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Evaluation of dynamic properties of rubber mounts by Levenberg–Marquardt method

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Abstract

Rubber mounts are widely used as vibration isolators as they are cheaper and available in different sizes. The performance of a rubber mount varies under different loading conditions, such as excitation force and frequency. The actual dynamic properties of a rubber mount cover linear and non-linear regions; thus, a characterization method is required that can capture and identify the dynamic properties for both regions. This paper proposes a method to identify the dynamic properties of rubber mounts by comparing the Levenberg–Marquardt method and the classical hysteresis loop method. The rubber mounts are excited under different excitation forces and frequencies. The excitation condition where the rubber mounts behave non-linearly is identified. The dynamic properties from the rubber mounts are analysed by fitting the Levenberg–Marquardt method to identify the parameters from the measured hysteresis loop. The results show that the proposed approach can capture the stiffness and loss factor of rubber mounts, including both linear and non-linear regions. Then, the measured results are compared with the impact technique for validation, with low percentage differences found between the classical hysteresis loop method and impact technique. This study indicates that the dynamic characterisation of rubber mounts using the Levenberg–Marquardt method could provide an alternative solution for identification of rubber mount properties, including their hysteresis behaviour. Overall, this work represents an important contribution in understanding the non-linear identification of rubber mount properties.

Keywords: *hysteresis loop; loss factor; non-linearity; rubber mount; stiffness.*

1. Introduction

Rubber mounts are simple yet effective devices for various noise and vibration control applications. Rubber is a unique material, in that its viscoelastic behaviour can deform linearly and non-linearly under different excitation frequencies and amplitudes (Mallamace, Micali, & Vasi, 1990). Although several passive and active mounting systems have been developed, including elastomeric and hydraulic mounts, rubber mounts are still widely used due to their compact structure, simple usage, high damping ratio and consistent performance (Fan, Lee, Kang, & Kim, 1998; Tárrago, Kari, Vinolas, & Gil-Negrete, 2007). One of the applications for rubber mounts is in reducing the vibration of machines caused by rotational force or repetitive force (Ibrahim, 2008). Despite that rubber is common material, further insights into the characteristics of rubber mounts are still required.

In general, the performance of rubber mounts is influenced by the dynamic properties of the rubber mount components (Yu, Naganathan, & Dukkipati, 2001). The dynamic properties of rubber mounts include the stiffness, which contributes to energy storage in the system, and the loss factor, which describes the energy loss from the system. In recent years, there have been increasing studies into the loss factor of elastomers, such as a study performed to investigate multi-directional properties for applications such as rotational machinery (Asokan & Hussain, 2018). However, the stiffness and loss factor of rubber mounts can be affected by different excitation frequency and vibration amplitude values. The dependency of the stiffness and loss factor over different frequencies and loading could change from linear to non-linear due to different excitation conditions (Ferry, 1980). Thus, measuring both linear and non-linear viscoelastic rubber properties are important for a range of rubber applications (Tolpekina, Pyckhout-Hintzen, & Persson, 2019).

Several characterisation techniques have been carried out to obtain good description of viscoelastic behaviour of rubber mount. Researchers have proposed several models to analyse and predict the dynamic characterisation of rubber mounts. For example, the Kelvin–Voigt model was proposed to investigate the behaviour of linear characteristics of rubber mounts (Medalia, 1978). Subsequently, some researchers have also used the Kelvin–Voigt model to study the nonlinear behaviour of rubber mounts; however, their findings show that the Kelvin–Voigt model is incapable of reproducing experiment results, and the nonlinear behaviour of rubber mounts is described (Amabili, 2016; Balasubramanian, Ferrari, Amabili, & del Prado, 2017; Zaitsev, Shtempluck, Buks, & Gottlieb, 2012).

Two models, Maxwell and standard linear solid models have also been used to investigate the viscoelastic behaviour of rubber mounts. These models have only two or three parameters, thus their capabilities of representing nonlinear behaviour are limited (Johnson & Quigley, 1992; Kari, 2003; Sommer & Meyer, 1974; Yin, Hu, & Song, 2018; Zhang & Richards, 2007). The generalized Kelvin–Voigt, Maxwell and SLS models are introduced to improve the capabilities of this model (Tschoegl, & Tschoegl, 2011). Lin, Bengisu, & Mourelatos. (2011) and Höfer and Lion (2009) used the generalised Kelvin–Voigt and Maxwell models, respectively, to study the dynamic behaviour of rubber-like materials. However, these viscoelastic models are limited by the excessive number of materials parameters required to fit the experimental data.

In addition, to significantly reduce the number of parameters in the generalised models, the fractional derivative was used by introducing a fractional-order operator instead of an integer-order operator (Arikoglu, 2014; Yin, Hu, Luo, & Song, 2017). The nonlinear behaviour of rubber mounts can be more accurately fitted with the experimental data using this model. Previous studies have reported that the fractional derivative model is superior to the generalised model for fitting non-linear behaviour (Haupt & Lion, 2002; Wollscheid & Lion, 2013). However, the fractional derivative model requires different fractional operators for different cases, even for the same materials. Hence, the fractional derivative model needs more complicated mathematical models to characterise the nonlinear behaviour of rubber mounts.

Recent studies have shown interest in characterizing the hysteresis properties of rubber elements, including experimental and modelling work. For example, an experimental device was set up for identifying the elastic–hysteresis properties of rubber elements, and the energy loss of rubber elements was investigated (Nasonov, Ilichev, & Raevsky, 2021). The energy dissipation of the rubber mounts' hysteresis properties was also modelled for a power train study (Penas, Gaudin, Kreis, & Balmes, 2019). In addition, the hysteresis loss of rubber elements was assessed under cyclic deformation conditions, including the effect of temperature (Luo et al., 2021). This approach may better estimate the fatigue life of rubber elements. The hysteresis characteristics of rubber dampers were also recently investigated using the hysteresis loop cyclic method to understand the relationship between rubber stiffness and hardness (Yang & Zhou, 2020). Overall, studies in recent years have shown that the hysteresis properties of rubber elements are an important characteristic, thus developing measurement and modelling techniques to better understand these hysteresis properties of rubber is of particular importance.

The study of rubber mounts' properties is still highly important, especially for the measurement of the stiffness and loss factor (Liu et al., 2021; Sun, Chen, Zhang, & Eberhard, 2011; Ucar & Basdogan, 2018). These parameters are usually investigated to test the performance of rubber-made devices, including vibration dampers or damping layers in space-related devices (Busse, Sinclair, Redda, & Wondimu, 2021; Luo et al., 2021). However, identification of the rubber mounts' properties including their non-linear region remains very limited. This paper aims to propose a measurement method integrated with the Levenberg–Marquardt (L-M) method to better identify the dynamic properties of rubber mounts. The proposed method could help identify the rubber mounts' linear and non-linear regions' properties. The proposed method provides a better estimation of rubber mount properties and will help to widen their application in rubber-related industries. Constraining these properties is important as it could provide improved insights into both the development of and studies relating to rubber-made devices, including rubber mounts or elastomers. The experimental measurements are performed on the rubber mounts using a shaker. The L-M method is used to identify the relevant parameters from the measured data, and the results are compared with the classical hysteresis loop method. The findings from the proposed method are then later validated with the impact technique.

2. Objectives

The main objective of this research article is to present a method for evaluating the dynamic properties of rubber mounts using the L-M method. The stiffness and loss factors of the selected rubber mounts are calculated and compared with the classical hysteresis loop method and impact technique. The dynamic properties cover from the excitation force between 3 N and 10 N to observe the changing in the loop shape from the linear to the non-linear region (S-shape) of the rubber mounts' properties. The linear and non-linear regions of the rubber mounts properties are identified, and the corresponding stiffness and loss factors are calculated.

3. Methodology

3.1 Levenberg–Marquardt method

Three solid rubber-to-metal rubber mount units are used for measurement. The rubber part of each rubber mount unit is approximately 15 mm wide and 20 mm height. The L-M method is used for parameter identification in determining the stiffness and loss factor values from the experimental data. The stiffness and loss factor play a vital role in the performance of rubber mounts. Identifying the stiffness and loss factor values of the system for optimum performance can be achieved by varying the excitation frequency and excitation amplitude. The mathematical model for nonlinear behaviour of rubber mounts is developed considering the frequency- and amplitude-dependent stiffness and loss factor.

The stiffness is defined as follow: $k(A,\omega)=F(A,\omega)/x(A,\omega)$, where the stiffness, force and displacement are affected by the excitation frequency (ω) and amplitude (x). The measurement system of rubber mounts has been modelled as a single degree of freedom (SDOF). The frequency- and amplitude-dependent system of damped forced vibration for the SDOF model can be written as follows:

$$F(A,\omega)=m(A,\omega)\ddot{x}(A,\omega)+c(A,\omega)\dot{x}(A,\omega)+k(A,\omega)x(A,\omega) \quad (1)$$

where $F(A,\omega)$ represents the excitation force at different frequencies and amplitudes, m is the pre-loading mass of the system, $\ddot{x}(A,\omega)$ is acceleration, $c(A,\omega)$ is the damping factor, $\dot{x}(A,\omega)$ represents velocity, $k(A,\omega)$ represents stiffness and $x(A,\omega)$ represents displacement.

The hysteresis loop consists of two lines, namely an upper and a lower line, which are anti-symmetric about the origin. In the nonlinear mounting system, the force (A) changes with displacement $x(A,\omega)$, producing an S-shaped hysteresis loop, as shown in Figure 1 (Sun et al., 2011, Xiao et al., 2021).

The forces $F_U(x)$ and $F_L(x)$ are related by:

$$F_U(x)=-F_L(x) \quad (2)$$

The forces $F_U(x)$ and $F_L(-x)$ can be described by power functions as follows:

$$F_U(x)=\sum_{i=0}^{q} a_i x^i, \dot{x}>0 \quad (3)$$

$$F_L(x)=\sum_{i=0}^{q} (-1)^{i+1} a_i x^i, \dot{x}<0 \quad (4)$$

where $F_U(x)$ is the upper force and $F_L(x)$ is the lower force, a_i is the power function coefficient and q is the number of the power function depending on the shape of the hysteresis loop and fitting precision.

Then, by combining equations (3) and (4), we obtain the following:

$$F(x,\omega)=\sum_{i=1}^{(M+1)/2} a_{2i-1}x^{2i-1}+\sum_{i=o}^{\frac{M-1}{2}} a_{2i}x^{2i}\,sign(\dot{x})=F_s(x)+F_d(x,\dot{x}) \quad (5)$$

where M should be set as an odd number and $sign(.)$ is the signum function. The nonlinear force F is next considered as the combination of a spring force, Fs, and a damping force, F_d.

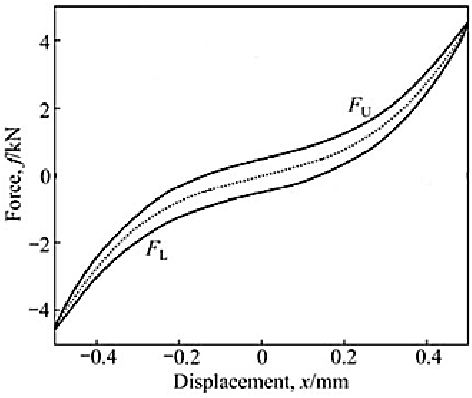

Figure 1 S-shape non-linear hysteresis loop (Sun et al., 2011)

Furthermore, Fs and F_d can be related to the excitation vibration amplitude and frequency by expressing them using the spring coefficient and damping coefficient, as shown in equations (6) and (7), as follows:

$$F_S=(x,A,\omega)=\sum_{i-1}^{n} k_{2i-1}(A,\omega)x^{2i-1} \quad (6)$$

$$F_d=(x,\dot{x},A,\omega)=d(A,\omega)|\dot{x}|^{p(A,\omega)-1}\dot{x} \quad (7)$$

where $k_{2i-1}(A,\omega)$ is the stiffness, $d(A,\omega)$ is the damping coefficient and $p(A,\omega)$ indicates different damper types.

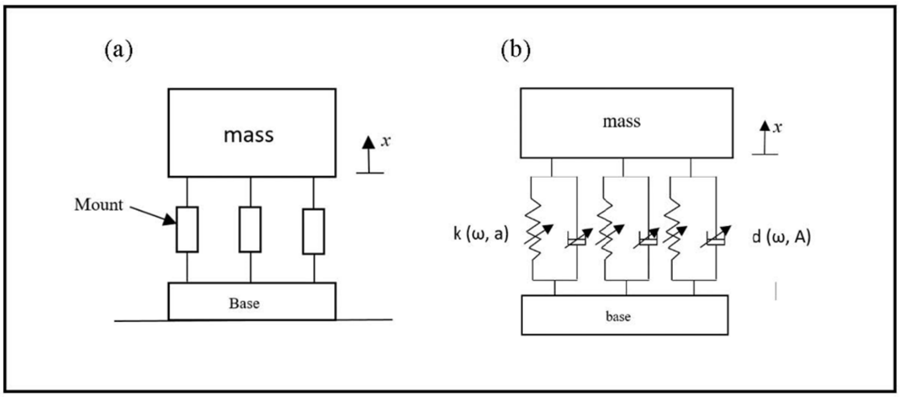

Figure 2 Concept of (a) general rubber mounting system and (b) rubber mounting system with amplitude- and frequency-dependent properties

Figure 2 shows the relationship between F_d and F_S with the excitation amplitude and frequency. Three rubber mount units are used, which are excited at different force amplitude and frequency values. The excitation force and corresponding responses of the system are captured. The L-M method is applied to identify the coefficients of the spring, damper, and loss factor. To do this, first, an order of n=1 is assumed in equations (6) and (7). This means that the force measurements are simulated by linear and nonlinear springs and damping force based on the experiment data. The force is then calculated as follows:

$$F(x,\dot{x},\omega,A)=k_1(A,\omega)x+d(A,\omega)|\dot{x}|^{p(A,\omega)-1}\dot{x} \quad (8)$$

The loss factor, corresponding to the energy dissipated in the system, can be defined as follows:

$$\eta=\frac{d\omega}{k_1} \quad (9)$$

The parameter identification method determined the k, d and η values (defined below) in the sine vibration processes, where the frequency and amplitude are found based on the nonlinear least-squares method using MATLAB. The objective function used is as follows:

$$\min f(k,d,\eta)=\frac{1}{2}\sum_{i=1}^{N}\left(F_{cal}(x)\text{-}F_{mea}(x)\right)^2 \quad (10)$$

where F_{cal} (x) is the calculated force from equation (10) and F_{mea} (x) is the experimentally measured force. By repeating this identification process, a different set of k, d and η values will be obtained from each different excitation conditions.

3.2 Classical Hysteresis Loop Method

The stiffness and loss factor can be used to represent the behaviour of rubber mounts and are modelled as an SDOF system with frequency- and amplitude-dependent properties, as follows as follows:

$$F(A,\omega)=m\ddot{x}+k(A,\omega)[1+j\eta(A,\omega)] \quad (11)$$

where $F(A,\ \omega)$ is the amplitude- and frequency-dependent force, x is the displacement, m is the pre-loading mass of the system, $\ddot{x}$ is the acceleration, $k(A,\omega)$ is the stiffness and $\eta(A,\omega)$ is the loss factor. The stiffness is as the energy storage in the system and the loss factor is the energy loss from the system.

For the classical hysteresis loop method, the stiffness is determined by finding the gradient (slope) of the loop. A straight line is drawn for each force per displacement (Nasonov et al., 2021). The loss factor is calculated from each loop using equation (12), as follows.

$$\eta=\frac{D}{2\pi W} \quad (12)$$

where D is the energy dissipated and W is the energy of the system, which can be written as follows:

$$W=\frac{1}{2}kx^2 \quad (13)$$

By inserting equation (13) into equation (12), the loss factor of the rubber mount can be obtained, as follows:

$$\eta=\frac{D}{\pi kx^2} \quad (14)$$

3.3 Experimental measurement

Three solid rubber mounts with a diameter of 15 mm and 20 mm length are used as a test object in this paper, as shown in Figure 3. The rubber mounts are obtained from a commercially available grass trimmer (Tanaka SUM 328 SE II Japan). These are rubber-to-metal mounts that are commonly used for small power motors. Figure 4 shows the experimental equipment and the setup used in this paper. The experimental setup consists of an accelerometer (Kistler, type 8776A50), force transducer (Kistler, type 9212), 0.9 kg preload mass, power amplifier, LMS Scadas data acquisition system, stringer and shaker. The shaker is used as the input excitation force to the system. Different levels of excitation forces are applied and varied at different excitation frequencies. The LMS Scadas system is used as an interface with a controller to provide signals to the system and collect data. The controller is used to generate a voltage sinusoidal wave signal to drive the shaker. The force transducer is attached at the end of the stringer to measure the input force from the shaker. The accelerometer is located below the preloaded mass plate to measure the acceleration response of the rubber mounts. This paper focused on the measurement in the vertical direction only.

Hysteresis loops are plotted from the measured excitation force and response at different frequencies and force amplitude values. The stiffness and loss factor at different frequencies and force amplitudes are calculated using the slope of each loop and the area under the curve respectively. Figure 5 summarises the flow for the stiffness and loss factor calculation using both the L-M method and the Classical hysteresis loop method.

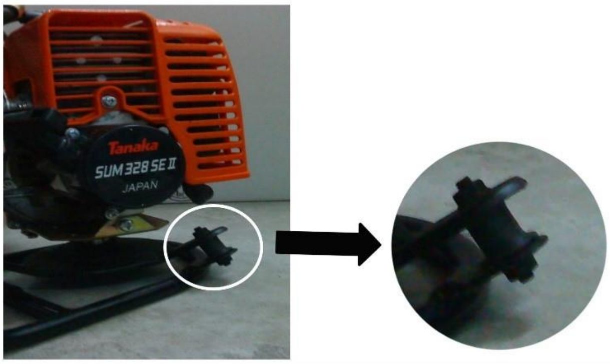

Figure 3 Rubber mount unit obtained from the commercial grass trimmer

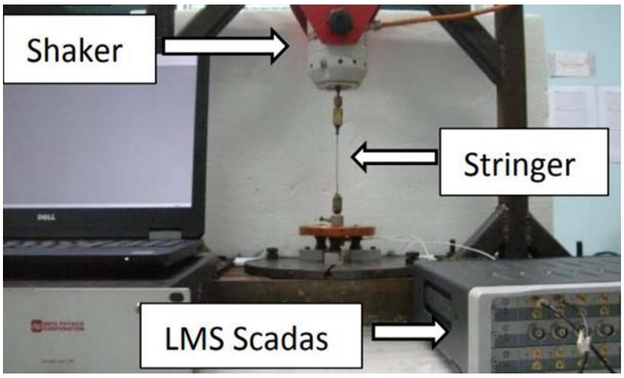

Figure 4 Experimental setup for generating the hysteresis loops of the rubber mounts

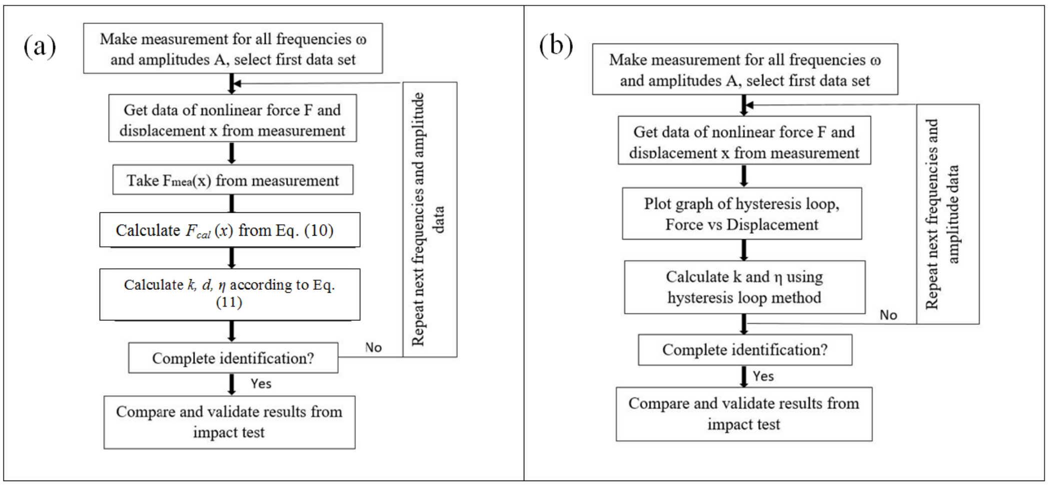

Figure 5 Summary of workflow for (a) the L-M method and (b) the classical hysteresis loop method

3.4 Validation using impact technique

To further validate the results obtained by the L-M and classical hysteresis loop methods, the impact technique is used to validate the stiffness and loss factor values. The details of the impact technique, which is used in the measurement of rubber mount properties, are described in Lin, Farag, & Pan (2005) and Ooi and Ripin (2011). This measurement setup consists of three rubber mounts, an impact hammer, an accelerometer and an analyser (LMS spectral testing). A preload mass of 0.9 kg is mounted on the rubber mount. The hammer is used to excite the test specimen, and the accelerometer is used to record the response from the system. The sensors are calibrated prior to each measurement.

4. Results and discussion

Figure 6 shows an example result for an excitation force of 3 N and the corresponding response of a rubber mount, captured using the method described in Section 3.3. In total, 10 cycles of data were collected for each value, and the average is calculated for the loop of each excitation force. Accordingly, the results presented in Figure 6 are based on the average of 10 cycles. Both the excitation force and response presented in Figure 6 were then used to plot the hysteresis loop shown in Figure 7. The same experimental procedure was repeated for different excitation forces, covering a range from 3 N to 10 N. All captured excitation forces and responses are used to plot the hysteresis loops shown in Figure 7.

Figure 7 shows the hysteresis loops obtained at 80Hz with different excitation force levels between 3N and 10N. The excitation force values of 1 N and 2 N are not included in this paper because the hysteresis loops produced by these excitation forces are almost a straight line (i.e. no significant area under the loop) given the small excitation force and response. As shown in Figure 7, the elliptical shape of the loop changes from a linear loop to an S-shape loop as the excitation force increases from 3N to 10N. The changes are significant, especially for excitation force values of 5N and above. The slope of the hysteresis loops also changes at different excitation force levels. The slope of the hysteresis loop represents the stiffness of the rubber mount; thus, the stiffness of the rubber mount changes according to different excitation force levels. Additionally, the slope of the hysteresis loops shown in Figure 7 exhibits a change from linear to non-linear, indicating that the dynamic properties of rubber mounts also transform from linear to non-linear behaviour as the excitation force level increases. S-shape hysteresis loops were also reported by Kikuchi and Aiken (1998) based on analytical modelling of the hysteresis loop model; these authors also suggest that, based on these loops, the dynamic properties of rubber mounts change from linear to non-linear with increasing excitation force level.

The hysteresis loops shown in Figure 7 were then used for parameter identification using the L-M and classical hysteresis loop methods. The stiffness and loss factors of the rubber mounts are calculated using the L-M method based on equations (8)–(10) and compared with the classical hysteresis loop method. The comparison of the stiffness and loss factor values obtained by both methods are plotted in Figure 8. As shown, the values obtained from both methods exhibit a similar trend of decreasing stiffness with increasing excitation force. The stiffness of the rubber mount is around 230 kN/mm when the excitation force is 3 N and decreases to approximately 200 kN/mm when the excitation force is 10 N. Figure 8 also shows that the loss factor of rubber mounts increases as the excitation force increases. This finding is similar to the results reported by Berg (1998); however, Berg's study was based on controlled excitation displacement amplitude instead of the excitation force amplitude. Based on Figure 8, both the stiffness and loss factor obtained using both L-M and classical hysteresis loop methods showed good correlation, with the L-M method able to accurately predict the dynamic properties of rubber mounts. This finding also demonstrates the success of the L-M method in identifying the stiffness and loss factors from the measured hysteresis loops. The hysteresis loops shown in Figure 7 were then used for parameter identification using the L-M and classical hysteresis loop methods. The stiffness and loss factors of the rubber mounts are calculated using the L-M method based on equations (8)–(10) and compared with the classical hysteresis loop method. The comparison of the stiffness and loss factor values obtained by both methods are plotted in Figure 8. As shown, the values obtained from both methods exhibit a similar trend of decreasing stiffness with increasing excitation force. The stiffness of the rubber mount is around 230 kN/mm when the excitation force is 3 N and decreases to approximately 200 kN/mm when the excitation force is 10 N. Figure 8 also shows that the loss factor of rubber mounts increases as the excitation force increases. This finding is similar to the results reported by Berg (1998); however, Berg's study was based on controlled excitation displacement amplitude instead of the excitation force amplitude. Based on Figure 8, both the stiffness and loss factor obtained using both L-M and classical hysteresis loop methods showed good correlation, with the L-M method able to accurately predict the dynamic properties of rubber mounts. This finding also demonstrates the success of the L-M method in identifying the stiffness and loss factors from the measured hysteresis loops.

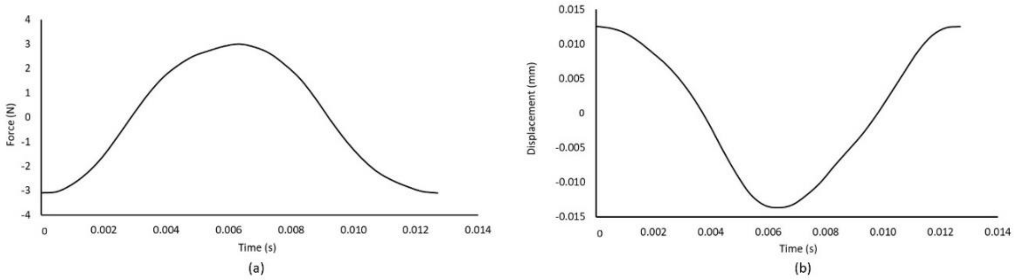

Figure 6 (a) Excitation force and (b) corresponding response from rubber mount captured at 3 N and 80 Hz

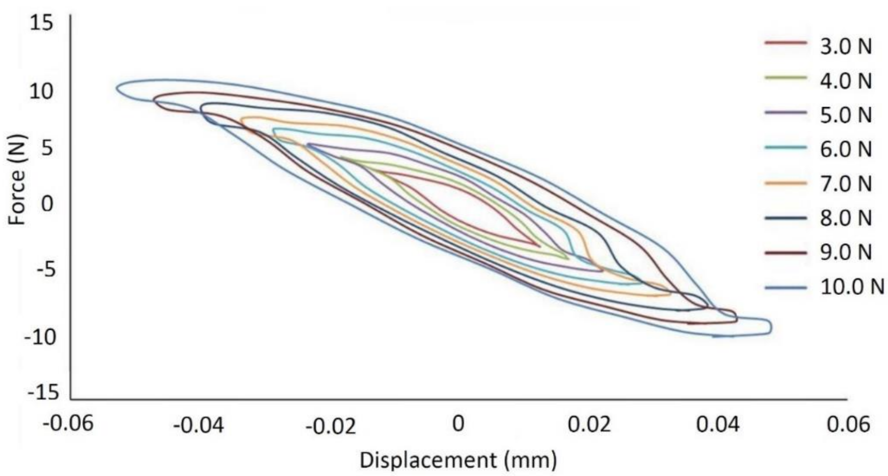

Figure 7 Experimental measured hysteresis loops at 80 Hz for different excitation force levels

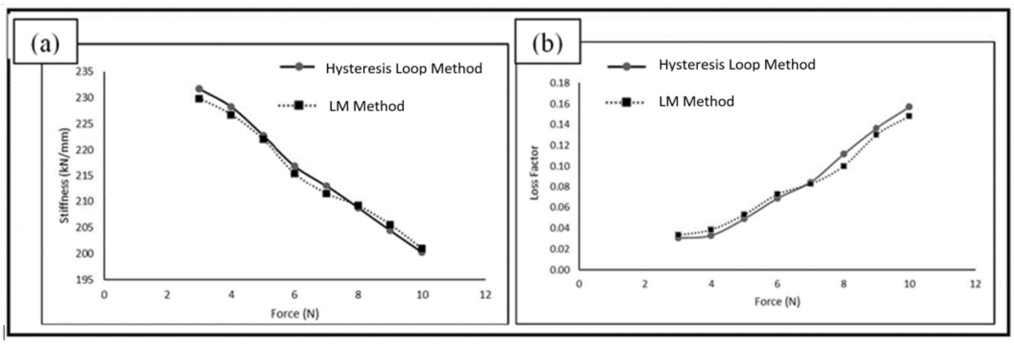

Figure 8 Comparison of (a) stiffness and (b) loss factor for the L-M method and classical hysteresis loop method

Table 1 Comparison of stiffness values obtained by the L-M method and classical hysteresis loop method

Force (N)	L-M Method	Classical hysteresis loop method	Percentage difference (%)
3	229.81	231.67	0.80
4	226.73	228.26	0.67
5	222.01	222.78	0.35
6	215.49	216.91	0.65
7	211.54	213.05	0.71
8	209.21	208.81	0.19
9	205.58	204.46	0.55
10	200.96	200.26	0.35

Table 2 Comparison of loss factor values obtained by the L-M method and classical hysteresis loop method

Force (N)	L-M Method	Classical hysteresis loop method	Percentage difference (%)
3	0.0335	0.0307	9.12
4	0.0387	0.0332	14.21
5	0.0532	0.0491	8.35
6	0.0726	0.0688	5.52
7	0.0829	0.0846	2.01
8	0.1004	0.1117	10.12
9	0.1305	0.1366	4.47
10	0.1482	0.1571	5.67

Tables 1 and 2 summarise the comparison of the stiffness and loss factor values obtained by the L-M method and classical hysteresis loop method. From Table 1, there is consistently less than 1% difference between both approaches. This shows that the L-M can identify the stiffness value from measurement data using the experimentally generated hysteresis loops. For the loss factor values of rubber mounts recorded in Table 2, the percentage difference is slightly higher compared to stiffness. However, these values are still considered to fall within an acceptable range, i.e. less than 15% difference. The calculation of the loss factor using the classical hysteresis loop method is based on the estimation of the area of the loop. The accuracy in estimating the area of the hysteresis loop may affect the actual loss factor value. This condition may cause slightly higher percentage differences between the loss factors calculated by both methods.

To further check the applicability of both methods, the L-M method and classical hysteresis loop method were later applied again on the same rubber mounts to obtain the dynamic properties as measured under a constant excitation force of 3 N. The excitation frequency was instead varied, using values of 50 Hz, 100 Hz, 150 Hz and 200 Hz. The measurement results are then compared with the stiffness and loss factor values obtained by the impact technique for validation (Lin et al., 2005; Ooi & Ripin, 2011). Tables 3 and 4 show a comparison of the stiffness and loss factor values obtained by the L-M method and hysteresis loop method versus those obtained from the impact technique. The results obtained by both methods show a good correlation with the impact technique results. However, the percentage differences of both stiffness and loss factors obtained using the L-M method are lower than those obtained using the classical hysteresis loop method. Both the hysteresis loop and L-M methods were again applied on another commercially available rubber mount (shown in Figure 9) for evaluating the capability of the techniques in identifying the rubber mount’s properties. All the methodology steps were repeated on these new rubber mounts. The applied excitation force was 3 N, with excitation frequencies of 50 Hz, 100 Hz, 150 Hz, 200 Hz, 250 Hz and 300 Hz. From Figure 9, the stiffness and loss factor values captured using both methods were close to each other. Similar observations are recorded to those of the previous rubber mount samples, indicating the applicability of the proposed method to different rubber mounts.

Table 3 Comparison of the stiffness and loss factor obtained by the L-M method with the impact technique

	Stiffness			Loss factor		
Frequency (Hz)	L-M method	Impact technique	% difference	L-M method	Impact technique	% difference
50	95.28	94.80	0.51	0.039	0.035	11.43
100	78.65	77.85	1.03	0.043	0.041	4.88
150	103.14	103.26	0.12	0.032	0.036	11.11
200	172.08	172.22	0.08	0.024	0.022	9.10

Table 4 Comparison of the stiffness and loss factor obtained by the classical hysteresis loop method with the impact technique method

	Stiffness			Loss factor		
Frequency (Hz)	Classical hysteresis loop method	Impact technique	% difference	Classical hysteresis loop method	Impact technique	% difference
50	95.71	94.80	0.96	0.041	0.035	17.14
100	78.48	77.85	0.81	0.045	0.041	9.56
150	102.82	103.26	0.43	0.033	0.036	8.33
200	171.12	172.22	0.64	0.025	0.022	13.64

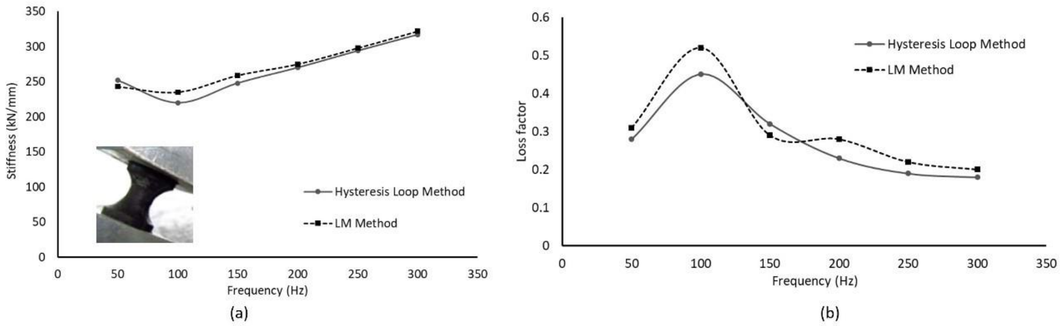

Figure 9 (a) Stiffness and (b) loss factor captured using the hysteresis loop method and L-M method for an additional rubber mount sample

5. Conclusions

The dynamic properties of rubber mounts are obtained by measuring different excitation force values and the corresponding responses from the system. Hysteresis loops are generated for excitation forces from 3 N to 10 N. Parameter identification is performed using the L-M method and then later compared with the classical hysteresis loop method. This parameter identification is used to calculate the dynamic properties of the rubber mounts, mainly targeting values of the stiffness and loss factor. The dynamic properties identified by both methods are then compared with the stiffness and loss factor as measured by the impact technique. The comparison shows that the L-M method achieves lower percentage difference values compared to the classical hysteresis loop method. The impact technique is one of the common techniques used in identifying the dynamic properties of small rubber mounts; thus, this comparison evidences that the L-M method can obtain the dynamic properties of small rubber mounts consistently with the impact technique. To further validate the proposed method, the L-M method is again applied to the rubber mounts to obtain the dynamic properties for an excitation force value of 3 N with excitation frequency values of 50 Hz, 100 Hz, 150 Hz and 200 Hz. The results obtained show good consistency and only small differences between approaches. The proposed method is also later applied to another commercially available rubber mount to evaluate its applicability. Similar observations are recorded for this application, with only small differences found between the LM method and the classical hysteresis loop method. Below are the summaries of the finding:

- The highest percentage differences recorded between the L-M method and classical hysteresis loop method are 0.80% for the stiffness and 14.21% for the loss factor.
- This finding demonstrates that the L-M method can effectively estimate the rubber mounts' dynamic properties and provides an alternative solution for the dynamic characterization of rubber mounts.
- Further validation of the L-M method at different excitation frequency also showed good consistency.
- The application of the L-M method on other commercially available rubber mount proved that the method able to capture the dynamic properties of the rubber mounts.

6. Acknowledgements

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