balami et al., 2018; Das, 2018; Akin, 2020) are good for solving DMPs based on FSMS, but there have some limitations. IFSSs can effectively represent and simulate the uncertainty and diversity of judgment information offered by decision makers. In comparison to FSSs, IFSSs are highly beneficial for expressing vagueness and uncertainty more accurately. As a result, we offer an approach for solving group-DMPs with p-sets by extending the FSMS-based DMM. All the methods given in (Alkhazaleh, & Salleh, 2012; Mukherjee, & Das, 2015a; 2015b; 2015c; Balami et al., 2018; Das, 2018; Akin, 2020) are good for solving DMPs, but in their methods they used FSMS evaluated by only one decision maker and importance of membership degrees of parameters are not considered, so these methods are may not be useful in the modelling of group-DMPs, but the constructed method in this paper is very advantageous for group-DMPs. A real-life example is given to show how our DMM can be used in practical applications. First, we'll go over some definitions and outcomes that will assist us to continue our discussion (Section 2). The concept of a p-set has been introduced in section 3, and its basic qualities are being investigated. Next, we have characterized the aggregate FS and defined several forms of t-norm product (TNP) and t-conorm product (TCP) of p-sets (Section 4). In section 5, we provide an adjustable DMM to solve p-set based DMPs using these products, and some real-life examples demonstrate the practicality of our proposed p-set based DMM in practice (Section 6). In section 7, we compare our DMM to other FSMS-based DMMs that are already available.

2. Preliminary

First, we'll go over some definitions and outcomes that will assist us to continue our discussion. Let V_U stand for the initial universe, E_V for the parameter arrangement, $P(V_U)$ for the power set of V_U and also, let $A_E, B_E, C_E \subseteq E_V$

Definition 2.1 (Zadeh, 1965) An FS ψ on V_U is a set having the form $\psi=\left\{\left(v,\mu_\psi(v)\right):v\in V_U\right\}$, where the function $\mu_\psi:V_U\to[0,1]$ is said to be the membership function and $\mu_\psi(v)$ means the degree of membership of each member $v\in V_U$.

If $\mu_\psi(v)=1,\forall v\in V_U$, then ψ becomes a crisp (ordinary) set. We represent the collection of all FSs over V_U by $FS(V_U)$.

Definition 2.2 (Zadeh, 1965) Let $\psi,\phi\in FS(V_U)$. Then the FS-union of ψ and φ is an FS denoted by $\psi\cup\phi$ and defined as

$$\psi\cup\phi=\left\{\left(v, \max\{\mu_\psi(v),\mu_\phi(v)\}\right):v\in V_U\right\}.$$

Definition 2.3 (Zadeh, 1965) Let $\psi,\phi\in FS(V_U)$. Then FS-intersection of ψ and φ is an FS, denoted by $\psi\cap\phi$ and defined as

$$\psi\cap\phi=\left\{\left(v, \min\{\mu_\psi(v),\mu_\phi(v)\}\right):v\in V_U\right\}.$$

Definition 2.4 (Zadeh, 1965) Let $\psi\in FS(V_U)$. Then complement of ψ is denoted by ψ^C and defined as $\psi^C=\left\{\left(v,1-\mu_\psi(v)\right):v\in V_U\right\}$.

Definition 2.5 (Zadeh, 1965) Let $\psi,\phi \in FS(V_U)$. Then ψ is said to be a fuzzy subset of φ, denoted by $\psi \subseteq \phi$ if $\mu_\psi(v) \le \mu_\phi(v)$, $\forall v \in V_U$.

Definition 2.6 (Atanassov, 1986) An IFS ψ is the structure $\psi = \{\langle v, \mu_\psi(v), \nu_\psi(v)\rangle : v \in V_U\}$, where $\mu_\psi: V_U \to [0,1]$. and $\nu_\psi: V_U \to [0,1]$ are real valued functions satisfying the condition $0 \le \mu_\psi(v) + \nu_\psi(v) \le 1$, $\forall v \in V_U$. We represent the class of all IFSs on V_U by $IFS(V_U)$.

Definition 2.7 (Molodtsov, 1999) A soft set on V_U refers to a couple (ψ_S, A_E), where $\psi_S: A_E \to P(V_U)$ is a mapping.

Definition 2.8 (Maji, Biswas, & Roy, 2001a; 2001b) A pair (ψ_S, A_E) is called an *IFSS* over V_U, where ψ_S is a function given by $\psi_S: A_E \to IFS(V_U)$. We represent the class of all IFSSs on V_U by $IFSS(V_U)$.

Definition 2.9 (Maji, Biswas, & Roy, 2001a; 2001b) Let $(\psi_S, A_E), (\phi_S, B_E) \in IFSS(V_U)$ Then (ψ_S, A_E) is said to be a sub-IFSS of (ϕ_S, B_E), denoted by $(\psi_S, A_E) \subseteq (\phi_S, B_E)$ if

(i). $A_E \subseteq B_E$

(ii). $\forall r \in A_E$, $\mu_{\psi_S(r)}(v) \le \mu_{\varphi_s(r)}$ and $\nu_{\varphi_s(r)}(v) \ge V_{\varphi_s(r)}(v), \forall v \in V_U$

Definition 2.10 (Maji, Biswas, & Roy, 2001a; 2001b b) The union of two IFSSs $(\psi_S, A_E), (\phi_S, B_E) \in IFSS(V_U)$ is an IFSS (σ_S, C_E), where $C_E = A_E \cup B_E$ and $\forall r \in C_E$, $v \in V_U$,

$$\mu_{\sigma_S(r)}(v) = \begin{cases} \mu_{\psi_S(r)}(v), & \text{if } r \in A_E - B_E \\ \mu_{\phi_S(r)}(v), & \text{if } r \in B_E - A_E \\ \max\{\mu_{\psi_S(r)}(v), \mu_{\phi_S(r)}(v)\}, & \text{if } r \in A_E \cap B_E, \end{cases}$$

$$\nu_{\sigma_S(r)}(v) = \begin{cases} \nu_{\psi_S(r)}(v), & \text{if } r \in A_E - B_E \\ \nu_{\phi_S(r)}(v), & \text{if } r \in B_E - A_E \\ \min\{\nu_{\psi_S(r)}(v), \nu_{\phi_S(r)}(v)\}, & \text{if } r \in A_E \cap B_E. \end{cases}$$

We write $(\psi_S, A_E) \cup (\phi_S, B_E) = (\sigma_S, C_E)$.

Definition 2.11 (Maji, Biswas, & Roy, 2001a; 2001b) The intersection of two IFSSs(ψ_S, A_E), $(\phi_S, B_E) \in IFSS(V_U)$ is an IFSS (σ_S, C_E), where $C_E = A_E \cup B_E$ and $\forall r \in C_E$, $v \in V_U$,

$$\mu_{\sigma_S(r)}(v) = \begin{cases} \mu_{\psi_S(r)}(v), & \text{if } r \in A_E - B_E \\ \mu_{\phi_S(r)}(v), & \text{if } r \in B_E - A_E \\ \min\{\mu_{\psi_S(r)}(v), \mu_{\phi_S(r)}(v)\}, & \text{if } r \in A_E \cap B_E, \end{cases}$$

$$\nu_{\sigma_S(r)}(v) = \begin{cases} \nu_{\psi_S(r)}(v), & \text{if } r \in A_E - B_E \\ \nu_{\phi_S(r)}(v), & \text{if } r \in B_E - A_E \\ \max\{\nu_{\psi_S(r)}(v), \nu_{\phi_S(r)}(v)\}, & \text{if } r \in A_E \cap B_E. \end{cases}$$

We write *$(\psi_S, A_E) \cap (\phi_S, B_E) = (\sigma_S, C_E)$*.

Definition 2.12 (Maji, Biswas, & Roy, 2001a; 2001b) Let $(\psi_S, A_E) \in IFSS(V_U)$. Then Complement of (ψ_S, A_E), denoted by $(\psi_S, A_E)^C$ and defined as $(\psi_S, A_E)^C = (\psi_S^C, A_E)$, where $\psi_S^C(r) = (\psi_S(r))^C$, for $r \in A_E$.

Definition 2.13 (Mukherjee & Das, 2014) Suppose $\{V_i : i \in \Lambda\}$ represent a collection of nonempty universes and $\{S_{V_i} : i \in \Lambda\}$ represent a collection of nonempty sets of parameters, such that $\cap_{i \in \Lambda} V_i = \varphi$. Let $V_U = \prod_{i \in \Lambda} IFS(V_i)$, where $IFS(V_i)$ denote the arrangement of all the sub-IFSs of V_i, $E_V = \prod_{i \in \Lambda} E_{V_i}$ and $A_E \subseteq E_V$. Then an IFSMS on V_U refers to a couple (F, A_E), where $F: A_E \to V_U$ is a mapping defined by $F(e) = \left(\left\{v^{(\mu_{F(e)}(v), \nu_{F(e)}(v))} : v \in V_\lambda\right\} : \lambda \in \Lambda\right)$. Thus an IFSMS (F, A_E) over V_U can be represented by $(F, A_E) = \left\{\left(e, \left(\left\{v^{(\mu_{F(e)}(v), \nu_{F(e)}(v))} : v \in V_\lambda\right\} : \lambda \in \Lambda\right)\right) : e \in A_E\right\}$.
We represent the class of all IFSMSs on V_U by $IFSMS(V_U, A_E)$, where the parameter set A_E is fixed.

Definition 2.14 (Mukherjee & Das, 2014) For any IFSMS $(F, A_E) \in IFSMS(V_U, A_E)$, a pair (F^λ, A_E) is said to be a V_λ-IFSMS-part (IFSMSP) of (F, A_E), where $F^\lambda: A_E \to V_\lambda$ is a mapping defined by $F(e) = \left\{v^{(\mu_{F(e)}(v), \nu_{F(e)}(v))} : v \in V_\lambda\right\}$ for $e \in A_E$. Thus an *IFSMSP (F^λ, A_E)* over V_U can be represented by $(F^\lambda, A_E) = \left\{\left(e, \left\{v^{(\mu_{F(e)}(v), \nu_{F(e)}(v))} : v \in V_\lambda\right\}\right) : e \in A_E\right\}$.

Definition 2.15 (Mukherjee & Das, 2014) An IFSMS $(F,A_E)\in IFSMS(V_U,A_E)$ is called a null IFSMS, denoted by Φ_A, if for all $e\in A_E$. $\mu_{F(e)}(v)=0$ and $\nu_{F(e)}(v)=1$, $\forall v\in V_\lambda$, $\lambda\in\Lambda$,

i.e. $\Phi_A=\left\{\left(e,\left(\left\{v^{(0,1)}:v\in V_\lambda\right\}:\lambda\in\Lambda\right)\right):e\in A_E\right\}$.

Definition 2.16 (Mukherjee & Das, 2014) Let $(F,A_E)\in IFSMS(V_U,A_E)$. If for every $e\in A_E$, $\mu_{F(e)}(v)=1$ and $\nu_{F(e)}(v)=0$, $\forall v\in V_\lambda$, $\lambda\in\Lambda$, then (F,A_E) is called an absolute IFSMS, denoted by V_A, i.e. $V_A=\left\{\left(e,\left(\left\{v^{(1,0)}:v\in V_\lambda\right\}:\lambda\in\Lambda\right)\right):e\in A_E\right\}$.

Definition 2.17 (Mukherjee & Das, 2014) For two IFSMSs $(F,A_E),(G,A_E)\in IFSMS(V_U,A_E)$, we say that (F,A_E) is an IFSM-subset of (G,A_E) if $\forall e\in A_E$, $\mu_{F(e)}(v)\leq\mu_{G(e)}(v)$ and $\nu_{F(e)}(v)\geq\nu_{G(e)}(v)$, $\forall v\in V_\lambda$, $\lambda\in\Lambda$. We write $(F,A_E)\widetilde{\subseteq}(G,A_E)$.

Definition 2.18 (Mukherjee & Das, 2014) Union between two IFSMSs *$(F,A_E),(G,A_E)\in IFSMS(V_U,A_E)$* is denoted by *$(F,A_E)\cup(G,A_E)$* and defined as

$$(F,A_E)\cup(G,A_E)=\left\{\left(e,\left(\left\{v^{\left(\max\left\{\mu_{F(e)}(v),\mu_{G(e)}(v)\right\},\min\left\{\nu_{F(e)}(v),\nu_{G(e)}(v)\right\}\right)}:v\in V_\lambda\right\}:\lambda\in\Lambda\right)\right):e\in A_E\right\}.$$

Definition 2.19 (Mukherjee & Das, 2014) Intersection between two IFSMSs *$(F,A_E),(G,A_E)\in IFSMS(V_U,A_E)$* is denoted by *$(F,A_E)\cap(G,A_E)$* and defined as

$$(F,A_E)\cap(G,A_E)=\left\{\left(e,\left(\left\{v^{\left(\min\left\{\mu_{F(e)}(v),\mu_{G(e)}(v)\right\},\max\left\{\nu_{F(e)}(v),\nu_{G(e)}(v)\right\}\right)}:v\in V_\lambda\right\}:\lambda\in\Lambda\right)\right):e\in A_E\right\}.$$

Definition 2.20 (Mukherjee & Das, 2014) The complement of an IFSMSs *$(F,A_E)\in IFSMS(V_U,A_E)$* can be represented by

$$(F,A_E)^C=\left\{\left(e,\left(\left\{v^{\left(\nu_{F(e)}(v),\mu_{F(e)}(v)\right)}:v\in V_\lambda\right\}:\lambda\in\Lambda\right)\right):e\in A_E\right\}.$$

3. p-set and its properties

In this present section, we have proposed the idea of a p-set, as well as its basic qualities are now being investigated. Let $\{V_\lambda:\lambda\in\Lambda\}$ be a collection of nonempty universes, such that $\cap_{\lambda\in\Lambda}V_\lambda=\varphi$ and $\{E_{V_\lambda}:\lambda\in\Lambda\}$ be a set of nonempty collections of parameters. Let $V_U=\prod_{\lambda\in\Lambda}IFS(V_\lambda)$, where $IFS(V_\lambda)$ signifies the arrangement of every single IF subsets of V_λ, $E_V=\prod_{\lambda\in\Lambda}E_{V_\lambda}$ and $A_E\subseteq E_V$. $X=\left\{e^{\mu_X(e)}:e\in A_E\right\}$, be an FS over A_E.

Definition 3.1 A p-set F_X over V_U is a mapping F_X: $A_E\to V_U$, defined by

$$F_X(e)=\left(\left\{u^{\left(\mu_{F_{X(e)}}(u),\nu_{F_{X(e)}}(u)\right)}:u\in V_\lambda\right\}:\lambda\in\Lambda\right)\text{ for } e\in A_E.$$

Thus a p-set F_X over V_U can be represented by

$$F_X=\left\{\left(e^{\mu_X(e)},\left(\left\{u^{\left(\mu_{F_{X(e)}}(u),\nu_{F_{X(e)}}(u)\right)}:u\in V_\lambda\right\}:\lambda\in\Lambda\right)\right):e\in A_E\right\}$$

Or

$$F_X=\left\{\left(e^{\mu_X(e)},\left(\left\{u^{\binom{\mu_{F_{X(e)}}(u)}{\nu_{F_{X(e)}}(u)}}:u\in V_\lambda\right\}:\lambda\in\Lambda\right)\right):e\in A_E\right\},$$

If $\mu_X(e)=1$, $\forall e\in A_E$, then X will be generated as a regular FS, and F_X will be generated as a traditional IFSMS. Simply, we denote the collection of all p-sets over V_U by $p_S(V_U,A_E)$, where the parameter set A_E is fixed.

Example 3.2 We assume that there are three universes $V_1=\{o_1, o_2, o_3\}$, $V_2=\{p_1, p_2\}$ and $V_3=\{r_1, r_2\}$, each of which contains a collection of flats, vehicles, and inns. Suppose that Dr. Roy has a budget for buying a flat, a vehicle and renting a location for a wedding festival. Consider a p-set F_X that shows some flats, vehicles, and inns that Dr. Roy is considering for settlement, transportation, and a wedding festival location, respectively. Let $\left\{S_{V_1},S_{V_2},S_{V_3}\right\}$ be a set of collections of decision parameters associated with the universes mentioned above, where

$S_{V_1}=\{s_{V_1,1}=\text{Price}, \ s_{V_1,2}=\text{Carpet area}, \ s_{V_1,3}=\text{Location}, \ s_{V_1,4}=\text{Parking space}\}$,
$S_{V_2}=\{s_{V_2,1}=\text{Safety rating}, \ s_{V_2,2}=\text{Model}, \ s_{V_2,3}=\text{Creature comfort}, \ s_{V_2,4}=\text{Ownership cost}\}$,
$S_{V_3}=\{s_{V_3,1}=\text{Expensive}, \ s_{V_3,2}=\text{Available transport options}, \ s_{V_3,3}=\text{Near to place of stay}, \ s_{V_3,4}=\text{Parking space}\}$.

Let $V=\prod_{i=1}^{3} IFS(V_i)$, $S=\prod_{i=1}^{3} S_{V_i}$ and $A\subseteq S$, such that

$$A=\left\{\begin{array}{l} a=(s_{V_1,1},s_{V_2,1},s_{V_3,1}) = (\text{Price, Safety rating, Expensive}),\\ b=(s_{V_1,3},s_{V_2,2},s_{V_3,1}) = (\text{Location, Model, Expensive}),\\ c=(s_{V_1,2},s_{V_2,3},s_{V_3,2}) = (\text{Carpet area, Creature comfort, Available transport options}),\\ d=(s_{V_1,3},s_{V_2,2},s_{V_3,1}) = (\text{Location, Model, Expensive}) \end{array}\right\}$$

Suppose Dr. Roy is tasked with selecting objects from the arrangements of given objects based on the arrangements of choice parameters. If we chose X be an FS over A with membership values for the parameters in A as

$a=(s_{V_1,1},s_{V_2,1},s_{V_3,1})$ = (Price, Safety rating, Expensive), $\mu_X(a)=0.4$;

$b=(s_{V_1,3},s_{V_2,2},s_{V_3,1})$=(Location, Model, Expensive), $\mu_X(b)=0.5$;
$c=(s_{V_1,2},s_{V_2,3},s_{V_3,2})$=(Carpet area, Creature comfort, Available transport options),$\mu_X(c)=0.6$;
$d=(s_{V_1,3},s_{V_2,2},s_{V_3,1})$=(Location, Model, Expensive) $\mu_X(d)=0.4$;

i.e. if we chose X be an FS over A as $X=\{a^{0.4},b^{0.5},c^{0.6},d^{0.4}\}$. Then we have a *p*-set

$$F_X=\Big\{\Big(a^{0.4},(\{o_1{}^{(0.2,0.5)},o_2{}^{(0.4,0.5)},o_3{}^{(0.2,0.3)}\},\{p_1{}^{(0.1,0.2)},p_2{}^{(0.5,0.7)}\},\{r_1{}^{(0.6,0.8)},r_2{}^{(0.2,0.4)}\})\Big),$$
$$\Big(b^{0.5},(\{o_1{}^{(0.5,0.6)},o_2{}^{(0.2,0.3)},o_3{}^{(0.3,0.5)}\},\{p_1{}^{(0.2,0.5)},p_2{}^{(0.6,0.7)}\},\{r_1{}^{(0.1,0.4)},r_2{}^{(0.2,0.3)}\})\Big),$$
$$\Big(c^{0.6},(\{o_1{}^{(0.1,0.3)},o_2{}^{(0.3,0.5)},o_3{}^{(0.2,0.4)}\},\{p_1{}^{(0.1,0.5)},p_2{}^{(0.2,0.3)}\},\{r_1{}^{(0.2,0.4)},r_2{}^{(0.1,0.3)}\})\Big),$$
$$\Big(d^{0.4},(\{o_1{}^{(0.3,0.4)},o_2{}^{(0.2,0.5)},o_3{}^{(0.2,0.5)}\},\{p_1{}^{(0.2,0.3)},p_2{}^{(0.4,0.6)}\},\{r_1{}^{(0.2,0.5)},r_2{}^{(0.2,0.3)}\})\Big),$$

The tabular form of the *p*-set F_X can be represented as in Table 1.

Table 1 The *p*-set F_X

V_λ		**a** **0.4**	**b** **0.5**	**c** **0.6**	**d** **0.4**
V_1	o_1	(0.2,0.5)	(0.5,0.6)	(0.1,0.3)	(0.3,0.4)
	o_2	(0.4,0.5)	(0.2,0.3)	(0.3,0.5)	(0.2,0.5)
	o_3	(0.2,0.3)	(0.3,0.5)	(0.2,0.4)	(0.2,0.5)
V_2	p_1	(0.1,0.2)	(0.2,0.5)	(0.1,0.5)	(0.2,0.3)
	p_2	(0.5,0.7)	(0.6,0.7)	(0.2,0.3)	(0.4,0.6)
V_3	r_1	(0.6,0.8)	(0.1,0.4)	(0.2,0.4)	(0.2,0.5)
	r_2	(0.2,0.4)	(0.2,0.3)	(0.1,0.3)	(0.2,0.3)

Definition 3.3 For any p-set $F_X \in p_S(V_U,A_E)$, a V_λ-part (or λ-part) F_X^λ over V_U is the structure $F_X^\lambda=\Big\{\Big(e^{\mu_X(e)},\Big\{u^{\left(\mu_{F_{X(e)}}(u),\nu_{F_{X(e)}}(u)\right)}:u\in V_\lambda\Big\}\Big):e\in A_E\Big\}$, where $\forall e\in A_E$, $F_X^\lambda(e)=\Big\{u^{\left(\mu_{F_{X(e)}}(u),\nu_{F_{X(e)}}(u)\right)}:u\in V_\lambda\Big\}$.

Example 3.4 If we consider the *p*-set F_X as in Example 3.2, then the V_1-part, V_2-part, and V_3-part are as follows

$F_X^1 = \{(a^{0.4},\{o_1{}^{(0.2,0.5)},o_2{}^{(0.4,0.5)},o_3{}^{(0.2,0.3)}\}),(b^{0.5},\{o_1{}^{(0.5,0.6)},o_2{}^{(0.2,0.3)},o_3{}^{(0.3,0.5)}\}),$
$(c^{0.6},\{o_1{}^{(0.1,0.3)},o_2{}^{(0.3,0.5)},o_3{}^{(0.2,0.4)}\}),(d^{0.4},\{o_1{}^{(0.3,0.4)},o_2{}^{(0.2,0.5)},o_3{}^{(0.2,0.5)}\})\},$
$F_X^2 = \{(a^{0.4},\{p_1{}^{(0.1,0.2)},p_2{}^{(0.5,0.7)}\}),(b^{0.5},\{p_1{}^{(0.2,0.5)},p_3{}^{(0.6,0.7)}\}),$
$(c^{0.6},\{p_1{}^{(0.1,0.5)},p_2{}^{(0.2,0.3)}\}),(d^{0.4},\{p_1{}^{(0.2,0.3)},p_3{}^{(0.4,0.6)}\})\},$
$F_X^3 = \{(a^{0.4},\{r_1{}^{(0.6,0.8)},r_2{}^{(0.2,0.4)}\}),(b^{0.5},\{r_1{}^{(0.1,0.4)},r_2{}^{(0.2,0.3)}\}),$
$(c^{0.6},\{r_1{}^{(0.2,0.4)},r_2{}^{(0.1,0.3)}\}),(d^{0.4},\{r_1{}^{(0.2,0.5)},r_2{}^{(0.2,0.3)}\})\}$

and their tabular representation as shown in Tables 2, 3, and 4 respectively

Table 2 V_1-part F_X^1 of F_x

V_1	**a** **0.4**	**b** **0.5**	**c** **0.6**	**d** **0.4**
o_1	(0.2,0.5)	(0.5,0.6)	(0.1,0.3)	(0.3,0.4)
o_2	(0.4,0.5)	(0.2,0.3)	(0.3,0.5)	(0.2,0.5)
o_3	(0.2,0.3)	(0.3,0.5)	(0.2,0.4)	(0.2,0.5)

Table 3 V_2-part F_X^2 of F_x

V_1	**a** **0.4**	**b** **0.5**	**c** **0.6**	**d** **0.4**
o_1	(0.1,0.2)	(0.2,0.5)	(0.1,0.5)	(0.2,0.3)
o_2	(0.5,0.7)	(0.6,0.7)	(0.2,0.3)	(0.4,0.6)

Table 4 The V_3-part F_X^3 of F_x

V_1	**a** **0.4**	**b** **0.5**	**c** **0.6**	**d** **0.4**
r_1	(0.6,0.8)	(0.1,0.4)	(0.2,0.4)	(0.2,0.5)
r_2	(0.2,0.4)	(0.2,0.3)	(0.1,0.3)	(0.2,0.3)

Definition 3.5 For two p-sets $F_X, F_Y \in p_S(V_U, A_E)$, $\mathbf{F_X}$ is a p-subset of F_Y if
(*i*). X is a fuzzy subset of Y, i.e.
$\forall e \in A_E$, $\mu_X(e) \leq \mu_Y(e)$
(*ii*). $\forall e \in A_E$, $\mu_{F_{Y(e)}}$ $(u) \leq \mu_{F_{Y(e)}}$ and
$(u) \leq \mu_{F_{Y(e)}}$, $\forall u \in V_\lambda$, $\lambda \in \Lambda$
We write $F_X \tilde{\subseteq} F_Y$

Definition 3.6 A p-set $F_X \in p_S(V_U, A_E)$ is said to be a null p-set, denoted by Φ_A, if $\forall e \in A_E$, $\mu_X(e)=0$, $\mu_{F_{X(e)}}(u) = 0$ and $\nu_{F_{X(e)}}(u)=1$, $\forall u \in V_\lambda$, $\lambda \in \Lambda$,
i.e. $\Phi_A=\left\{\left(e^0,\left(\left\{u^{(0,1)}:u \in V_\lambda\right\}:\lambda \in \Lambda\right)\right):e \in A_E\right\}$.

Definition 3.7 Let $F_X \in p_S(V_U, A_E)$. If for every $e \in A_E$, $\mu_{F_{X(e)}}(u)=0$ and $\nu_{F_{X(e)}}(u)=1$, $\forall u \in V_\lambda$, $\lambda \in \Lambda$, then F_X is called an X-null p-set, denoted by X_Φ, i.e.
$X_\Phi=\left\{\left(e^{\mu_X(e)},\left(\left\{u^{(0,1)}:u \in V_\lambda\right\}:\lambda \in \Lambda\right)\right):e \in A_E\right\}$.

Definition 3.8 Let $F_X \in p_S(V_U, A_E)$. If for every $e \in A_E$, $\mu_X(e)=1$, $\mu_{F_{X(e)}}(u)=1$ and $\nu_{F_{X(e)}}(u)=0$, $\forall u \in V_\lambda$, $\lambda \in \Lambda$, then F_X is said to be an absolute p-set, denoted by V_A, i.e.
$V_A=\left\{\left(e^1,\left(\left\{u^{(1,0)}:u \in V_\lambda\right\}:\lambda \in \Lambda\right)\right):e \in A_E\right\}$.

Definition 3.9 Let $F_X \in p_S(V_U, A_E)$. If for every $e \in A_E$, $\mu_{F_{X(e)}}(u)=1$ and $\nu_{F_{X(e)}}(u)=0$, $\forall u \in V_\lambda$, $\lambda \in \Lambda$, then F_X is said to be an X-absolute p-set, denoted by X_V, i.e.
$X_V=\left\{\left(e^{\mu_X(e)},\left(\left\{u^{(1,0)}:u \in V_\lambda\right\}:\lambda \in \Lambda\right)\right):e \in A_E\right\}$.

Example 3.10 Let us consider Example 3.2 and if we chose X is an FS over A as
$X=\{a^{0.4},b^{0.5},c^{0.6},d^0\}$. Then we have a *p*-set

$$F_X=\Big\{\Big(a^{0.4},\big(\{o_1{}^{(0.2,0.5)},o_2{}^{(0.4,0.5)},o_3{}^{(0.2,0.3)}\},\{p_1{}^{(0.1,0.2)},p_2{}^{(0.5,0.7)}\},\{r_1{}^{(0.6,0.8)},r_2{}^{(0.2,0.4)}\}\big)\Big),\ \Big(b^{0.5},(\varphi,\varphi,\varphi)\Big),\Big(c^{0.6},(V_1,V_2,V_3)\Big),\Big(d^{0},(\varphi,\varphi,\varphi)\Big)\Big\}.$$

If $Y=\{a^0,b^0,c^0,d^0\}$ and $F_Y(a)=(\varphi,\varphi,\varphi)$, $F_Y(b)=(\varphi,\varphi,\varphi)$, $F_Y(c)=(\varphi,\varphi,\varphi)$,$F_Y(d)=(\varphi,\varphi,\varphi)$, then the p-set $F_Y$is a null p-set.

If $W=\{a^{0.4},b^{0.5},c^{0.6},d^{0.5}\}$ and $F_W(a)=(\varphi,\varphi,\varphi)$, $F_W(b)=(\varphi,\varphi,\varphi)$,$F_W(c)=(\varphi,\varphi,\varphi)$,$F_W(d)=(\varphi,\varphi,\varphi)$, then the p-set F_W is a W-null p-set.

If $Z=\{a^1,b^1,c^1,d^1\}$ and $F_Z(a)=(V_1,V_2,V_3)$, $F_Z(b)=(V_1,V_2,V_3)$, $F_Z(c)=(V_1,V_2,V_3)$,$F_Z(d)=(V_1,V_2,V_3)$, then the p-set F_Z is an absolute p-set.

If $K=\{a^{0.4},b^{0.5},c^{0.6},d^{0.5}\}$ and $F_K(a)=(V_1,V_2,V_3)$, $F_K(b)=(V_1,V_2,V_3)$, $F_K(c)=(V_1,V_2,V_3)$,$F_K(d)=(V_1,V_2,V_3)$, then the p-set F_K is a K-absolute p-set.

Proposition 3.11 Let $F_X,F_Y\in p_S(V_U,A_E)$. Then

[i]. $F_X\tilde{\subseteq}F_X$;
[ii]. $\Phi_A\tilde{\subseteq}X_\Phi\tilde{\subseteq}F_X$;
[iii]. $F_X\tilde{\subseteq}X_U\tilde{\subseteq}U_A$.

Definition 3.12 Let $F_X,F_Y\in p_S(V_U,A_E)$. Then F_X and F_Y are equal-set, denoted by $F_X=F_Y$, if and only if $\forall e\in A_E$,
$[i]\ \mu_X(e)=\mu_Y(e)$;
[ii] $F_X(e)=F_Y(e)\Leftrightarrow\ \mu_{F_{X(e)}}(u)=\mu_{F_{Y(e)}}(u)$ and $\nu_{F_{X(e)}}(u)=\nu_{F_{Y(e)}}(u)$, $\forall u\in V_\lambda$, $\lambda\in\Lambda$.

Proposition 3.13 Let $F_X,F_Y,F_Z\in p_S(V_U,A_E)$. Then

[i]. $F_X=F_Y$ and $F_Y=F_Z\Rightarrow F_X=F_Z$;
[ii]. $F_X\tilde{\subseteq}F_Y$ and $F_Y\tilde{\subseteq}F_X\Leftrightarrow F_X=F_Y$;
[iii]. $F_X\tilde{\subseteq}F_Y$ and $F_Y\tilde{\subseteq}F_Z\Rightarrow F_X=F_Z$.

Definition 3.14 The complement of a *p*-set $F_X\in p_S(V_U,A_E)$ can be represented by

$$F_X^C=\left\{\left(e^{1-\mu_X(e)},\left(\left\{u^{\left(\nu_{F_{X(e)}}(u),\mu_{F_{X(e)}}(u)\right)}:u\in V_\lambda\right\}:\lambda\in\Lambda\right)\right):e\in A_E\right\}$$

Proposition 3.15 For a p-set $F_X\in p_S(V_U,A_E)$,

(a) $\left(F_X^C\right)^C=F_X$,
(b) $\Phi_A^C=V_A$
(C) $U_A^C=\Phi_A$

Proof. (c) From the definition of an absolute p-set $V_A=\left\{\left(e^1,\left(\left\{u^{(1,0)}:u\in V_\lambda\right\}:\lambda\in\Lambda\right)\right):e\in A_E\right\}$,
Then $V_A^C=\left\{\left(e^0,\left(\left\{u^{(0,1)}:u\in V_\lambda\right\}:\lambda\in\Lambda\right)\right):e\in A_E\right\}=\Phi_A$
Similarly, (a) and (b) easily can be made.
Remark 3.16 In general, $X_V^C\neq X_\Phi$ and $X_\Phi^C\neq X_V$. For example, we consider the $FSX=\{a^{0.4},b^{0.5},c^{0.6},d^0\}$ as in example 3.10. If
$X_\Phi=\left\{\left(a^{0.4},(\varphi,\varphi,\varphi)\right),\left(b^{0.5},(\varphi,\varphi,\varphi)\right),\left(c^{0.6},(\varphi,\varphi,\varphi)\right),\left(d^0,(\varphi,\varphi,\varphi)\right)\right\}$ and

$X_V=\left\{\left(a^{0.4},(V_1,V_2,V_3)\right),\left(b^{0.5},(V_1,V_2,V_3)\right),\left(c^{0.6},(V_1,V_2,V_3)\right),\left(d^{0},(V_1,V_2,V_3)\right)\right\}$, then
$X_\Phi^C=\left\{\left(a^{0.6},(V_1,V_2,V_3)\right),\left(b^{0.5},(V_1,V_2,V_3)\right),\left(c^{0.4},(V_1,V_2,V_3)\right),\left(d^{1},(V_1,V_2,V_3)\right)\right\}\neq X_V$ and
$X_V^C=\left\{\left(a^{0.6},(\varphi,\varphi,\varphi)\right),\left(b^{0.5},(\varphi,\varphi,\varphi)\right),\left(c^{0.4},(\varphi,\varphi,\varphi)\right),\right\}\left(d^{1},(\varphi,\varphi,\varphi)\right)\neq X_\Phi$.

Definition 3.17 Union between two p-sets $F_X,F_Y \in p_S(V_U, A_E)$ is denoted by $F_X \tilde{\cup} F_Y$ and defined as $F_X \tilde{\cup} F_Y = F_Z$, where $Z = X \cup Y$, and $\cup$ denotes the fuzzy union and $\mathbf{F_Z}$ can be represented as

$$F_Z=\left\{\left(e^{\max\{\mu_X(e),\mu_Y(e)\}},\left(\left\{u^{\begin{pmatrix}\max\{\mu_{F_{X(e)}}(u),\mu_{F_{Y(e)}}(u)\}\\ \min\{\nu_{F_{X(e)}}(u),\nu_{F_{Y(e)}}(u)\}\end{pmatrix}}:u\in V_\lambda\right\}:\lambda\in\Lambda\right)\right):e\in A_E\right\}.$$

Proposition 3.18 If $F_X \in p_S(V_U, A_E)$, then
(a) $F_X\tilde{\cup}F_X=F_X$,
(b) $F_X\tilde{\cup}\Phi_A=F_X$,
(c) $F_X \tilde{\cup} V_A = V_A$.

Definition 3.19 Intersection between two p-sets $F_X,F_Y \in p_S(V_U, A_E)$ is denoted by $F_X \tilde{\cap} F_Y$ and defined as $F_X \tilde{\cap} F_Y = F_Z$, where $Z = X \cap Y$, where $\cap$ denotes the fuzzy intersection and F_Z can be represented as

$$F_Z=\left\{\left(e^{\min\{\mu_X(e),\mu_Y(e)\}},\left(\left\{u^{\begin{pmatrix}\min\{\mu_{F_{X(e)}}(u),\mu_{F_{Y(e)}}(u)\}\\ \max\{\nu_{F_{X(e)}}(u),\nu_{F_{Y(e)}}(u)\}\end{pmatrix}}:u\in V_\lambda\right\}:\lambda\in\Lambda\right)\right):e\in A_E\right\}.$$

Proposition 3.20 If $F_X \in p_S(V_U, A_E)$, then
(a) $F_X\tilde{\cap}F_X=F_X$,
(b) $F_X\tilde{\cap}\Phi_A=\Phi_A$,
(c) $F_X\tilde{\cap}V_A=F_X$.

Proposition 3.21 Let $F_X,F_Y,F_Z \in p_S(V_U, A_E)$, then:

1. **Associative Laws**
$$F_X\tilde{\cup}(F_Y\tilde{\cup}F_Z)=(F_X\tilde{\cup}F_Y)\tilde{\cup}F_Z$$
$$F_X\tilde{\cap}(F_Y\tilde{\cap}F_Z)=(F_X\tilde{\cap}F_Y)\tilde{\cap}F_Z$$
2. **Distributive Laws**
$$F_X\tilde{\cap}(F_Y\tilde{\cup}F_Z)=(F_X\tilde{\cap}F_Y)\tilde{\cup}(F_X\tilde{\cap}F_Z)$$
$$F_X\tilde{\cup}(F_Y\tilde{\cap}F_Z)=(F_X\tilde{\cup}F_Y)\tilde{\cap}(F_X\tilde{\cup}F_Z)$$

Proposition 3.22 Let $F_X,F_Y\in p_S(V_U,A_E)$, then:
$$(F_X\tilde{\cap}F_Y)^C=F_X^C\tilde{\cup}F_Y^C$$
$$(F_X\tilde{\cup}F_Y)^C=F_X^C\tilde{\cap}F_Y^C$$

Proof. Let $F_X,F_Y\in p_S(V_U,A_E)$.
Then

$$F_X \tilde{\cap} F_Y = F_Z = \left\{ \left(e^{\min\{\mu_X(e),\mu_Y(e)\}}, \left(\left\{ u^{\begin{pmatrix} \min\{\mu_{F_{X(e)}}(u),\mu_{F_{Y(e)}}(u)\} \\ \max\{\nu_{F_{X(e)}}(u),\nu_{F_{Y(e)}}(u)\} \end{pmatrix}} : u \in V_\lambda \right\} : \lambda \in \Lambda \right) \right) : e \in A_E \right\}.$$

Therefore,

$$\left(F_X \tilde{\cap} F_Y\right)^C = F_Z^C = \left\{ \left(e^{1-\min\{\mu_X(e),\mu_Y(e)\}}, \left(\left\{ u^{\begin{pmatrix} \max\{\nu_{F_{X(e)}}(u),\nu_{F_{Y(e)}}(u)\} \\ \min\{\mu_{F_{X(e)}}(u),\mu_{F_{Y(e)}}(u)\} \end{pmatrix}} : u \in V_\lambda \right\} : \lambda \in \Lambda \right) \right) : e \in A_E \right\}.$$

Now

$$F_X^C \tilde{\cup} F_Y^C = F_W = \left\{ \left(e^{\mu_W(e)}, \left(\left\{ u^{\left(\mu_{F_{W(e)}}(u),\nu_{F_{W(e)}}(u)\right)} : u \in V_\lambda \right\} : \lambda \in \Lambda \right) \right) : e \in A_E \right\},$$

where $\forall e \in A_E$,

$$\mu_W(e) = \max\{1-\mu_X(e), 1-\mu_Y(e)\}$$
$$= 1 - \min\{\mu_X(e), \mu_Y(e)\} = \mu_Z^C(e)$$

and

$$\mu_{F_{W(e)}}(u) = \max\left\{\nu_{F_{X(e)}}(u), \nu_{F_{Y(e)}}(u)\right\} = \nu_{F_{Z(e)}}(u)$$
$$\nu_{F_{W(e)}}(u) = \min\left\{\mu_{F_{X(e)}}(u), \mu_{F_{Y(e)}}(u)\right\} = \mu_{F_{Z(e)}}(u)$$

Thus $\left(F_X \tilde{\cap} F_Y\right)^C = F_X^C \tilde{\cup} F_Y^C$.
[ii] has proof that is similar to [i].

Definition 3.23 Let $F_X, F_Y \in p_S(V_U, A_E)$. Then min-union of F_X and F_Y is denoted by $F_X \tilde{\cup} Y_{Z_{\min}}$ and defined as

$$F_Z = \left\{ \left(e^{\min\{\mu_X(e),\mu_Y(e)\}}, \left(\left\{ u^{\begin{pmatrix} \max\{\mu_{F_{X(e)}}(u),\mu_{F_{Y(e)}}(u)\} \\ \min\{\nu_{F_{X(e)}}(u),\nu_{F_{Y(e)}}(u)\} \end{pmatrix}} : u \in V_\lambda \right\} : \lambda \in \Lambda \right) \right) : e \in A_E \right\}.$$

Definition 3.24 Let $F_X, F_Y \in p_S(V_U, A_E)$. Then max-intersection of F_X and F_Y is denoted by $F_X \tilde{\cap} Y_{Z_{\max}}$ and defined as

$$F_Z = \left\{ \left(e^{\max\{\mu_X(e),\mu_Y(e)\}}, \left(\left\{ u^{\begin{pmatrix} \min\{\mu_{F_{X(e)}}(u),\mu_{F_{Y(e)}}(u)\} \\ \max\{\nu_{F_{X(e)}}(u),\nu_{F_{Y(e)}}(u)\} \end{pmatrix}} : u \in V_\lambda \right\} : \lambda \in \Lambda \right) \right) : e \in A_E \right\}.$$

Proposition 3.25 Let $F_X, F_Y \in p_S(V_U, A_E)$. Then
[*i*] $F_X \tilde{\cup}_{\min} F_X = F_X$;
[*ii*] $F_X \tilde{\cap}_{\max} F_X = F_X$;
[*iii*] $F_X \tilde{\cup}_{\min} \Phi_A = \Phi_A$;
[*iv*] $F_X \tilde{\cap}_{\max} X_\Phi = X_\Phi$;
[*v*] $F_X \tilde{\cup}_{\min} V_A = X_U$;
[*vi*] $F_X \tilde{\cup}_{\min} F_Y = F_Y \tilde{\cup}_{\min} F_X$;
[*vii*] $F_X \tilde{\cap}_{\max} F_Y = F_Y \tilde{\cap}^{\max} F_X$.

Proposition 3.26 Let $F_X,F_Y\in p_S(V_U,A_E)$. Then
[i] $V_X\tilde{\cup}_{min}YV_Y\tilde{\subseteq} V_X\ \tilde{\cup}\ V_Y$
[ii] $V_X\tilde{\cap}V_Y\tilde{\subseteq}V_X\tilde{\cap}_{max}\ V_Y$

ARemark 3.27 Let $F_X\in p_S(V_U,A_E)$. Then in general $F_X\tilde{\cap}_{max}V_A= F_X$ and $F_X\tilde{\cup}_{min}V_A=V_A$ are not true. For example, we consider $X=\{a^{0.4},b^{0.5},c^{0.6},d^0\}$ as shown in example 3.10, then we have a *p*-set

$$F_X=\Big\{\Big(a^{0.4},\big(\{o_1{}^{(0.2,0.5)},o_2{}^{(0.4,0.5)},o_3{}^{(0.2,0.3)}\},\{p_1{}^{(0.1,0.2)},p_2{}^{(0.5,0.7)}\},\{r_1{}^{(0.6,0.8)},r_2{}^{(0.2,0.4)}\}\big)\Big),$$
$$\Big(b^{0.5},(\varphi,\varphi,\varphi)\Big),\Big(c^{0.6},(V_1,V_2,V_3)\Big),\Big(d^0,(\varphi,\varphi,\varphi)\Big)\Big\},$$

Then

$$F_X\ \tilde{\cap}_{max}V_A=\Big\{\Big(a^1,\big(\{o_1{}^{(0.2,0.5)},o_2{}^{(0.4,0.5)},o_3{}^{(0.2,0.3)}\},\{p_1{}^{(0.1,0.2)},p_2{}^{(0.5,0.7)}\},\{r_1{}^{(0.6,0.8)},r_2{}^{(0.2,0.4)}\}\big)\Big)$$
$$\Big(b^1,(\varphi,\varphi,\varphi)\Big),\Big(c^1,(V_1,V_2,V_3)\Big),\Big(d^0,(\varphi,\varphi,\varphi)\Big)\Big\}\neq V_X \text{ and}$$
$$F_X\ \tilde{\cup}_{min}V_A=\Big\{\Big(a^{0.4},(V_1,V_2,V_3)\Big)\ \Big(b^{0.5},(V_1,V_2,V_3)\Big)\Big(c^{0.6},(V_1,V_2,V_3)\Big)\Big(d^0,(V_1,V_2,V_3)\Big)\Big\}$$

Proposition 3.28 Let $F_X,F_Y,F_Z\in p_S(V_U,A_E)$. Then
[i] $F_X\tilde{\cup}_{min}\ (F_Y\tilde{\cup}_{min}F_Z)=(F_X\tilde{\cup}_{min}F_Y)\ \tilde{\cup}_{min}F_Z$
[ii] $F_X\tilde{\cap}_{max}\ (F_Y\tilde{\cap}_{max}Z)=(F_X\tilde{\cap}_{max}F_Y)\tilde{\cup}_{max}F_Z$
[iii] $F_X\tilde{\cup}_{min}(F_Y\tilde{\cap}_{max}U_Z)=(F_X\tilde{\cup}_{min}F_Y)\ \tilde{\cap}_{max}(F_X\tilde{\cup}_{min}F_Z)$
[iv] $F_X\tilde{\cap}_{max}\ (F_Y\tilde{\cup}_{min}F_Z)=(F_X\tilde{\cap}_{max}F_Y)\ \tilde{\cup}_{min}(F_X\tilde{\cap}_{max}F_Z)$

Proposition 3.29 Let $F_X,F_Y\in p_S(V_U,A_E)$. Then
[i] $(F_X\tilde{\cap}_{max}F_Y)^C= F_X^C\tilde{\cup}_{min}F_Y^C$
[ii] $(F_X\tilde{\cup}_{min}F_Y)^C= F_X^C\tilde{\cap}_{max}F_Y^C$

Definition 3.30 Let $F_X,F_Y\in p_S(V_U,A_E)$. Then the AND operation between F_X and F_Y is the *p*-set denoted by $F_X\wedge F_Y$ and defined as

$$F_X\wedge F_Y=\left\{\left(e^{\min\{\mu_X(\beta),\mu_Y(\beta)\}},\left(\left\{u^{\left(\begin{smallmatrix}\min\{\mu_{F_X(\alpha)}(u),\mu_{F_Y(\beta)}(u)\},\\ \max\{\nu_{F_X(\alpha)}(u),\nu_{F_Y(\beta)}(u)\}\end{smallmatrix}\right)}:u\in V_\lambda\right\}:\lambda\in\Lambda\right)\right):\alpha,\beta\in A_E\right\}.$$

Definition 3.31 Let $F_X,F_Y\in p_S(V_U,A_E)$. Then the OR operation between F_X and F_Y is the *p*-set denoted by $F_X\vee F_Y$ and defined as

$$F_X\vee F_Y=\left\{\left(e^{\max\{\mu_X(\beta),\mu_Y(\beta)\}},\left(\left\{u^{\left(\begin{smallmatrix}\max\{\mu_{F_X(\alpha)}(u),\mu_{F_Y(\beta)}(u)\},\\ \min\{\nu_{F_X(\alpha)}(u),\nu_{F_Y(\beta)}(u)\}\end{smallmatrix}\right)}:u\in V_\lambda\right\}:\lambda\in\Lambda\right)\right):\alpha,\beta\in A_E\right\}.$$

Proposition 3.32 Let $F_X,F_Y\in p_S(V_U,A_E)$ then
(1). $(F_X\wedge F_Y)^C=F_X^C\vee F_Y^C$
(2). $(F_X\vee F_Y)^C=F_X^C\wedge F_Y^C$

Proof. (1). Let $F_X,F_Y\in p_S(V_U,A_E)$, then

$$F_X \wedge F_Y = \left\{ \left(e^{\min\{\mu_X(\beta),\mu_Y(\beta)\}}, \left(\left\{ u^{\begin{pmatrix} \min\{\mu_{F_{X(\alpha)}}(u),\mu_{F_{Y(\beta)}}(u)\}, \\ \max\{\nu_{F_{X(\alpha)}}(u),\nu_{F_{Y(\beta)}}(u)\} \end{pmatrix}} : u \in V_\lambda \right\} : \lambda \in \Lambda \right) \right) : \alpha,\beta \in A_E \right\}.$$

Thus

$$(F_X \wedge F_Y)^C = \left\{ \left(e^{1-\min\{\mu_X(\beta),\mu_Y(\beta)\}}, \left(\left\{ u^{\begin{pmatrix} \max\{\nu_{F_{X(\alpha)}}(u),\nu_{F_{Y(\beta)}}(u)\}, \\ \min\{\mu_{F_{X(\alpha)}}(u),\mu_{F_{Y(\beta)}}(u)\} \end{pmatrix}} : u \in V_\lambda \right\} : \lambda \in \Lambda \right) \right) : \alpha,\beta \in A_E \right\}.$$

Again,

$$F_X^C \vee F_Y^C = \left\{ \left(e^{\max\{1-\mu_X(\beta),1-\mu_Y(\beta)\}}, \left(\left\{ u^{\begin{pmatrix} \max\{\nu_{F_{X(\alpha)}}(u),\nu_{F_{Y(\beta)}}(u)\}, \\ \min\{\mu_{F_{X(\alpha)}}(u),\mu_{F_{Y(\beta)}}(u)\} \end{pmatrix}} : u \in V_\lambda \right\} : \lambda \in \Lambda \right) \right) : \alpha,\beta \in A_E \right\}$$

$$= \left\{ \left(e^{1-\min\{\mu_X(\beta),\mu_Y(\beta)\}}, \left(\left\{ u^{\begin{pmatrix} \max\{\nu_{F_{X(\alpha)}}(u),\nu_{F_{Y(\beta)}}(u)\}, \\ \min\{\mu_{F_{X(\alpha)}}(u),\mu_{F_{Y(\beta)}}(u)\} \end{pmatrix}} : u \in V_\lambda \right\} : \lambda \in \Lambda \right) \right) : \alpha,\beta \in A_E \right\}$$

$$= (F_X \wedge F_Y)^C.,$$

Hence proved.

Proposition 3.33 Let $F_X,F_Y,F_Z \in p_S(V_U,A_E)$. Then

[i] $F_X \vee (F_Y \vee F_Z) = (F_X \vee F_Y) \vee F_Z$
[ii] $F_X \wedge (F_Y \wedge F_Z) = (F_X \wedge F_Y) \wedge F_Z$
[iii] $F_X \vee (F_Y \wedge U_Z) = (F_X \vee F_Y) \wedge (F_X \vee F_Z)$
[iv] $F_X \wedge (F_Y \vee F_Z) = (F_X \wedge F_Y) \vee (F_X \wedge F_Z)$

4. TNP and TCP of p-sets

In this part, we have characterized the aggregate FS and define several forms of TNP and TCP of p-sets, such as AND-TNP, AND-TCP, OR-TNP, and OR-TCP.

Definition 4.1 Let $F_X,F_Y \in p_S(V_U,A_E)$. Then the AND−TNP of F_X and F_Y is the *p*-set denoted by $F_X \otimes F_Y$ and defined as $F_X \otimes F_Y = F_Z = \left\{ \left(e^{\mu_Z(e)}, \left(\left\{ u^{(\mu_{F_{Z(e)}}(u),\nu_{F_{Z(e)}}(u))} : u \in V_\lambda \right\} : \lambda \in \Lambda \right) \right) : e \in A_E \right\}$, where for all $e \in A_E$, $\mu_Z(e) = \frac{\mu_X(e).\mu_Y(e)}{2-[\mu_X(e)+\mu_Y(e)-\mu_X(e).\mu_Y(e)]}$ and

$$\mu_{F_{Z(e)}}(u) = min\{\mu_{F_{X(e)}}(u), \mu_{F_{Y(e)}}(u)\}, \nu_{F_{Z(e)}}(u) = max\{\nu_{F_{X(e)}}(u), \nu_{F_{Y(e)}}(u)\}, \forall u \in V_\lambda, \lambda \in \Lambda.$$

Definition 4.2 Let $F_X, F_Y \in p_S(V_U, A_E)$. Then the AND−TCP of F_X and F_Y is the *p*-set denoted by $F_X \oplus F_Y$ and defined as $F_X \oplus F_Y = F_Z = \left\{ \left(e^{\mu_Z(e)}, \left(\left\{ u^{(\mu_{F_{Z(e)}}(u),\nu_{F_{Z(e)}}(u))} : u \in V_\lambda \right\} : \lambda \in \Lambda \right) \right) : e \in A_E \right\}$, where for all $e \in A_E$, $\mu_Z(e) = \frac{\mu_X(e)+\mu_Y(e)}{1+\mu_X(e).\mu_Y(e)}$ and

$$\mu_{F_{Z(e)}}(u) = min\{\mu_{F_{X(e)}}(u), \mu_{F_{Y(e)}}(u)\}, \nu_{F_{Z(e)}}(u) = max\{\nu_{F_{X(e)}}(u), \nu_{F_{Y(e)}}(u)\}, \forall u \in V_\lambda, \lambda \in \Lambda.$$

Definition 4.3 Let $F_X, F_Y \in p_S(V_U, A_E)$. Then the OR−TNP of F_X and F_Y is the *p*-set denoted by $F_X \bar{\otimes} F_Y$ and defined as $F_X \bar{\otimes} F_Y = F_Z = \left\{ \left(e^{\mu_Z(e)}, \left(\left\{ u^{(\mu_{F_{Z(e)}}(u),\nu_{F_{Z(e)}}(u))} : u \in V_\lambda \right\} : \lambda \in \Lambda \right) \right) : e \in A_E \right\}$, where for all $e \in A_E$, $\mu_Z(e) = \frac{\mu_X(e).\mu_Y(e)}{2-[\mu_X(e)+\mu_Y(e)-\mu_X(e).\mu_Y(e)]}$ and

$$\mu_{F_{Z(e)}}(u) = max\{\mu_{F_{X(e)}}(u), \mu_{F_{Y(e)}}(u)\}, \nu_{F_{Z(e)}}(u) = min\{\nu_{F_{X(e)}}(u), \nu_{F_{Y(e)}}(u)\}, \forall u \in V_\lambda, \lambda \in \Lambda.$$

Definition 4.4 Let $F_X, F_Y \in p_S(V_U, A_E)$. Then the $OR-TCP$ of F_X and F_Y is the p-set denoted by $F_X \bar{\oplus} F_Y = F_Z$ and defined as $F_X \bar{\oplus} F_Y = F_Z = \left\{\left(e^{\mu_Z(e)}, \left(\left\{u^{\left(\mu_{F_{Z(e)}}(u), \nu_{F_{Z(e)}}(u)\right)}: u \in V_\lambda\right\}: \lambda \in \Lambda\right)\right): e \in A_E\right\}$,
where for all $e \in A$, $\mu_Z(e) = \frac{\mu_X(e)+\mu_Y(e)}{1+\mu_X(e).\mu_Y(e)}$ and

$$\mu_{F_{Z(e)}}(u) = max\{\mu_{F_{X(e)}}(u), \mu_{F_{Y(e)}}(u)\}, \nu_{F_{Z(e)}}(u) = min\{\nu_{F_{X(e)}}(u), \nu_{F_{Y(e)}}(u)\}, \forall u \in V_\lambda, \lambda \in \Lambda.$$

Definition 4.5 Let $F_X \in p_S(V_U, A_E)$ and $\alpha, \beta \in [0,1]$. Then a soft fuzzification operator $S_{(\alpha,\beta)}$ on F_X, denoted by $S_{(\alpha,\beta)}(F_X)$ and defined as

$$S_{(\alpha,\beta)}(F_X) = \left\{(u, \mu_{S_{(\alpha,\beta)}(X)}(u)): u \in \underset{e \in A_E}{\cup} (F_X(e))_{(\alpha,\beta)}, \alpha, \beta \in [0,1]\right\}, \text{where}$$

$$\mu_{S_{(\alpha,\beta)}(X)}(u) = \frac{1}{|A_E|}\sum_{e \in A_E} \mu_X(e).\mu_{(F_X(e))_{(\alpha,\beta)}}(u),$$

$$(F_X(e))_{(\alpha,\beta)} = \{u \in V_\lambda: \mu_{F_{X(e)}}(u) \geq \alpha, \nu_{F_{X(e)}}(u) \leq \beta, \lambda \in \Lambda\}, \alpha, \beta \in [0,1].$$

5. Applications of *p*-sets in DMPs

In this present section, we have introduced a new machine learning algorithm to solve p-set dependent DMIs using aggregate FS and our newly defined operations (TNP and TCP).

5.1. p-sets based DMM

The steps of our new DMM are listed below:

Algorithm 1.

Step1. Enter the group of experts (decision makers) {M1, M2,...,Mn} and their corresponding opinions (p-sets) $F_{X_1}, F_{X_2}, \ldots, F_{X_3} \in p_S(V_U, A_E)$

Step2. Compute the resultant p-set F_Z using any p-set operation (union or intersection or any TNP or TCP).

Step3. Input the fixed values of $\alpha, \beta \in [0,1]$.

Step4. Compute aggregate FS $S_{(\alpha,\beta)}(F_Z)$ and present in tabular form.

Step5. For each $k \in \Lambda$, if the associated value $\mu_{S_{(\alpha,\beta)}(X)}(u)$ is maximized from V_k, then the decision z_k is to choose u from V_k.

Step6. If u has many values, the decision-maker can be chosen from any of them.

Step7. The final optimal decision is $(\boldsymbol{z_k}: \boldsymbol{k} \in \boldsymbol{\Lambda})$.

Remark 5.2 If there are lots of ideal choices to be selected in the 7th step, we can return to the 2nd and 3rd steps and adjust the operation or values of $\alpha, \beta \in [0,1]$, so that we can find few optimal choices.

6. Results and discussions

In this present part, we adopt some real-life examples to demonstrate the proposed algorithm to solve p-sets based DMPs.

Example 6.1 We assume that there are three universes V_1= {o_1, o_2, o_3, o_4}, V_2= {p_1, p_2, p_3} and V_3= {r_1, r_2, r_3}, which are the collections of some flats, vehicles, and inns. Suppose that Dr. Roy has a budget for buying a flat, a vehicle, and renting a location for a wedding festival. Consider a p-set F_X that shows some flats, vehicles and inns that Dr. Roy is considering for settlement, transportation, and a wedding festival location, respectively. Assume $\{S_{V_1}, S_{V_2}, S_{V_3}\}$ be a set of collections of decision parameters associated with the universes mentioned above, where

$S_{V_1}=\{s_{V_1,1}$=Price, $s_{V_1,2}$=Carpet area, $s_{V_1,3}$=Location, $s_{V_1,4}$=Parking space$\}$,
$S_{V_2}=\{s_{V_2,1}$=Safety rating, $s_{V_2,2}$=Model, $s_{V_2,3}$=Creature comfort, $s_{V_2,4}$=Ownership cost$\}$,
$S_{V_3}=\{s_{V_3,1}$=Expensive, $s_{V_3,2}$=Available transport options, $s_{V_3,3}$=Near to place of stay, $s_{V_3,4}$=Parking space$\}$.

Let V=$\prod_{i=1}^{3}$IFS(V_i), S=$\prod_{i=1}^{3}S_{V_i}$and A⊆S, such that

$$A=\begin{Bmatrix} a_A=(s_{V_1,1},s_{V_2,1},s_{V_3,1})=(\text{Price, Safety rating, Expensive}), \\ b_A=(s_{V_1,3},s_{V_2,2},s_{V_3,1})=(\text{Location, Model, Expensive}), \\ c_A=(s_{V_1,2},s_{V_2,3},s_{V_3,2})=(\text{Carpet area, Creature comfort, Available transport options}), \\ d_A=(s_{V_1,4},s_{V_2,2},s_{V_3,1})=(\text{Parking space, Model, Expensive}), \\ e_A=(s_{V_1,1},s_{V_2,4},s_{V_3,3})=(\text{Price, Ownership cost, Near to place of stay}), \\ f_A=(s_{V_1,3},s_{V_2,3},s_{V_3,4})=(\text{Location, Creature comfort, Parking space}) \end{Bmatrix}$$

Suppose Dr. Roy is tasked with selecting objects from the arrangements of given objects based on the arrangements of choice parameters. If two experts chose X and Y are two FSs over A with membership values for the parameters in A as

μ_X(Price, Safety rating, Expensive)=0.7;
μ_X(Location, Model, Expensive)=0.8;
μ_X(Carpet area, Creature comfort, Available transport options)=0.7;
μ_X(Parking space, Model, Expensive)=0.5;
μ_X(Price, Ownership cost, Near to place of stay)=0.9;
μ_X(Location, Creature comfort, Parking space)=0.8;

and

μ_Y(Price, Safety rating, Expensive)=0.5;
μ_Y(Location, Model, Expensive)=0.6;
μ_Y(Carpet area, Creature comfort, Available transport options)=0.9;
μ_Y(Parking space, Model, Expensive)=0.8;
μ_Y(Price, Ownership cost, Near to place of stay)=0.7;
μ_Y(Location, Creature comfort, Parking space)=0.5.

We consider two expert's observations (p-sets)F_X and F_Y regarding some flats, vehicles, and inns are as in Table 5 and Table 6 respectively.

Table 5 p-set $\boldsymbol{F_x}$

$\mathbf{V_i}$		***a_A*** 0.7	***b_A*** 0.8	***c_A*** 0.7	***d_A*** 0.5	***e_A*** 0.9	***f_A*** 0.8
V_1	o_1	(0.3,0.5)	(0.8,0.2)	(0.7,0.2)	(0.8,0.2)	(0.3,0.5)	(0.7,0.2)
	o_2	(0.4,0.4)	(0.9,0.1)	(0.8,0.1)	(0.9,0.1)	(0.4,0.4)	(0.6,0.3)
	o_3	(0.9,0.1)	(0.3,0.5)	(1,0)	(0.3,0.5)	(0.9,0.1)	(0.9,0.1)
	o_4	(0.7,0.2)	(0.8,0.1)	(0,1)	(0.8,0.1)	(0.7,0.2)	(0.5,0.4)
V_2	p_1	(0.8,0.2)	(0.8,0.1)	(0.6,0.3)	(0.8,0.1)	(0.9,0.1)	(0.6,0.3)
	p_2	(0.6,0.2)	(0.8,0.2)	(0.8,0.2)	(0.8,0.2)	(1,0)	(0.8,0.2)
	p_3	(0.6,0.3)	(0.5,0.2)	(0.3,0.4)	(0.5,0.2)	(0.9,0.1)	(0.3,0.4)
V_3	r_1	(0.9,0.1)	(0.9,0.1)	(0.5,0.4)	(0.9,0.1)	(0.8,0.1)	(0.9,0.1)
	r_2	(0.7,0.2)	(0.7,0.2)	(0.5,0.3)	(0.7,0.2)	(0.5,0.4)	(0.8,0.2)
	r_3	(0.9,0)	(0.9,0)	(0.7,0.2)	(0.9,0)	(0.4,0.3)	(1,0)

Table 6 p-set $\boldsymbol{F_Y}$

$\mathbf{V_i}$		***a_A*** 0.7	***b_A*** 0.8	***c_A*** 0.7	***d_A*** 0.5	***e_A*** 0.9	***f_A*** 0.8
V_1	o_1	(0.7,0.2)	(0.7,0.2)	(0.8,0.2)	(0.7,0.2)	(0.7,0.2)	(0.3,0.5)
	o_2	(0.6,0.3)	(0.8,0.1)	(0.9,0.1)	(0.8,0.1)	(0.6,0.3)	(0.4,0.4)
	o_3	(0.9,0.1)	(1,0)	(0.3,0.5)	(1,0)	(0.9,0.1)	(0.9,0.1)
	o_4	(0.5,0.4)	(0,1)	(0.8,0.1)	(0,1)	(0.5,0.4)	(0.7,0.2)
V_2	p_1	(0.8,0.1)	(0.5,0.4)	(0.9,0.1)	(0.5,0.4)	(0.9,0.1)	(0.9,0.1)

V_i		***a**A* 0.7	***b**A* 0.8	***c**A* 0.7	***d**A* 0.5	***e**A* 0.9	***f**A* 0.8
	p_2	(0.5,0.4)	(0.5,0.3)	(0.7,0.2)	(0.5,0.3)	(0.8,0.2)	(0.7,0.2)
	p_3	(0.4,0.3)	(0.7,0.2)	(0.9,0)	(0.7,0.2)	(1,0)	(0.9,0)
V_3	r_1	(0.6,0.3)	(0.6,0.3)	(0.9,0.1)	(0.6,0.3)	(0.8,0.1)	(0.6,0.3)
	r_2	(0.8,0.2)	(0.8,0.2)	(0.7,0.2)	(0.8,0.2)	(0.8,0.2)	(0.8,0.2)
	r_3	(0.3,0.4)	(0.3,0.4)	(0.9,0)	(0.3,0.4)	(0.5,0.2)	(0.3,0.4)

We consider the resultant p-set $F_X \otimes F_Y$ using *AND−TNP* as shown in Table 7. Now, we chose α=0.7 and β=0.2, then we find $S_{(\alpha,\beta)}(F_X \otimes F_Y)$ as in Table 8.

Table 7 p-set $\boldsymbol{F_X} \otimes \boldsymbol{F_Y}$

V_i		***a**A* 0.7	***b**A* 0.8	***c**A* 0.7	***d**A* 0.5	***e**A* 0.9	***f**A* 0.8
V_1	o_1	(0.3,0.5)	(0.7,0.2)	(0.7,0.2)	(0.7,0.2)	(0.3,0.5)	(0.3,0.5)
	o_2	(0.4,0.4)	(0.8,0.1)	(0.8,0.1)	(0.8,0.1)	(0.4,0.4)	(0.4,0.4)
	o_3	(0.9,0.1)	(0.3,0.5)	(0.3,0.5)	(0.3,0.5)	(0.9,0.1)	(0.9,0.1)
	o_4	(0.5,0.4)	(0,1)	(0,1)	(0,1)	(0.5,0.4)	(0.5,0.4)
V_2	p_1	(0.8,0.2)	(0.5,0.4)	(0.6,0.3)	(0.5,0.4)	(0.9,0.1)	(0.6,0.3)
	p_2	(0.5,0.4)	(0.5,0.3)	(0.7,0.2)	(0.5,0.3)	(0.8,0.2)	(0.7,0.2)
	p_3	(0.4,0.3)	(0.5,0.2)	(0.3,0.4)	(0.5,0.2)	(0.9,0.1)	(0.3,0.4)
V_3	r_1	(0.6,0.3)	(0.6,0.3)	(0.5,0.4)	(0.6,0.3)	(0.8,0.1)	(0.6,0.3)
	r_2	(0.7,0.2)	(0.7,0.2)	(0.5,0.3)	(0.7,0.2)	(0.5,0.4)	(0.8,0.2)
	r_3	(0.3,0.4)	(0.3,0.4)	(0.5,0.2)	(0.3,0.4)	(0.4,0.3)	(0.3,0.4)

Table 8 $S_{(a,\beta)}(F_X \otimes F_Y)$, α=0.7, β=0.2

V_i		***a**A* 0.304	***b**A* 0.444	***c**A* 0.612	***d**A* 0.364	***e**A* 0.612	***f**A* 0.364	$\mu_{S_{(0.7,0.2)}(X\otimes Y)}(u)$
V_1	o_1	0	1	1	1	0	0	0.237
	o_2	0	1	1	1	0	0	0.237
	o_3	1	0	0	0	1	1	0.213
	o_4	0	0	0	0	0	0	0
V_2	p_1	1	0	0	0	1	0	0.153
	p_2	0	0	1	0	1	1	0.265
	p_3	0	0	0	0	1	0	0.102
V_3	r_1	0	0	0	0	1	0	0.102
	r_2	1	1	0	1	0	1	0.264
	r_3	0	0	0	0	0	0	0

From Table 8, we see that for the V_1-part of $F_X \otimes F_Y$, flats o_1 and o_2 have the largest value $\mu_{S_{(0.7,0.2)}(X\otimes Y)}(o_1)=\mu_{S_{(0.7,0.2)}(X\otimes Y)}(o_2)$ =0.237; hence Dr. Roy can be selected o_1 flat or o_2 flat. For the V_2-part of $F_X \otimes F_Y$, vehicle p_2 has the largest value $\mu_{S_{(0.7,0.2)}(X\otimes Y)}(p_2)$ =0.265; hence vehicle p_2 is the best suit. Also, for the V_3-part of $F_X \otimes F_Y$, inn r_2 has the largest value $\mu_{S_{(0.7,0.2)}(X\otimes Y)}(r_2)$ =0.264; hence r_2 inn is the best suit. As a result, the best option for Dr. Roy is (o_1, p_2, r_2) or (o_2, p_2, r_2).

Table 9 Table for$S_{(\alpha,\beta)}(F_X \otimes F_Y)$, α=0.8, β=0.2

V_i		a_A 0.304	b_A 0.444	c_A 0.612	d_A 0.364	e_A 0.612	f_A 0.364	$\mu_{S_{(0.7,0.2)}(X\otimes Y)}(u)$
V_1	o_1	0	0	0	0	0	0	0
	o_2	0	1	1	1	0	0	0.237
	o_3	1	0	0	0	1	1	0.213
	o_4	0	0	0	0	0	0	0
V_2	p_1	1	0	0	0	1	0	0.153
	p_2	0	0	0	0	1	0	0.102
	p_3	0	0	0	0	1	0	0.102
V_3	r_1	0	0	0	0	1	0	0.102
	r_2	0	0	0	0	0	1	0.061
	r_3	0	0	0	0	0	0	0

From Table 9, we see that for the V_1-part of $F_X \otimes F_Y$, flat o_2 has the largest value $\mu_{S_{(0.8,0.2)}(X\otimes Y)}(o_2)$ =0.237; hence flat o_3 is the best suit. For the V_2- part of $F_X \otimes F_Y$, vehicle p_1 has the largest value $\mu_{S_{(0.8,0.2)}(X\otimes Y)}(p_1)$ =0.153; hence vehicle p_1 is the best suit. Also, for the V_3-part of $F_X \otimes F_Y$, inn r_1 has the largest value $\mu_{S_{(0.8,0.2)}(X\otimes Y)}(r_1)$ =0.102; hence r_1 is the best suit. As a result, the best option for Dr. Roy is (o_2, p_1, r_1).

Example 6.2 Now, let us consider the p-sets as in Table 5 and Table 6, then the resultant p-set $F_X \bar{\oplus} F_Y$ using OR−TCP as shown in Table 10 and we chose α=0.8 and β=0.1, we find $S_{(\alpha,\beta)}(F_X \bar{\oplus} F_Y)$ as shown in Table 11. From Table 11, we can see that for the V_1- part of $F_X \bar{\oplus} F_Y$, flat o_3 has the largest value $\mu_{S_{(0.8,0.1)}(X\otimes Y)}(o_3)$ =0.943; hence flat o_3 is the best suit. For the V_2- part of $F_X \bar{\oplus} F_Y$, vehicle p_1 has the largest value $\mu_{S_{(0.8,0.1)}(X\otimes Y)}(p_1)$ =0.943; hence vehicle p_1 is the best suit. Also, for the V_3- part of $F_X \bar{\oplus} F_Y$, inn r_1 has the largest value $\mu_{S_{(0.8,0.1)}(X\otimes Y)}(r_1)$ =0.943; hence r_1 inn is the best suit. Therefore the final optimal decision for Dr. Roy is (o_3, p_1, r_1).

Table 10 p-set $F_X \bar{\oplus} F_Y$

V_i		a_A 0.889	b_A 0.946	c_A 0.982	d_A 0.929	e_A 0.982	f_A 0.929
V_1	o_1	(0.7,0.2)	(0.8,0.2)	(0.8,0.2)	(0.8,0.2)	(0.7,0.2)	(0.7,0.2)
	o_2	(0.6,0.3)	(0.9,0.1)	(0.9,0.1)	(0.9,0.1)	(0.6,0.3)	(0.6,0.3)
	o_3	(0.9,0.1)	(1,0)	(1,0)	(1,0)	(0.9,0.1)	(0.9,0.1)
	o_4	(0.7,0.2)	(0.8,0.1)	(0.8,0.1)	(0.8,0.1)	(0.7,0.2)	(0.7,0.2)
V_2	p_1	(0.8,0.1)	(0.8,0.1)	(0.9,0.1)	(0.8,0.1)	(0.9,0.1)	(0.9,0.1)
	p_2	(0.6,0.2)	(0.8,0.2)	(0.8,0.2)	(0.8,0.2)	(1,0)	(0.8,0.2)
	p_3	(0.6,0.3)	(0.7,0.2)	(0.9,0)	(0.7,0.2)	(1,0)	(0.9,0)
V_3	r_1	(0.9,0.1)	(0.9,0.1)	(0.9,0.1)	(0.9,0.1)	(0.8,0.1)	(0.9,0.1)
	r_2	(0.8,0.2)	(0.8,0.2)	(0.7,0.2)	(0.8,0.2)	(0.8,0.2)	(0.8,0.2)
	r_3	(0.9,0)	(0.9,0)	(0.9,0)	(0.9,0)	(0.5,0.2)	(1,0)

Table 11 $S_{(\alpha,\beta)}(F_X \bar{\oplus} F_Y)$, α=0.8 and β=0.1

V_i		a_A 0.889	b_A 0.946	c_A 0.982	d_A 0.929	e_A 0.982	f_A 0.929	$\mu_{S_{(0.8,0.1)}(X\bar{\oplus}Y)}(u)$
V_1	o_1	0	0	0	0	0	0	0
	o_2	0	1	1	1	0	0	0.476
	o_3	1	1	1	1	1	1	0.943
	o_4	0	1	1	1	0	0	0.476
V_2	p_1	1	1	1	1	1	1	0.943
	p_2	0	0	0	0	1	0	0.164

V_i		a_A *0.889*	b_A *0.946*	c_A *0.982*	d_A *0.929*	e_A *0.982*	f_A *0.929*	$\mu_{S_{(0.8,0.1)}(X \bar{\oplus} Y)}(u)$
	p_3	0	0	1	0	1	1	0.482
V_3	r_1	1	1	1	1	1	1	0.943
	r_2	0	0	0	0	0	0	0
	r_3	1	1	1	1	0	1	0.779

Remark 6.3 From Example 6.1, we can see that applying $AND\!-\!TNP$ and for α=0.7 and β=0.2, we have the final optimal decision for Dr. Roy is (o_1, p_2, r_2) or (o_2, p_2, r_2), but if we chose α=0.8 and β=0.2, then the final optimal decision for Dr. Roy is (o_2, p_1, r_1), which is unique. Also, in Example 6.2, applying the $OR\!-\!TCP$ and choosing α=0.8 and β=0.1, we have flat o_3, vehicle p_1, and inn r_1 are the best suits. Thus, we can obtain a unique solution by changing operations on p-sets and the values of α and β.

Advantages 6.4 When we use Algorithm1, we get fewer object choices, which makes it easier for us to make a decision. However, by using Algorithm1, we can obtain more detailed data, which will assist leaders in making decisions. If there are lots of "ideal choices" to be selected in the 7th step, we can return to the 2nd and 3rd steps and adjust the operation or the values of α,β∈[0,1], that he once utilized in order to confirm the last ideal choice, particularly when there are too much "optimal decisions" to be selected.

7. Comparison analysis

IFSSs can effectively represent and simulate the uncertainty and diversity of judgment information offered by decision makers. In comparison to FSSs, IFSSs are highly beneficial for expressing vagueness and uncertainty more accurately. As a result, in this paper, we offer an approach for solving group DMPs with p-sets by extending the FSMS based DMM. FSMS is a fantastic and a helpful tool for dealing with decision making and all the existing FSMS-based DMMs given in (Alkhazaleh, & Salleh, 2012; Mukherjee, & Das, 2015a; 2015b; 2015c; Balami et al., 2018; Das, 2018; Akin, 2020) are good for solving DMPs, but in their methods, they used FSMS evaluated by only one decision maker, so these methods are may not be useful in the modeling of group-DMPs, but the constructed method in this paper is very advantageous for group-DMPs. Also, the importance of membership degrees of parameters are not considered in (Alkhazaleh, & Salleh, 2012; Mukherjee, & Das, 2015a, b or c; Balami et al., 2018; Das, 2018; Akin, 2020), but we allow the importance of membership degrees with the parameters so that every decision makers can give the importance of parameter selections according to their choice.

8. Conclusion and future work

In this study, we offer an approach for solving group DMPs with p-sets by extending the FSMS based DMM. FSMS is a fantastic and useful tool to deal with DMPs and all the existing FSMS-based DMMs are good for solving DMPs, but in their methods, they used FSMS evaluated by only one decision maker and the importance of membership degrees of parameters are not considered, so these methods are may not be useful in the modelling of group-DMPs, but the constructed method in this paper is very advantageous for solving group-DMPs. Some real-life examples are utilized to demonstrate the attainability of our DMM in helpful applications.

However, we can see that by utilising Algorithm1, we can get an empty set of alternatives for items, which is horrible for our decision. Furthermore, we recognize that determining the value of $\alpha, \beta \in [0,1]$ is critical in making a better decision. If we choose the estimates of $\alpha \in [0,1]$ is too little and $\beta \in [0,1]$ is too substantial in the Definition 4.5 formula, we may receive a lot of various possibilities to choose from. However, this is frequently bad for our decision because the decision-maker has a tendency to look over fewer options. The more options available, the more difficult it is to choose. As a result, the choices we make should not be too important. However, if we choose α's estimations to be too large and β's to be too small, we may end up with fewer alternatives, and in some cases, we may end up with an empty set of object options, indicating that our judgments were unsuccessful. We need the new selection to provide us with the $\alpha, \beta \in [0,1]$ estimations so that we can choose.

In a future study, we will use q-ROFS (Garg & Aurora, 2021a; 2021b) to extend this proposed DMM to other real-life applications in the field of pattern recognition and medical diagnostics.

Abbreviations:

DMM	Decision making method
DMP	Decision making problem

FS	Fuzzy set
FSMS	Fuzzy soft multi set
FSS	Fuzzy soft set
IFS	Intuitionistic fuzzy set
IFSMS	Intuitionistic fuzzy soft multiset
IFSMSP	Intuitionistic fuzzy soft multi set part
IFSS	Intuitionistic fuzzy soft set
IVFS	Interval-valued fuzzy set
IVIFS	Interval-valued intuitionistic fuzzy set
SS	Soft set
p-set	Fuzzy parametrized intuitionistic fuzzy soft multiset
RS	Rough set
TNP	t-norm product
TCP	t-conorm product

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11. Conflict of interest

The authors declare that they have no conflict of interest

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